ELECTRICAL ENGINEERING TEXTS

ELECTRICAL MEASUREMENTS

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BY

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FIRST EDITION

McGRAW-HILL BOOK COMPANY, INC.
239 WEST 39TH STREET. NEW YORK

LONDON: HILL PUBLISHING CO., LTD.
6 & 8 BOUVERIE ST., E. C.
1917

THE MAPLE PRESS YORK PA

PREFACE

In this book it is intended to give a general treatment of the subject of electrical measurements, special emphasis being placed on those matters which are important to the student of electrical engineering.

In preparing a book of this character one has to consider, not only the mature reader who may desire a compendium of methods together with certain practical suggestions, but the student who is beginning the study of Electrical Engineering and who should acquire early in his course a sound knowledge of the process of electrical measurement. This knowledge is fundamental not only to much of the work which the student is required to do in the dynamo laboratory as a matter of engineering training, but to an adequate understanding of electrical testing as it is encountered in the practice of the electrical engineering profession. In the preparation of the text this second class of readers has been particularly in mind.

It is assumed that those who use this text have had courses in physics, the theory of electricity, and in mathematics, such as are given to third year students in technical schools of the first rank.

The choice of material and the method of treatment have been determined by the author's experience in directing for many years the work of the laboratory for Technical Electrical Measurements at the Massachusetts Institute of Technology, and the book is intended as a text for the guidance of students working in such a laboratory as well as a general reference book on the subject.

It is expected that those using the book for purposes of instruction will select such portions as are best suited to their purposes, for more material is presented than can be utilized in both the class room and the laboratory in the time which can properly be allotted to this particular phase of electrical engineering instruction. In this connection it is suggested that interest and discussion are stimulated if the laboratory work is so arranged that

the various members of the class while engaged with the same general topic carry out the experimental work by alternative methods.

For the use of those who desire a more detailed discussion of the various methods, references to certain important papers are appended to each chapter. While no attempt had been made to form a bibliography of the subject of electrical measurements it is thought that these references should be of value in directing the students' attention to the original sources of information and thus assisting him to a broad view of this particular part of his professional training.

The various commercial instruments described in the text have been selected simply as giving good illustrations of the application of the particular principles under discussion. No attempt has been made to discuss instruments made by different makers which differ merely in minor details.

The author wishes to thank Professor H. E. Clifford, Gordon McKay Professor of Electrical Engineering at Harvard University and the Massachusetts Institute of Technology, for his continued interest in the work and for the many and valuable suggestions which he has made during its preparation.

F. A. Laws.

Massachusetts Institute of Technology,
July 5, 1917.

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ELECTRICAL MEASUREMENTS

CHAPTER I

THE MEASUREMENT OF CURRENT

Classes of Instruments.—Electrical measuring instruments may be divided into two classes:

1. Absolute instruments.
2. Secondary instruments.

An absolute instrument is one so designed and built that it gives results expressed in the absolute, or c.g.s., system of units. This implies that the numbers of turns in the coils and all the dimensions which are electrically important have been determined, so that the factor which connects the force or the turning moment acting on the movable member, and the numerical value of the quantity under measurement can be calculated.

A secondary instrument is one so constructed that the relation between its indications and the quantity under measurement must be established experimentally; that is, the instrument must be calibrated.

Examples of absolute instruments are the tangent galvanometer and the Rayleigh current balance (see page 89). The ordinary portable ammeters, voltmeters, and wattmeters are examples of secondary instruments.

Absolute instruments are not adapted for general use and are rarely employed outside of such establishments as the Bureau of Standards, the National Physical Laboratory or the Reichsanstalt. Their particular field of usefulness is in the determination of the fundamental electrical constants; for example, the international ampere.

GALVANOMETERS

The Tangent Galvanometer.—This is an absolute instrument. It consists essentially of a circular coil of insulated wire, having

a radius which is large compared with the dimensions of its cross-section, together with a small magnetic needle which is so suspended at the center of the coil that it can move about a vertical axis. The needle is provided with a pointer which moves over a scale graduated in degrees. The coil is so placed that its plane is vertical and in the magnetic meridian. When no current is

FIG. 1.—Tangent galvanometer.

flowing, the pointer stands at zero for the needle is then controlled by the horizontal component of the local magnetic field, H.

On the passage of a current the coil sets up a magnetic field, which at the needle is perpendicular to the plane of the coil and of magnitude

$$F = \frac{2\pi n}{r} I$$

where n is the number of turns in the coil, r the mean radius of the coil and I the current in absolute units. The needle will turn

through an angle θ and take up a position along the resultant of F and H

Then

$$\tan\theta = \frac{F}{H} = \frac{2\pi n}{Hr} I$$

or

$$I = \frac{H}{\frac{2\pi n}{r}} \tan\theta \qquad \text{in absolute units} \qquad (1)$$

$$I = \frac{10H}{\frac{2\pi n}{r}} \tan\theta \qquad \text{in amperes} \qquad (1a)$$

The quantity $\frac{2\pi n}{r}$ depends on the dimensions of the instrument and is the strength of field at the center of the coil due to unit current. It is frequently called the galvanometer constant of the coil and denoted by G.

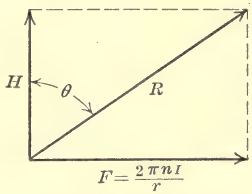


FIG. 2.—Fields at needle of tangent galvanometer.

In this elementary demonstration it has been assumed that:

1. The coil is perfectly circular.
2. The mean radius of the windings correctly represents the effective radius.
3. The needle is exactly at the center of the coil, and the field acting on a finite needle is the same as that at the mathematical center of the coil.
4. The needle is in a uniform field and consequently as it deflects, its poles do not swing into a field of strength differing from that at its zero position.
5. The plane of the coil is in the magnetic meridian and truly vertical.
6. The factor H, or the horizontal intensity of the local magnetic field, has been determined at the place occupied by the instrument and is constant.
7. Only magnetic forces act on the needle; that is, there is no friction and no torsional rigidity in the suspension.

In a careful study of the instrument it would be necessary to discuss each of these items and to determine the numerical effect

on the measured value of the current of the unavoidable departures from the assumed conditions.

In absolute electrical measurements this part of the work often calls for mathematical ability of a high order.

The Helmholtz Galvanometer.—In this instrument the needle is suspended on the axis midway between two equal coils, whose distance apart is equal to their radius.

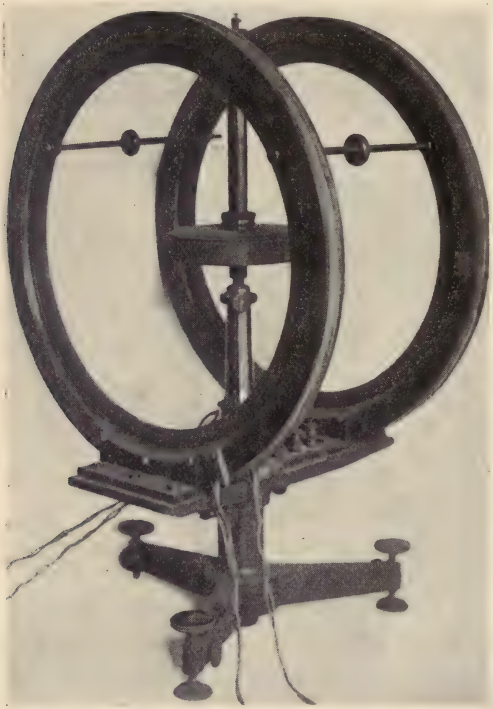


FIG. 3.—Helmholtz tangent galvanometer.

The reason for this construction is that it renders the field in which the needle swings very uniform, so that the correction for the finite length of the needle is much reduced.

This arrangement of coils is frequently employed in other instruments where a uniform magnetic field is desired.

These absolute galvanometers depend for their directive force on the horizontal component of the local field at the place where

they are used. This quantity is subject to great and erratic variations due to the proximity of electric cars, feeders, and structural iron work, and as it enters as a direct factor, present-day conditions have rendered these instruments practically useless.

Any of these absolute galvanometers will be reduced to secondary forms, if, in the attempt to gain sensitiveness, the coils are brought close to the needle.

The Thomson or Kelvin Reflecting Galvanometer.[4]—This form of galvanometer is the most sensitive instrument for the

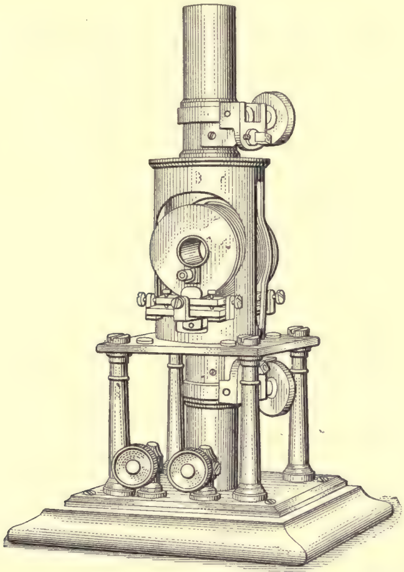


Fig. 4.—The original Thomson galvanometer used on board the Niagara in 1858, the first instrument by which a signal was received through a transatlantic cable.

detection and measurement of direct currents. It was invented by Lord Kelvin (then Professor William Thomson) for use originally as a signalling device in submarine cable work. It had been noticed that signals when transmitted through the short submarine cables then in use (1854) lost their sharpness and in some of the experiments became unintelligible. The first adequate explanation of this was given by Professor Thomson in 1854. He showed that it was due to the electrostatic capacity and resist-

ance of the cable and demonstrated that, after the key at the sending end is depressed, a definite time, a, must elapse before any current is received at the far end, that the received current then gradually rises, attaining 90 per cent. of its full value only after the lapse of a time equal to $10a$ and that when the key is released the received current gradually falls to zero. He also showed that the time which must elapse before any signal is received depends upon the square of the length of the cable. These were weighty matters, for the possibility of a transatlantic cable was then under discussion. Having demonstrated the shape of the "arrival" curve, Professor Thomson invented his galvanometer as a form of receiving instrument adapted to cope with these difficulties and render an Atlantic cable an economic success by increasing the speed of signalling.[1]*

What he desired was an instrument which would be deflected by minute currents and follow their every fluctuation. The original instrument used on the frigate Niagara in 1858 is shown in Fig. 4.

The essential features of the Thomson galvanometer are:

1. A very small and light movable magnetic system delicately suspended within a coil so proportioned that its turns are in close proximity to the needle.

2. A small needle, controlled by the combined action of the local magnetic field and the field due to a permanent magnet.

3. The magnification of the motion of the needle by the use of a beam of light as an index. (A small concave mirror is attached to the needle and throws an image of the filament of an incandescent lamp on a graduated scale from which the deflections are read. The equivalent of a very long pointer with a small moment of inertia is thus obtained. If a plane mirror is used, the reading is effected by a telescope and scale.)

In the older forms of the instrument a simple magnetic needle was used. This was cemented to the back of the mirror which was suspended by a very short silk fiber in a tube of diameter slightly greater than that of the mirror. The tube was closed at one end by a piece of plate glass and at the other by a movable plug. As the suspended system swung in a constricted space its motions were damped by air friction.

*Numbers refer to references at end of chapter.

This simple form of instrument is unsatisfactory, for it is not possible to attain an exceedingly high degree of sensitivity owing to the loss of effective space near the needle, which results from putting a comparatively large mirror within the coil. The torsion of the short suspension is also troublesome. More important still is the fact that the usable sensitivity is limited by the variability of the local field, due to outside magnetic disturbances.

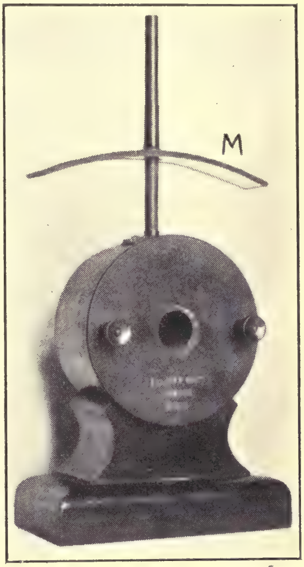


FIG. 5.—Simple Thomson galvanometer. Very susceptible to variations of the local field.

The galvanometer being of fundamental importance in all sorts of electrical testing, later experimenters have devoted much time to the improvement of the details of the Thomson type of instrument, the desire being to attain a very high degree of sensitivity, freedom from outside magnetic disturbances, a minimum but definite torsional control due to the suspension, proportionality of scale reading to current, and convenience of adjustment.

A form of Thomson galvanometer in which a simple magnetic system is used, is shown in Fig. 5. The needle is controlled and held in its zero position by the combined action of the local field and the magnet M. The amount of the control and thus the sensitivity, as well as the position of the zero reading of the instrument may be altered by raising or lowering M and turning it in azimuth.

A simple instrument like this one is no longer useful, for under conditions now well-nigh universal, the local magnetic field is subject to such erratic variations both of magnitude and direction that neither the zero nor the deflected readings can be taken with certainty. This trouble increases as the sensitivity of the galvanometer is raised by adjusting the magnet M. The difficulties may be minimized in two ways:

1. A truly astatic needle system may be used.

2. Magnetic shields may be employed.

An astatic suspended system is an arrangement of two sets of magnets of equal moment lying in the same plane and rigidly connected. The north poles of the upper element are directly over the south poles of the lower element. A perfect system of this sort if placed in a uniform field will experience no directive force, and it is necessary to resort to the use of a controlling magnet. This magnet is placed in a horizontal position, usually above the system, and creates a field which is stronger at the upper than at the lower needles. The controlling force becomes less as the fields at the upper and lower needles approach equality.

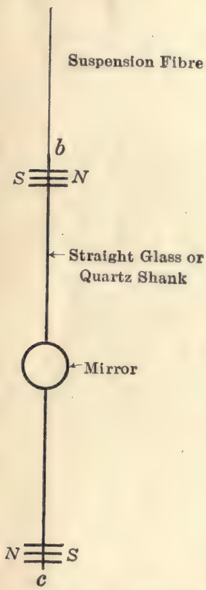


FIG. 6.—Astatic needle system.

It will be seen that fluctuations of H, the local field, will affect the directive fields at the upper and lower needles equally and have no influence on the sensitiveness or the zero reading of the instrument.

In practice, it is not possible to adjust the two sets of magnets so that they are exactly in the same plane, nor is it possible to make their moments exactly equal; so the theoretically perfect system cannot be realized. The actual system will have a small resultant polarity and when it is suspended the needles will take up an approximately east and west direction. Such a system will not be free from the effects of variations of the local field. Again, the magnetism of so-called permanent magnets gradually deteriorates so that the system cannot be expected to remain properly magnetized even though its initial adjustment is perfect. For this reason, some better expedient has been sought and recourse has been had to magnetic shielding.

Magnetic Shields.[2]—In order that a galvanometer may be effectively protected from variations of the local field by means of iron shields, it must be of small size so that the volume of the space to be shielded is reduced to a minimum. The shields may be either spherical or cylindrical; for mechanical reasons the latter form is to be preferred. The cylindrical shields are left

open at the ends and the diameter is made small compared with the length; for instance, if the internal diameter is 2 or 3 cm., the length may very well be about 30 cm. See Figs. 7 and 10.

The shielding ratio, that is, the ratio of the strength of the external local field to the corresponding field within the inner shield is greatly affected by the arrangement of the iron. If the iron is all concentrated in a single cylindrical shield, having an outside radius five times the inner radius, the shielding ratio will be about 98 per cent. of that for a shield of infinite thickness. If a single shield of permeability 202 be used, the maximum possible shielding ratio is about 50. This shows the futility of trying to thoroughly protect a galvanometer by a single shield of great weight.

If a given weight of iron is used in several concentric shields, with air spaces between them, its effectiveness is vastly increased.

Given the innermost and outermost radii of a system of three shields, the shielding ratio is a maximum when the radii of the shells are in geometrical progression.

Cyclic annealing of the shields at high temperatures materially increases their effectiveness through an increase of the permeability. The following experimentally determined values give an idea of the protection afforded by a multiple shield. No joints or lateral openings are allowable.

Data for Three Shields of Cast Silicon Steel[5]

Length 29.3 cm.

	Inner radius, centimeters	Outer radius, centimeters	Weight, grams
Shield No. 1	2.55	3.55	1,875
Shield No. 2	4.40	5.45	7,500
Shield No. 3	8.25	10.45	26,900

Shielding Ratios Obtained by Using the Above

Shields used	Shielding ratios	
	Before annealing	After annealing
1	18.62	20.6
1+2	240.4	317.6
1+2+3	2,900.0	4,274.0

A set of five shields cut from ordinary soft iron water pipes, and annealed, gave the results quoted below.

Data for Five Shields of Soft Iron Pipe

Length of shields 30 cm.

	Inner radius, centimeters	Outer radius, centimeters	Weight, grams
Shield No. 1	2.65	3.0	1,530
Shield No. 2	3.90	4.45	3,120
Shield No. 3	5.20	5.7	4,260
Shield No. 4	6.45	7.05	6,130
Shield No. 5	8.95	9.7	9,650

Shielding Ratios Obtained by Using the Above

Shields used	Shielding ratios
1	19.3
1+2	104.1
1+2+3	252.0
1+2+3+4	723.1
1+2+3+4+5	2,724.8

These figures show that a Thomson galvanometer can be rendered practically immune from the effects of local field variation by the use of multiple shields with air spaces.

A commercial form of galvanometer in which spherical shields are employed is shown in Fig. 7.

A simple and effective instrument, the one from which the data concerning the shields were taken, is shown in Fig. 10 on page 22.

Under present-day conditions it is useless to attempt to employ an unshielded Thomson galvanometer.

Suspensions.—In Thomson's original instrument, the needle system was suspended by a single silk fiber. This does well enough for instruments of moderate sensitivity. When the highest sensitivity is desired, the controlling field is reduced to a minimum. The torsional properties of the suspension fiber then become important. Silk fiber suspensions are exceedingly troublesome for they are greatly affected by changes of hygrometric con-

ditions, the result being a continual shifting of the zero point. To remove this trouble, C. Vernon Boys introduced the use of quartz-fiber suspensions.[3]

Quartz when melted is very viscous and may readily be drawn into long threads of uniform cross-section. Intrinsically, quartz fibers are very stiff but this is compensated for by their great strength which permits the use of exceedingly fine threads. A fiber 0.0014 cm. in diameter breaks under a weight of about 10 gm. and may be used to carry 5 gm.; finer threads break at even higher stresses per unit area. Fibers as long as 8 or 10 cm. may be employed for galvanometer suspensions.*

It is found that with quartz fibers the twist produced by a given turning moment is accurately proportional to the moment and independent of the previous history of the thread; this very important property allows quartz-fiber suspensions to be used in many sorts of instruments where a delicate torsional control is desired.

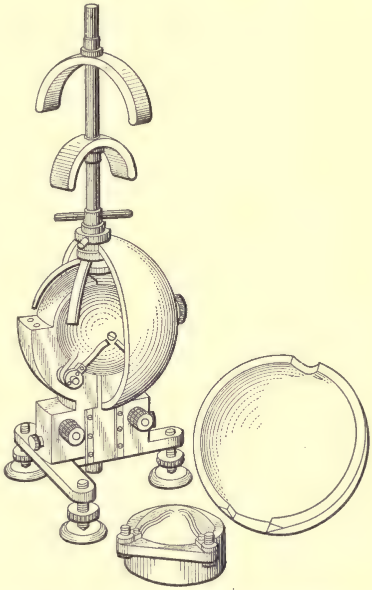


FIG. 7.—Du Bois-Rubens shielded galvanometer.

The Needle System.—The system must be light with the masses symmetrically placed with respect to the axis of rotation. For this reason the shank carrying the needle and the mirror (*bc* in Fig. 6) must be perfectly straight. A very slender glass or quartz rod such as is used for this purpose will be straightened, if, while held vertically under the tension of a weight, it is stroked up and down with a yellow gas flame.

*The manipulation of fused quartz is discussed in THRELFALL'S book "On Laboratory Arts," p. 196. Quartz fibers are now articles of commerce and may be obtained of the Hanovia Chemical Co., New York City.

Symmetry is essential in order that mechanical vibrations may not produce excessive disturbances of the needle system and thus render difficult the reading of the instrument. The moment of inertia of the non-magnetic parts of the system should be reduced to a minimum, so the mirror must be small and light. It may be cut from a (silvered) cover glass such as is used on microscope slides.

Each member of an astatic system may consist of six or seven magnets. Glass-hard tungsten steel is used for the magnets which may be about 1.2 mm. long and 0.2 mm. or less in diameter. The magnets are attached to the shank by minute drops of shellac. To preserve the symmetry of the system the magnets are placed on both sides of the shank. The total mass of a system constructed with great care, for use in research work, may be as small as 6 or 7 mg. Such a very light system will be over-damped by the air friction.

Damping.—In order to economize time, all galvanometers should be properly damped so that they will come promptly to rest. The various devices used, such as damping vanes or in moving coil galvanometers, damping loops, are arrangements for quickly dissipating the energy of motion of the movable system.

Arrangement of the Galvanometer Coils.—In order to attain a high sensitivity the windings must be so disposed that they produce the maximum field at the needle. Having given a definite length of wire of a certain size, it will be most effective when used on an astatic galvanometer. For suppose it to be wound in a coil adapted to a single-needle instrument, the outer layers will be at a considerable distance from the needle and therefore their effectiveness will be small. If an astatic system is used, these outer layers may be taken off and wound in coils which closely surround and act on the lower member of the needle system. Thus the wire as a whole is brought into a more advantageous position and the sensitivity of the instrument increased.

An additional advantage is derived from winding the wire on four rather than on two spools. For, to a certain extent, the galvanometer resistance may be changed to suit the work in hand by connecting the four coils in series, series-parallel or in parallel.

Galvanometer Constant.—It is natural to wind the coils so that their cross-sections will be rectangular. Let n' be the number of turns of wire per square centimeter of cross-section, the other quantities being defined by Fig. 8. Then, if the coil is wound throughout with the same size of wire the field at its center due to unit current is

$$G = 4\pi n'b \log_\epsilon \frac{\cot . \frac{\beta}{2}}{\cot . \frac{\gamma}{2}}.$$

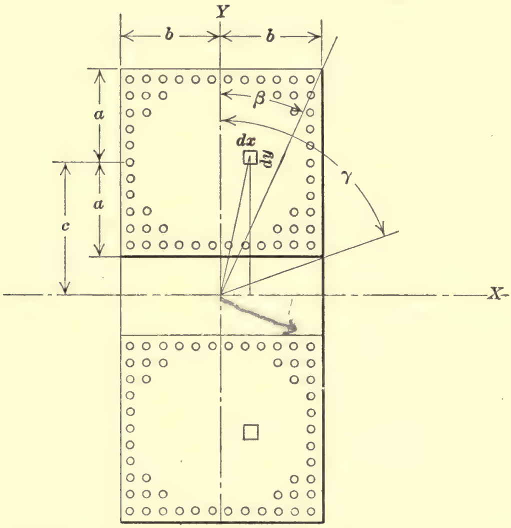


Fig. 8.—Pertaining to demonstration for galvanometer constant.

For, consider the filament of current, $dx\ dy$, the coördinates of whose trace on the plane of the paper are x and y. The magnetic force at the center of the coil due to the length ds of this filament when conveying unit current, will by the elementary laws of electromagnetic action be perpendicular to the radius vector of the point, x, y, and have for its value

$$\frac{n'dsdydx}{x^2 + y^2}.$$

When resolved along the axis this becomes

$$\frac{n'dsdydx}{x^2 + y^2} \times \frac{y}{\sqrt{x^2 + y^2}},$$

and for the whole filament is

$$\frac{2\pi n' y^2 dydx}{(x^2 + y^2)^{3/2}}.$$

Therefore, if c is the mean radius of the coil

$$G = 2\pi n' \int_{-b}^{b} \int_{c-a}^{c+a} \frac{y^2 dydx}{(x^2 + y^2)^{3/2}} = 4\pi n' b \log_\epsilon \frac{c + a + \sqrt{(c+a)^2 + b^2}}{c - a + \sqrt{(c-a)^2 + b^2}}$$

but

$$\cot \frac{\beta}{2} = \frac{1}{\sin \beta} + \cot \beta$$

$$\therefore G = 4\pi n' b \log_\epsilon \frac{\cot \frac{\beta}{2}}{\cot \frac{\gamma}{2}}.$$

Theoretically, there is a best ratio of total breadth to radius for the coil. If the coil has a fixed volume and it be assumed that the diameter of the core is zero, G will be a maximum when $\beta = 30°.8$. If the area of the cross-section be fixed, a given number of turns of wire of a definite diameter will render G a maximum when $\beta = 16°.66$.

Best Form of Cross-section.—Theoretically, the rectangular form of cross-section is not the best. With a definite length of wire of a certain size, the volume of the coil and its resistance are fixed. The question is, how shall this wire be arranged so that it will produce the maximum effect at the needle?

The effect at the needle of a unit length of wire, bent into an arc of a circle and carrying unit current, is $F = \frac{\sin \alpha}{r^2}$ where r and α are the polar coördinates of the traces of the wire on a plane including the axis. The origin of coördinates being the axis of the coil and the coil center, call $\frac{\sin \alpha}{r^2}$ the efficacy of a unit length of wire, denoted by e_v. Then every unit length of wire whose traces are on the curve $r^2 = \frac{1}{e_v} \sin \alpha$, where e_v has a definite value, will produce the same effect at the center of the coil. Suppose

the wire has been so wound that the boundary of the cross-section of the coil is given by $r^2 = \frac{1}{e_v} \sin \alpha$, where e_v has some particular numerical value. If an attempt be then made to alter the form of the cross-section by changing the position of a portion of the wire, the field at the needle will be reduced, for the only change possible is that to a region of less efficacy. Consequently, the equation is that of the boundary of the best form of cross-section. The curve is symmetrical about the maximum radius vector, the value of which is $\frac{1}{\sqrt{e_v}}$.

Similarly, to arrange a given number of turns to give the maximum value of G, their traces must be included within the curve, $r = \left(\frac{2\pi}{e_c}\right) \sin^2 \alpha$. The gain from using the best form of cross-section is small.

Graded Coils.—As the inner turns of the coil of a Thomson galvanometer are very near the needle they are much more effective than those in the outer portion of the coil, which, while they add much to the resistance of the instrument, contribute comparatively little to the galvanometer constant. This suggests that with a coil of a definite resistance, it might be best to concentrate the resistance in the turns near the needle, winding this part of the coil with a finer wire than that used for the outer portion. Maxwell has shown in his "Treatise on Electricity and Magnetism,"* Art. 719, that the diameter of the wire should increase with the diameter of the layer of which it forms a part, the exact law depending on the relation between the diameters of the covered and the bare wire.

As it is not possible to wind the coil with a wire having a cross-section which is a function of its distance from the end of the wire, it is customary to wind the coil in three or four sections, each of a single size of wire.

In a very sensitive galvanometer having a resistance of 25 ohms, the three sections were wound with No. 40, No. 34 and No. 26 B. & S. gage. The thickness of the insulation was 0.002 cm. The galvanometer constant with the instrument so wound was about 33 per cent. greater than if a No. 26 wire had been

* Third edition.

used for the entire coil, this being the size which would be most advantageous if the coil were uniformly wound.*

Relation between the Galvanometer Constant and the Resistance of a Thomson Galvanometer.—The magnetic effects obtained by using coils having the same dimensions but wound with wires of different sizes are proportional to the ampere-turns; the galvanometer constant, G, is therefore proportional to the total number of turns in the coil. Let A be the area of the cross-section of the coil, and n' the number of turns which thread through a square centimeter of the cross-section. Suppose first that the thickness of the insulation is zero and that the diameter of the wire is B_1. Then

$$G = kAn' = Kn' = \frac{K}{B_1^2}$$

where k and K are constants.

The resistance per unit volume of the coil, w, will be

$$w = \frac{K_1 n'}{B_1^2} = \frac{K_1}{B_1^4}$$

and the galvanometer resistance, if V is the volume of the coil, will be

$$R_G = Vw = \frac{K_2}{B_1^4}$$

$$\therefore G = K_3\sqrt{R_G} \tag{2}$$

So if the thickness of the insulation is zero, the galvanometer constant is proportional to the square root of the galvanometer resistance.

Suppose the bobbin to be filled with an insulated wire, the diameter outside of the insulation, designated by C, being the same as that of the bare wire just considered. Let the diameter of the wire itself be B. As the number of turns has remained the same, G is not changed but the resistance will be increased in the ratio $\frac{C^2}{B^2}$; call this ratio y^2, then to keep G the same if the *new* value of R_G be used,

$$G = K_3\frac{\sqrt{R_G}}{y} \tag{3}$$

* The construction of a graded coil of a definite resistance and having three sections is discussed by C. G. Abbot in the *Annals of the Astrophysical Observatory*, vol. 1, 1900, p. 244, where all the necessary formulæ are given.

or, if the ratio of the area of the bare to that of the covered wire be u,

$$G = K_3\sqrt{uR_G} \tag{4}$$

The Best Resistance for a Thomson Galvanometer.—It is well known that in the practice of most methods of electrical testing, the precision obtainable depends on the proper adjustment of the galvanometer resistance to the work in hand.

Consider the simple case of a galvanometer in series with a definite resistance, R, and a battery of electromotive force E. It will be assumed that the bobbin on which the galvanometer coil is wound is of fixed dimensions.

It is easy to see that if a very large wire is used the current will be considerable, on account of the low resistance, but as there are only a few turns, the ampere-turns or the magnetic effect at the needle, to which the deflection is proportional, will be small. If a very fine wire is used, the resistance will be large and the current small and while the number of turns is great, the ampere-turns and the resulting deflection will again be small. Between these two extremes there will be a size of wire which will correspond to a maximum deflection of the galvanometer.

The "best galvanometer resistance" is that obtained when the coil is wound with the size of wire which renders the deflection a maximum.

The deflection of the instrument, D, will be proportional to the magnetic force at the needle, or

$$D = KI_GG$$

and

$$I_G = \frac{E}{R + R_G}$$

$$\therefore D = \frac{K'E\sqrt{uR_G}}{R + R_G}$$

R is the resistance external to the galvanometer. It is to be noticed that R_G is a function of u. The deflection is to be made a maximum by varying R_G. If V is the fixed volume of the coil and w is the resistance per unit volume of the winding,

$$R_G = Vw$$

and

$$D = \frac{K''\sqrt{uw}}{R + Vw} \tag{5}$$

The value of D is to be made a maximum by properly choosing the size of wire. In order that this may be done data must be at hand concerning the resistance per unit volume of various sizes of the insulated wire, as well as the relation of the thickness of the insulation to the diameter of the conductor. In the preliminary discussion of methods of measurement it is sufficient to assume that $G = K\sqrt{R_G}$, in which case the maximum deflection is obtained when

$$R_G = R$$

that is, when the galvanometer resistance is equal to that of the remainder of the circuit. If the galvanometer constant be taken as

$$G = K(R_G)^n$$

the deflection will be a maximum when

$$R_G = \left(\frac{n}{1 - n}\right) R \tag{6}$$

Some authorities recommend that n be taken as $\frac{2}{5}$ instead of $\frac{1}{2}$ as just used.

A table of rough data concerning a particular make of double silk-covered wire is given below.

Table of Rough Data on D.S.C. Cu. Wire

B. & S. gage No.	w = ohms per cu. in.	y	u	Lb. per cu. in.	B. & S. gage No.
20	0.76	1.12	0.79	0.24	20
22	2.0	1.20	0.69	0.23	22
24	5.0	1.27	0.62	0.21	24
26	12.0	1.35	0.55	0.19	26
28	25.0	1.43	0.49	0.17	28
30	54.0	1.52	0.43	0.14	30
32	105.0	1.64	0.37	0.12	32
34	195.0	1.79	0.31	0.08	34
36	355.0	2.00	0.25	0.075	36
38	630.0	2.29	0.19	0.06	38
40	1050.0	2.77	0.13	0.05	40

To illustrate the influence of the size of wire on the deflection of a galvanometer suppose the volume of the coil to be 5 cu. in. and the external resistance 1,900 ohms. The relative deflections calculated by aid of formula (5) and the table are plotted in Fig. 9.

The figure shows that it is better to err on the side of too great resistance and that a considerable departure from the ideal

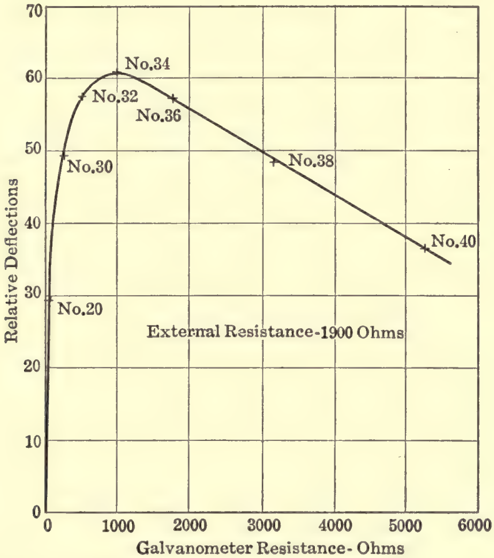


FIG. 9.—Illustrating effect of galvanometer resistance on the deflection.

resistance does not greatly affect the sensitiveness of the arrangement.

An important use of the reflecting galvanometer is in insulation testing, where the current is measured in a circuit of exceedingly high resistance. The galvanometer resistance cannot be more than a few thousand ohms so the current is practically independent of it. This case corresponds to that of the ordinary ammeter; every turn adds something to the deflection, the amount becoming smaller and smaller as the diameter of the coils increases. In this case, there is no "best resistance."

The Sensitiveness of Reflecting Galvanometers.—The sensitiveness of a Thomson galvanometer depends on the arrangement

and winding of the coils, on the construction of the suspended magnetic system, on the strength and position of the neutralizing magnets which produce the controlling field, on the torsional constant of the suspension fiber, and on its initial torsion. This last must be removed when the galvanometer is set up, as its presence necessitates a strong controlling field to bring the needle to its zero position.

Ampere Sensitivity. Microampere Sensitivity.—Quantitatively, the current sensitivity of a galvanometer is the deflection, as read from the scale, per unit current. This simple statement is not sufficiently definite, so in order to obtain results that admit of comparisons being made between different instruments, some convention must be adopted as to the conditions under which the sensitivity is to be measured.

It will be assumed that one of the mirror and scale methods of reading is used. The current, I_G, will be stated in amperes, the scale deflection D, in millimeters, and the scale distance L, in meters (1,000 scale divisions). Then, $S_I = \frac{D}{LI_G}$ is the deflection in millimeters which would be produced by unit current if the scale had been at a meter's distance. S_I will be a very large number; consequently, for convenience in writing, the microampere (10^{-6} ampere) is frequently used as the unit of current. This gives the microampere sensitivity.

Relation between Time of Vibration and Current Sensitivity.—By common consent the sensitivity is measured when the suspended system has a stated time of vibration.

The connection of the sensitivity with the time of vibration of the needle system will be seen from the following. If the damping due to air friction be neglected, the time of vibration, T_0 of a suspended magnetic system, having a moment of intertia P, and a magnetic moment M, when placed in a field of strength H, will be

$$T_0 = 2\pi\sqrt{\frac{P}{MH}}.$$

The current through the instrument is given nearly enough by

$$I_G = \frac{10H}{G}\,\frac{D}{2000L}$$

The deflection is

$$D = \frac{200 L I_G G}{H}$$

$$\therefore S_I = \frac{D}{I_G L} = \frac{200G}{H} = \frac{KMG}{P} T_0^2 = \frac{K'M\sqrt{R_G}T_0^2}{P} \qquad (7)$$

Therefore, with any given magnetic system, the sensitivity is proportional to the square of the time of vibration. If this be doubled by changing H, the sensitivity will be increased fourfold, for to double the time of vibration the directive force must be reduced to one-fourth of its previous value, so the deflection due to the same value of the current is quadrupled.

If the needle system is very light the damping due to air friction will be so great that there will be a considerable departure from the above relation. The sensitivity of modern research galvanometers with very delicate suspended systems is more nearly proportional to the first than to the second power of the time of swing. Instruments with exceedingly light systems are sometimes operated *in vacuo*.

In comparing instruments, the sensitivity must be reduced to the value it would have if the movable system had some definite time of vibration; 10 sec. for a complete swing is that commonly taken.

Normal Sensitivity.—It is unfair to compare instruments which have very different resistances and different times of vibration. It is desirable to reduce the sensitivities to the values they would have with the galvanometers wound to a standard resistance, 1 ohm, with a wire having an insulation of zero thickness, and with a needle system whose time of vibration is 10 sec. From the previous discussion the normal current sensitivity is

$$S_N = \frac{D}{I_G L} \frac{10^2}{T^2} \frac{y}{\sqrt{R_G}} \qquad (8)$$

Volt Sensitivity. Microvolt Sensitivity.—The volt sensitivity under any given conditions is the deflection per unit voltage and is consequently the current sensitivity divided by the resistance. When dealing with moving coil galvanometers (page 38) the condition is frequently imposed that the resistance of the circuit shall be such that the galvanometer is critically damped.

Megohm Sensitivity.—The megohm sensitivity is the number of megohms which must be inserted in a circuit containing an e.m.f. of 1 volt in order to obtain a galvanometer deflection of 1 mm., at a meter's scale distance. This is the same, numerically, as the microampere sensitivity, referred to on page 20.

Design for a Reflecting Galvanometer.[5]—A modern design for a reflecting galvanometer with magnetic shielding is shown in Fig. 10.

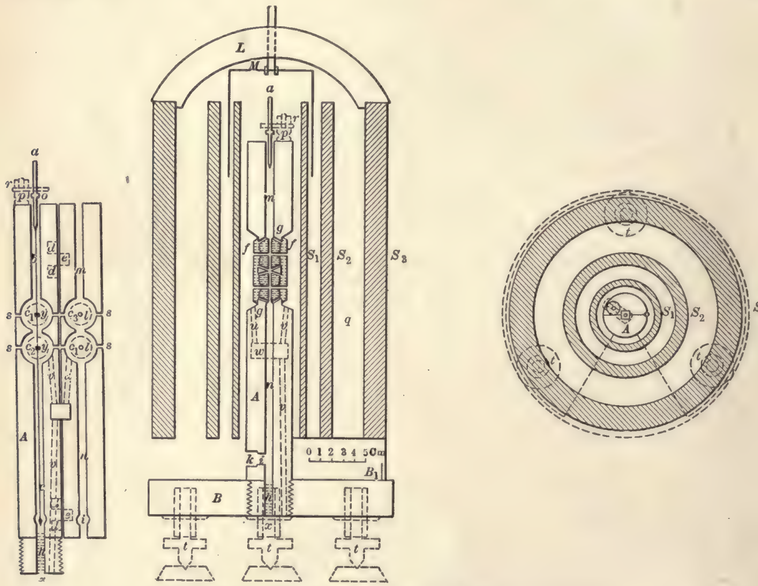


FIG. 10.—Shielded Thomson galvanometer.

The support for the coils and the suspended system is shown at the left hand of the figure. It consists of a brass rod, 3.8 cm. in diameter. This is divided longitudinally along the axis and the two parts are hinged together at *de*. A groove *mn* is milled lengthwise in both halves and serves to accommodate the suspended system. When the instrument is ready for use, the space between the coils is about 2½ mm., just sufficient to allow the needle system to turn around. There are small holes along the axis of the coils, denoted by *l* and transverse saw cuts in the uprights, denoted by *s*; by this means the needle system can be observed when the coils are in their proper position.

The window, k, allows the mirror to be observed, the hard rubber base, B_1, upon which the shields rest being cut away for that purpose, as shown in the cross-section.

The needle system is suspended by a quartz fiber, which is mounted on a swinging support at p, by which the system may be centered. When the coil support is opened, the needle system may be swung forward for examination.

The arched piece L carries the control magnet M, which is made of bent clock spring. By it, the zero point and the sensitiveness of the galvanometer may be changed. The needle system is not adjusted for astaticism.

The coils are wound in three sections, the inner consisting of 81 cm. of No. 38 wire, the middle section of 328 cm. of No. 32 wire and the outer section of 1,318 cm. of No. 26 wire. Each of the finished coils has a resistance of 5.6 ohms.

To guard against the effects of static charges, the plane faces of the coils are covered with tin foil. The terminal wires of each coil are twisted together and carried down through channels in the upright and each coil has its own set of terminals.

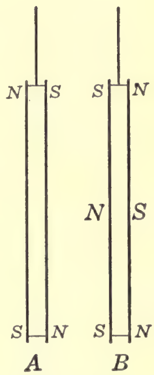


FIG. 11.—Movable system with vertical needles.

The coils are mounted in the support by means of an insulating wax. The shields are those referred to on page 9.

The sensitivity attained with all the coils in parallel is 3×10^9, the time of a complete swing being 6 seconds.

The Broca Galvanometer.[6]—Various experimenters have suggested the use of a needle system consisting of two slender vertical magnets as shown at A in Fig. 11.

In this case two pairs of coils would be used, acting respectively on the upper and lower pair of poles. Practically, it is very difficult, if not impossible, to astaticize such a system, for the magnets must be exactly parallel to the axis of rotation.

In the Broca instrument the vertical needles are magnetized with consequent poles at the middle of their lengths as shown at B in Fig. 11. Hence it is easy to astaticize the system by retouching the magnets, and this is its advantage.

In the instrument shown in Fig. 12, a single pair of coils acting principally on the consequent poles is used.

The control magnet is at B. By it the time of vibration may be varied from about 5 to 20 seconds. A clamping device by which the needle may be held in place is actuated by the knob D. Damping is accomplished by a light aluminum vane G which swings between two adjustable plates, manipulated by the knobs C. A quartz fiber suspension is used.

The coils are so mounted that they are readily changed and the instrument thus adapted to different pieces of work.

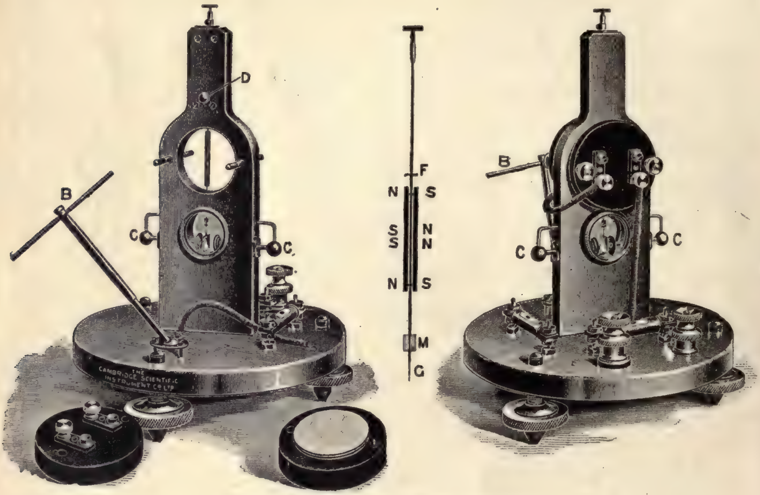


FIG. 12.—Broca galvanometer.

THE MOTION OF THE SUSPENDED SYSTEM OF A GALVANOMETER

Suppose that when the movable system of a galvanometer is at rest, the circuit is suddenly closed so that a current flows through the instrument. Experience shows that in some cases the movable system arrives at its final deflected position by a series of oscillations of diminishing amplitude, in other cases by a steady increase of the deflection without oscillations.

It is desirable to derive the equations which will represent the motion of the system sufficiently well for practical purposes, for the general considerations thus introduced are of importance in

dealing with both current and ballistic galvanometers. An important special case is that of the critically damped instrument of the D'Arsonval type, as ordinarily used, and also when the period has been so reduced that the instrument has become an oscillograph capable of following a complex wave in its variations.

The Equation of Motion.—1. The angular deflection of a reflecting galvanometer is always small so the deflecting moment at any instant may be taken as proportional to the instantaneous value of the current and represented by Ci where C is a constant depending on the construction of the instrument.

2. For small deflections the restoring moment will be proportional to the angle through which the system has been turned, that is, it will be equal to $\tau\theta$ where θ is the angle of deflection and τ is the restoring moment for unit angular deflection.

3. The system as it moves is retarded by air friction, etc., and in some cases by induced currents. It is customary to *assume* that this retarding moment is proportional to the angular velocity of the system, and is therefore represented by $k\,\frac{d\theta}{dt}$ where k is the coefficient of damping.*

Let P be the moment of inertia of the movable system. The total moment acting to change the angular velocity of a body rotating about a fixed axis is the product of the moment of inertia and the angular acceleration. On equating this product to the sum of the turning moments acting on the system

$$P\,\frac{d^2\theta}{dt^2} = Ci - \tau\theta - k\,\frac{d\theta}{dt}.$$

Consequently, the motion takes place according to the equation

$$P\,\frac{d^2\theta}{dt^2} + k\,\frac{d\theta}{dt} + \tau\theta = Ci \tag{9}$$

* This law of damping was introduced by GAUSS and W. WEBER in their study of the behavior of the vibrating magnets used in their magnetic measurements at Göttingen, 1836–37. With air damping, in order that this law be reasonably well fulfilled, the damping must be slight, the amplitude of the vibration small and the restoring moment due to the suspension large. That this law is not absolutely exact is apparent, for, according to it, the movable system when once set in vibration would continue to swing for an infinite time with a constantly decreasing amplitude. But it is a matter of common experience that the system comes to rest in a comparatively short time. However, the results obtained by GAUSS's theory are in close enough agreement with the observed facts to warrant its use.

The right-hand member of equation (9) may be a function of t, constant, or zero according to the conditions of the problem. In this section i will be taken as constant (circuit closed) or zero (circuit opened).

The mathematical form of equation (9) and the following discussion should be compared with that for the displacement of electricity in a circuit containing resistance, inductance, and capacity in series.

The deflection of a galvanometer may be oscillatory or non-oscillatory and its ultimate value will be most quickly attained if the conditions are such that the motion is just becoming non-oscillatory. This occurs when the relation $\frac{k^2}{\tau} = 4P$ is on the point of being fulfilled. The instrument is then said to be critically damped.

When $\frac{k^2}{\tau} > 4P$ the motion of the suspended system will be non-oscillatory and when $\frac{k^2}{\tau} < 4P$ it will be oscillatory. When the instrument is not critically damped the solution of (9) is*

$$\theta = C_1\epsilon^{m_1t} + C_2\epsilon^{m_2t} +$$

$$\left(\frac{C}{P(m_1 - m_2)}\right)\left[\epsilon^{m_1t}\int\epsilon^{-m_1t}\,idt - \epsilon^{m_2t}\int\epsilon^{-m_2t}\,idt\right] \quad (10)$$

m_1 and m_2 are the roots of the equation

$$Pm^2 + km + \tau = 0$$

or

$$m_1 = -\frac{k}{2P} + \sqrt{\frac{k^2}{4P^2} - \frac{\tau}{P}} \quad (11)$$

$$m_2 = -\frac{k}{2P} - \sqrt{\frac{k^2}{4P^2} - \frac{\tau}{P}} \quad (12)$$

The constants C_1 and C_2 must be determined to fit the conditions of the particular case under consideration. When a current of a definite strength is sent through the galvanometer

$$i = I, \text{ a constant.}$$

*See COHEN'S "Differential Equations," p. 105; CAMPBELL'S "Diffeential Equations," p. 56.

At $t = 0$ both the deflection and the angular velocity are zero, or

$$t = 0 \qquad \theta = 0$$

$$t = 0 \qquad \frac{d\theta}{dt} = 0.$$

Imposing the first of these conditions on (10) gives

$$0 = C_1 + C_2 + \frac{CI}{\tau}$$

but $\frac{CI}{\tau} = \theta_F$, the final value of the deflection.

$$\left.\frac{d\theta}{dt}\right]_{t=0} = C_1 m_1 + C_2 m_2 = 0$$

$$\therefore C_1 = -\frac{\theta_F m_2}{m_2 - m_1}, \text{ and } C_2 = \frac{\theta_F m_1}{m_2 - m_1}.$$

The value of θ which fulfils the conditions is therefore

$$\theta = \theta_F \left[-\frac{m_2}{m_2 - m_1} \epsilon^{m_1 t} + \frac{m_1}{m_2 - m_1} \epsilon^{m_2 t} \right] + \theta_F \tag{13}$$

If, when the needle is at rest in its deflected position, the circuit is broken, the deflecting moment due to the current becomes zero, so

$$P \frac{d^2\theta}{dt^2} + k \frac{d\theta}{dt} + \tau\theta = 0$$

$$\therefore \theta = C_1 \epsilon^{m_1 t} + C_2 \epsilon^{m_2 t}.$$

When

$$t = 0 \qquad \theta = \theta_F$$

and

$$t = 0 \qquad \frac{d\theta}{dt} = 0$$

$$\therefore \theta_F = C_1 + C_2$$

and

$$m_1 C_1 + m_2 C_2 = 0.$$

Therefore

$$\theta = \theta_F \left[\frac{m_2}{m_2 - m_1} \epsilon^{m_1 t} - \frac{m_1}{m_2 - m_1} \epsilon^{m_2 t} \right] \tag{15}$$

Non-oscillatory Deflection.—If $k^2 > 4\tau P$, both m_1 and m_2 are real and negative and (13) is the equation of a curve which gradually rises toward the value θ_F; in this case, the galva-

nometer is said to be over-damped. When the circuit is broken, the needle returns to zero according to equation (15). Equations (13) and (15) are plotted in Fig. 13 where the values $\tau = 0.2$, $P = 0.2$ and $k = 1.0$ are assumed.

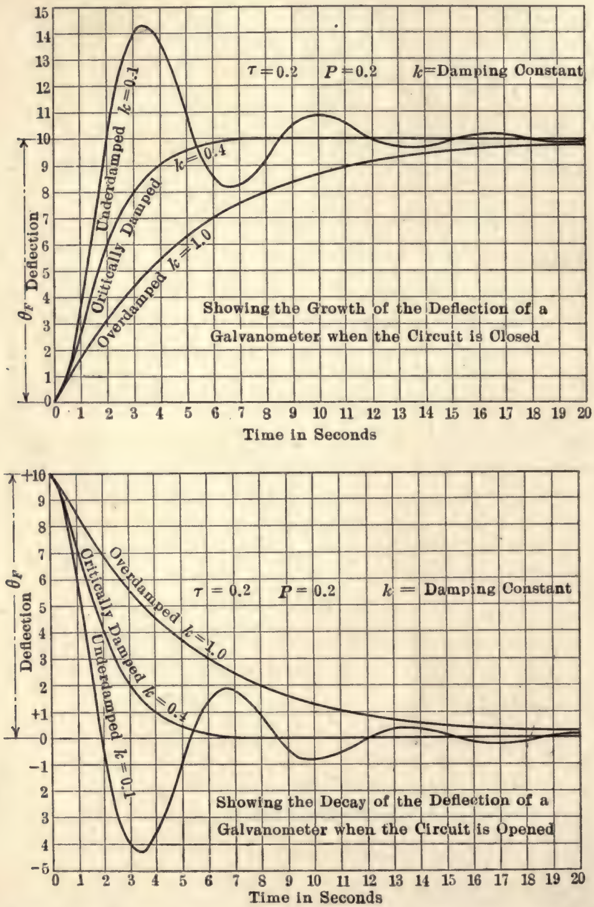


FIG. 13.—Illustrating the motion of galvanometer needle.

Oscillatory Deflection.—When $k^2 < 4\tau P$, m_1 and m_2 are complex, that is

$$m_1 = -a + jb$$
$$m_2 = -a - jb$$

where

$$a = \frac{k}{2P} \text{ and } b = \sqrt{\frac{\tau}{P} - \frac{k^2}{4P^2}}.$$

In this case, the final deflection is attained by a series of oscillations. If the values of m_1 and m_2 are substituted in (10) and the resulting equation simplified by the use of the exponential values of the sine and cosine and the relation

$$A \sin \beta + B \cos \beta = \sqrt{A^2 + B^2} \sin \left(\beta + \tan^{-1} \frac{B}{A}\right)$$

it will be found that

$$\theta = \theta_F - \theta_F \epsilon^{-at}\left[\sqrt{\frac{a^2 + b^2}{b^2}} \sin \left\{ bt + \tan^{-1} \frac{b}{a} \right\}\right] \tag{16}$$

The variable part of (16) represents an oscillation of constantly decreasing magnitude and having a period,

$$T = \frac{2\pi}{b} = \frac{2\pi}{\sqrt{\frac{\tau}{P} - \frac{k^2}{4P^2}}} \tag{17}$$

The time of a complete swing when no damping is present is

$$T_0 = 2\pi\sqrt{\frac{P}{\tau}}.$$

So
$$\theta = \theta_F - \theta_F \frac{T}{T_0} \epsilon^{-\frac{kt}{2P}} \sin \left[\frac{2\pi}{T} t + \tan^{-1} \frac{4\pi P}{kT}\right] \tag{18}$$

The elongations, or maximum and minimum values of the deflection, occur when

$$t = 0, t = \frac{T}{2}, t = T, \quad . \quad . \quad . \quad t = n\frac{T}{2}.$$

If, when the needle is at rest in its deflected position, the circuit is broken, the return to zero is by a series of oscillations according to the equation

$$\theta = \theta_F \frac{T}{T_0} \epsilon^{-\frac{kt}{2P}} \sin \left[\frac{2\pi}{T} t + \tan^{-1} \frac{4\pi P}{kT}\right] \tag{19}$$

Equations (18) and (19) are plotted in Fig. 13 where the values $\tau = 0.2$, $P = 0.2$ and $k = 0.1$ are assumed.

Logarithmic Decrement.—Let $\lambda = \frac{kT}{4P}$, then (18) and (19) become

$$\theta = \theta_F - \theta_F \frac{T}{T_0} \epsilon^{-\left(\frac{2\lambda}{T}\right)t} \sin\left[\frac{2\pi t}{T} + \tan^{-1}\frac{\pi}{\lambda}\right] \tag{18a}$$

and

$$\theta = \theta_F \frac{T}{T_0} \epsilon^{-\left(\frac{2\lambda}{T}\right)t} \sin\left[\frac{2\pi t}{T} + \tan^{-1}\frac{\pi}{\lambda}\right] \tag{19a}$$

The utility of this substitution lies in the fact that λ is much more easily determined than its components k and P.

λ is called the Napierian logarithmic decrement; it is a quantity of importance in the theory of damped vibrations.

The first elongation after the circuit is broken occurs when $t = \frac{T}{2}$. Substituting this value in (19a) and using (21) gives

$$\theta_1 = \theta_F \epsilon^{-\lambda} \cos \pi$$

The nth elongation, when $t = n\frac{T}{2}$, is

$$\theta_n = \theta_F \epsilon^{-n\lambda} \cos n\pi.$$

$$\therefore \frac{\theta_1}{\theta_n} = \epsilon^{\lambda(n-1)}$$

and

$$\lambda = \frac{1}{n-1} \log_\epsilon \frac{\theta_1}{\theta_n} \tag{20}$$

The method of determining λ is obvious: To obtain the necessary data one has only to set the movable system in motion, read an elongation, which will be called θ_1 and, after a counted number of elongations read θ_n.

Experiment shows that with air damping λ is slightly affected by the amplitude of vibration, but not enough to give rise to practical difficulties.

Influence of Damping on the Time of Vibration.—From the above,

$$T = \frac{2\pi}{\sqrt{\frac{\tau}{P} - \frac{k^2}{4P^2}}} = \frac{2\pi}{\sqrt{\frac{4\pi^2}{T_0^2} - \frac{4\lambda^2}{T^2}}}$$

$$\therefore \frac{T}{T_0} = \sqrt{\frac{\pi^2 + \lambda^2}{\pi^2}} \tag{21}$$

From this it is seen that λ must be large before it greatly affects the time of vibration.

Critical Damping.—A galvanometer is critically damped when the motion of the needle is just becoming non-oscillatory. In this case, when the current I is constant and $k^2 = 4\tau P$ the solution of (9) becomes

$$\theta = \theta_F + \epsilon^{-\frac{kt}{2P}} \{C_1 + C_2 t\} \tag{22}$$

To determine C_1 and C_2

$$t = 0 \qquad \theta = 0$$

$$t = 0 \qquad \frac{d\theta}{dt} = 0$$

$$\therefore C_1 = -\theta_F.$$

$$\left.\frac{d\theta}{dt}\right]_{t=0} = -\frac{kC_1}{2P} + C_2 = 0$$

$$\therefore C_2 = -\frac{\theta_F k}{2P}.$$

So

$$\theta = \theta_F - \theta_F \epsilon^{-\frac{kt}{2P}} \left\{1 + \frac{k}{2P} t\right\}. \tag{23}$$

For this case

$$\frac{k}{2P} = \frac{2\pi}{T_0}$$

and

$$\theta = \theta_F - \theta_F \epsilon^{-\left(\frac{2\pi}{T_0}\right)t} \left\{1 + \frac{2\pi}{T_0} t\right\}. \tag{24}$$

When the circuit is broken the needle returns to zero, in accordance with the equation

$$\theta = \theta_F \epsilon^{-\left(\frac{2\pi}{T_0}\right)t} \left\{1 + \frac{2\pi}{T_0} t\right\} \tag{25}$$

Fig. 13 shows the character of the motion in this case when $\tau = 0.2$ and $P = 0.2$.

Using the above equations the time required for the deflection to approach within a given percentage of its ultimate value may be calculated. The results for a particular case are shown in

Fig. 14; it is seen that the reading is most quickly obtained when the instrument is very nearly critically damped. The deflection of a critically damped galvanometer is within $\frac{1}{10}$ per cent. of its ultimate value, at a time which is approximately equal to $1.5T_0$, and within 1 per cent. at a time which is approximately equal to T_0.

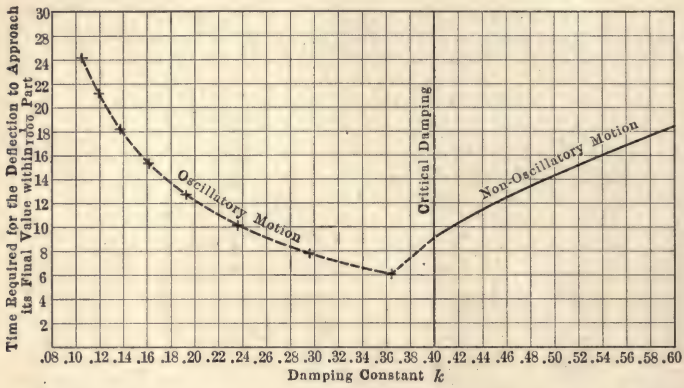


FIG. 14.—Showing the effect of damping as the time required by a galvanometer to attain its deflection.

The D'Arsonval or Moving-coil Galvanometer.[7]—Sir William Thomson used the suspended-coil principle in his siphon recorder (1870) and its application to galvanometers was later suggested by Maxwell in his Treatise on Electricity and Magnetism.

The name D'Arsonval is frequently applied to galvanometers of this class, attention having been recalled to them by Deprez and D'Arsonval in 1882.

The great practical advantage of this instrument lies in its freedom from the effects of stray fields and the ease with which a long and uniform scale may be attained. It is these things that have caused it to be adopted, in a modified form, for direct-current ammeters and voltmeters.

The essential features of a moving-coil galvanometer are shown in Fig. 15. The field is furnished by a strong permanent magnet and the movable coil swings in the air gaps between the poles of the magnet and a fixed iron core. The coil is hung by a fine wire suspension which also serves as a lead, while below the

coil the current is taken out by a loosely coiled metallic spiral. A proper torsion head for adjusting the coil vertically and setting the zero reading is provided. The entire coil system is mounted in a removable frame so that the coils may be readily changed.

Suppose the coil is rectangular, the total length of active wire is l and the half breadth of the coil is b, and that the field, H, in which the coil is placed is uniform. The turning moment due to the current, I, is

$$M = IHlb \cos \theta$$

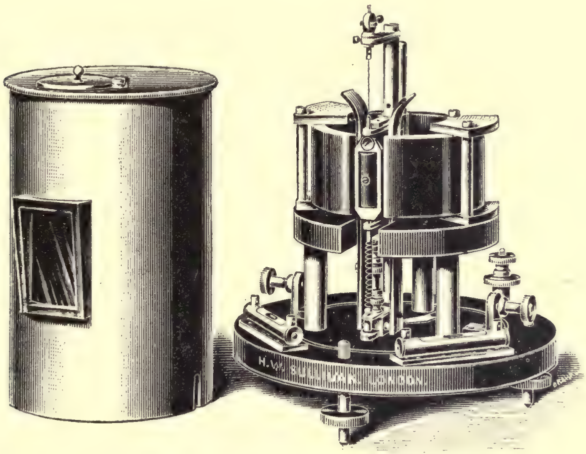


FIG. 15.—Moving-coil galvanometer.

where θ is the angle between the lines of force and the plane of the coil. If the motion of the coil be resisted by a spring, the latter will be twisted until the restoring moment due to it is equal to the deflecting moment due to the current. If the zero position of the plane of the coil is in the direction of the lines of force, the restoring moment will be $\tau\theta$ where τ is the restoring moment for unit angular deflection (a constant). Thus, at equilibrium

$$\tau\theta = IHlb \cos \theta$$

If there are n turns,

$$I = \frac{\tau}{nHlb} \left(\frac{\theta}{\cos \theta}\right)$$

The D'Arsonval galvanometer is a secondary instrument; that is, the relation between the deflection and the current is always

determined by calibration; but the formula serves to direct attention to certain quantities which are involved in its action.

The Magnets.—It is seen that if there were no modifying conditions the current sensitivity would increase proportionally to H. This indicates that the magnet should be very strong. However, strength is not the only thing to be considered, for it is well known that so-called permanent magnets gradually lose their strength, that is, they "age." The ageing depends upon the quality of the steel, the design of the magnetic circuit and upon the temperature variations and mechanical jarring to which the magnet is subjected. Any deterioration will influence the sensitiveness of the instrument, so in this and in many other cases, for instance, in the magnets used in direct-current ammeters and voltmeters, and in watt-hour meters, it is necessary to resort to artificial ageing. As pointed out by Strouhal and Barus this may be done by the proper heat treatment at moderate temperatures. Their procedure was, after the magnet had been hardened, to heat it in a steam bath at 100°C. for 20 or 30 hours, then magnetize it strongly and afterward heat it again in the steam bath for 4 or 5 hours. In addition, some makers resort to a partial demagnetization. The net result is that while the strength of the magnet is reduced, the remaining magnetization is very permanent.

The temperature coefficient of magnets such as are used in galvanometers and in direct-current ammeters and voltmeters is about -0.025 per cent. per degree C. rise of temperature; it varies with the magnet.

Chilled cast-iron magnets may be employed.[8] In general, they are useful where it is necessary to employ forms so complicated that forging would be difficult.

After the gray iron castings have been machined, they are heated to a bright red (just under the melting point) in a gas furnace provided with a power blast, and then plunged into a cold acid bath which is violently stirred. Care must be taken in the manipulation of the heated castings, for they are lacking in tenacity. It is important that the entire mass of the casting be hardened; consequently the heating must be prolonged until it is certain that the temperature is practically uniform throughout the mass.

After hardening, the magnets are heated in a steam bath for a

long time and then magnetized to saturation. The ageing is effected by alternately heating in steam and cooling in tap water. If the magnets are magnetized to saturation, the strength is reduced about 20 per cent. by this process.

When properly prepared, chilled cast-iron magnets have a small temperature coefficient. For magnets of the forms used in instruments, the average decrease of field strength per degree rise of temperature, between 10° and 100°, is about 0.04 per cent. Within the ordinary range of room temperatures it is much smaller, about 0.013 per cent. per degree.

The strength of aged cast-iron magnets is less than the strength of those of equal weight, made of special magnet steel.

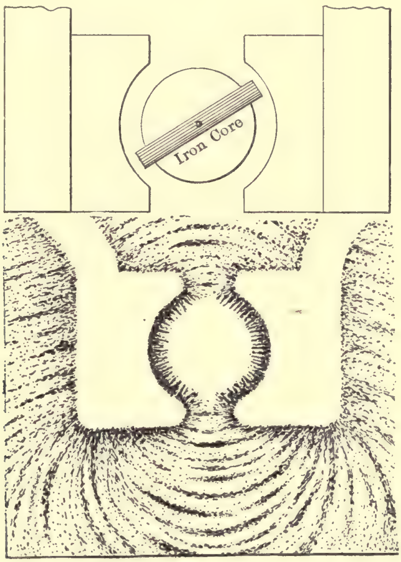


FIG. 16.—Pole pieces for producing a radial field and the resultant field.

Effect of Magnetic Impurities in the Coil.—Practically it is found that the increase in sensitiveness may not be proportional to the increase in H, for, as first pointed out by Ayrton and Mather, the coil itself may be slightly magnetic due to the presence of minute amounts of iron as an impurity in the metal or which have been worked into the insulation during the process of winding.

Consequently, in very strong uniform fields, the attraction between the magnet and the induced poles on the coil, when it is deflected from its initial position, may be strong enough to materially reduce the sensitiveness. In some cases the control thus exercised may be greater than that due to the suspension. The magnetic action of the coil also gives rise to indefiniteness of the zero reading which will be displaced in the direction of the next previous deflection. These difficulties are reduced to negligible amounts if a radial field is used as indicated in Fig. 16.

Obviously, induced poles, if they exist, do not affect the restoring moment when the coil changes its position. This construction also gives a long and uniform scale. It was introduced in the Weston direct-current ammeters and voltmeters in 1888.

When the coil moves in a radial field, the turning moment acting upon it is given by

$$M = IHlb.$$

Suspensions.—The materials commonly used for the suspension wires in commercial instruments are phosphor-bronze and steel. Silver gives a low resistance wire but is not as stable as phosphor-bronze.

The sensitivity will be increased by diminishing the torsion constant, τ, which is proportional to the fourth power of the diameter of the suspension wire. Of course, with a given coil, the stress per unit area on the suspension is inversely as the square of the diameter.

To increase the sensitivity without increasing the unit stress on the suspension, Ayrton and Perry suggested that a flat strip be substituted for the round wire. A strip having a breadth of about ten times its thickness has approximately one-fifth of the torsional rigidity of a round wire of the same length and sectional area. Such suspensions are very commonly used in commercial moving coil galvanometers. Northrup has suggested the employment of a cable of very fine wires as a means of supporting heavy coils. For a cable capable of supporting a given weight the torsional rigidity decreases in proportion as the number of strands is increased. This form of suspension is frequently used in portable galvanometers.

All connections about the suspended system should be *soldered*, as otherwise extraneous resistances may be introduced by loose

contacts or by corrosion. Rosin should be used as a flux, as it is difficult to remove the last traces of acid, which would seriously corrode the delicate wires. For the same reason, so-called non-corrosive soldering liquids should be avoided.

The use of the taut suspension employed in the original D'Arsonval instrument is not advisable unless there is a special provision for balancing the coil, for it is difficult to attach the suspension wires so that their axes pass through the center of gravity of the coil when the suspension is drawn taut. If this condition is not fulfilled, the center of gravity is coerced into taking up an abnormal position and the weight of the coil will cause a turning moment which will vary with the tightness of the wire and the level of the instrument. For these reasons, in sensitive instruments of the best design, the coil is allowed to hang free and the electrical connection at the bottom of the coil is made by a loose spiral which may have a very small torsional rigidity. This procedure also decreases the stress in the upper wire so that it may be made smaller. However, in special cases where the instrument is to be subjected to great changes of level, as on shipboard, the taut wire must be employed. In the Sullivan marine galvanometer and other similar instruments, the center of gravity of the moving coil can be adjusted to its proper position by bending bits of lead wire which project from the coil frame or by adjusting two sets of screws which project at right angles through and perpendicular to the shank supporting the coil.

Effect of Changes of Temperature.—If the instrument is to be used as an unshunted current galvanometer, a change of room temperature will alter the calibration. Experiments show that for phosphor-bronze strip the elasticity decreases about 0.05 per cent. per degree rise of temperature. The strength of the magnet also diminishes with an increase of temperature, the change being 0.01 or 0.02 per cent. per degree. It varies with different magnets. The tendency of these effects is toward compensation, but in general their relative magnitudes will not be such as to eliminate the error.

If the instrument is shunted, the multiplying power of the shunt will depend, to a certain extent, upon temperature conditions, for the galvanometer is wound with copper and the

shunt is most probably of a material having zero temperature coefficient.

When the D'Arsonval galvanometer is used as a potential galvanometer or a millivoltmeter, there is an additional source of variation due to the change of resistance. If the necessary series resistance is made partly of copper and partly of manganin in the proper proportion, its net temperature coefficient may be made such that the instrument is almost exactly compensated for variations of room temperature.

It should be remembered that when the temperature varies very rapidly, there will be a time lag in the change of resistance and in the change of the strength of the magnet.

Magnetic Damping.—To prevent loss of time when using any galvanometer, it is necessary that it be properly damped. In the D'Arsonval instrument the damping may be attained by use of closed loops of wire attached to the movable coil or by winding the coil on a very light metal bobbin, as is done in direct-current ammeters and voltmeters. The current induced in the closed circuit as the coil swings through the magnetic field promptly brings the coil to rest. Similarly a moving coil galvanometer, as will be seen below, will be appreciably damped if it is used in a closed circuit of moderate resistance and it is frequently possible, by adjusting the resistance of the circuit and the constants of the galvanometer, to attain critical damping.

When it is necessary that the coil be perfectly free to move from and be brought back quickly to its zero position, a short circuiting key, which must be free from thermo-electromotive forces, may be placed across the terminals of the galvanometer. The motion of the coil is promptly checked by depressing the key.

The damped D'Arsonval instrument is very useful as a ballistic galvanometer on account of its quick return to the zero reading.

The Critically Damped Moving-coil Galvanometer.[7]—The field in which the coil moves will be assumed to be radial.

SYMBOLS USED IN THE DISCUSSION

H = strength of field.
l = total length of active wire.
b = one-half the breadth of movable coil.

v = total length of vertical wire.
h = total length of horizontal wire, across ends of coil.
m = mass per unit length of bare wire.
m' = mass per unit length of covered wire.
$n = \frac{m'}{m}$.
a = area of bare wire.
ρ = resistivity of copper, 1.724 microhms centimeter at 20°C.
δ = density of copper, 8.89.
P = moment of inertia of entire moving system.
P' = moment of inertia of coil fittings.
τ = torsion constant of suspension, restoring moment for unit angular deflection.
θ = deflection at any instant.
θ_F = final value of θ when a constant current I_F flows in coil, or initial value of θ when circuit is broken.
i = current in coil at any instant.
I_F = final value of current after coil has come to rest in its deflected position.
T_0 = time of an undamped vibration of movable system.
R = total resistance of circuit, including galvanometer.
L = total inductance of circuit.
R_G = resistance of coil of galvanometer.
E = e.m.f. of battery.
E_B = back e.m.f. generated by movement of coil.
C = Hlb, turning moment which acts on coil when it carries unit current.
S_I = current sensitivity.
$S_V = \frac{S_I}{R}$ = voltage sensitivity.

The turning moment acting on the coil at any instant is $iHlb = iC$, while the restoring moment is $\tau\theta$. After the coil has come to rest

$$\tau\theta_F = I_F C$$

or

$$I_F = \frac{\tau\theta_F}{C} \tag{28}$$

An important use of the moving-coil instrument is in a closed circuit of moderate resistance as occurs with the Dieselhorst or the Brooks potentiometer or when the instrument is used in connection with thermo-electric junctions. In these cases the resistance of the circuit is constant or nearly so.

Consider the instrument to form a part of a galvanic circuit which includes a source of e.m.f. and a resistance external to the

galvanometer. When the circuit is closed the coil will begin to move and as it swings in the field, H, a back e.m.f. will be set up; its magnitude will be

$$E_B = Hlb\frac{d\theta}{dt} = C\frac{d\theta}{dt}.$$

Consequently the current at any instant will be

$$i = \frac{E - L\dfrac{di}{dt} - C\dfrac{d\theta}{dt}}{R}.$$

Let k be the damping coefficient when the galvanometer circuit is open. This damping may be due to currents induced in damping loops attached to the movable coil or in a metal frame upon which the coil is wound and, to a slight extent, to the air damping. From the above

$$P\frac{d^2\theta}{dt^2} = \left(\frac{E - L\dfrac{di}{dt} - C\dfrac{d\theta}{dt}}{R}\right)C - k\frac{d\theta}{dt} - \tau\theta$$

or

$$P\frac{d^2\theta}{dt^2} + \left(\frac{C^2}{R} + k\right)\frac{d\theta}{dt} + \tau\theta = \frac{EC}{R} - \frac{CL}{R}\frac{di}{dt} \tag{29}$$

The term containing $\dfrac{di}{dt}$ is negligible so

$$P\frac{d^2\theta}{dt^2} + \left(\frac{C^2}{R} + k\right)\frac{d\theta}{dt} + \tau\theta = \frac{EC}{R} = I_FC = \tau\theta_F \tag{29a}$$

Here $\left(\dfrac{C^2}{R} + k\right)$ is the damping constant and replaces k in the equation on page 25.

The case of special importance is when the constants of the circuit are such that the instrument is critically damped. Then if it is assumed that the damping is entirely due to actions in the movable coil itself,

$$\left(\frac{C^2}{R}\right)^2 = 4P\tau \tag{30}$$

Introducing T_0 and eliminating τ gives

$$\left(\frac{C^2}{R}\right)\frac{1}{2P} = \frac{2\pi}{T_0}.$$

Current and Voltage Sensitivity.—Using θ in radians and I_F in absolute units

$$S'_I = \frac{\theta_F}{I_F} = \frac{C}{\tau}$$

$$\therefore\ (S'_I)^2 = \frac{C^2}{\tau^2} = \frac{RT_0}{\pi\tau} \tag{31}$$

R is the resistance of the circuit necessary for critical damping. Using the microampere sensitivity (page 20), if R is expressed in ohms and the reading in millimeters, on a scale at a meter's distance,

$$S_I^2 = \left(\frac{T_0R}{\tau}\right)\left(\frac{10^9 \times 2{,}000^2}{\pi(10^7)^2}\right) = 12.7\frac{T_0R}{\tau} = 0.32\frac{T_0^3R}{P}$$

$$= 80\cdot\frac{R}{\tau}\sqrt{\frac{P}{\tau}} \tag{32}$$

$$S_I = 8\cdot\times 10^{14}\frac{R^2P}{C^3}$$

The voltage sensitivity or the deflection per microvolt applied to the circuit is

$$S_V = \frac{S_I}{R}$$

so

$$S_V^2 = 12.7\frac{T_0}{\tau R} = 0.32\frac{T_0^3}{PR} = \frac{80}{\tau R}\sqrt{\frac{P}{\tau}}$$

$$S_V = 8\cdot\times 10^{14}\frac{RP}{C^3} \tag{33}$$

In designing, the formulæ may of course be worked backward to find the values of P, τ, etc., corresponding to the conditions of the problem.

To illustrate the utility of (32) and (33), suppose that an instrument, already constructed, is to be used in a circuit having a fixed resistance and that the conditions are such that it is critically damped or dead beat but not sufficiently sensitive for the work in hand. There are two factors which may be altered to increase the sensitivity without using a new coil; the torsional control τ and the field strength H. If the field strength is increased, the ultimate deflection due to any current will be

increased in the same proportion but the instrument will no longer be dead beat; it will become sluggish in its action so that the time which must elapse before the reading can be taken is unduly increased. If τ is decreased the same is true, so *two* changes are necessary if the critical damping is to be preserved.

Suppose the restoring moment of the spring is reduced to one-sixteenth of its original value. Then by (32) and (33) both S_I and S_V will be increased eightfold, for T_0^3 is increased to 64 times its former value and both S_I and S_V are proportional to $\sqrt{T_0^3}$. In detail, by (30) if τ is reduced to one-sixteenth of its former value, in order to maintain critical damping C must be halved. As the coil is not to be altered, the quantity $C = Hbl$ must be halved by halving the strength of field H. If the field is halved and the restoring moment reduced to one-sixteenth of its first value, the sensitivity will become eight times its initial value.

The modification will render the instrument less prompt in its action, for the time of an undamped vibration T_0 will be increased to four times its original value.

Expression for the Field Required to Produce Critical Damping.[7]

$$H^2 = \frac{C^2}{l^2b^2} = \frac{TR_0\tau}{\pi l^2 b^2}.$$

Expressing R in ohms

$$H = \sqrt{\frac{10^9}{\pi}}\sqrt{\frac{TR_0\tau}{l^2b^2}} = \frac{17{,}800\cdot\sqrt{TR_0\tau}}{lb} = \frac{112{,}000}{lb}\sqrt{\frac{RP}{T_0}} \quad (34)$$

This relation can be transformed so that H is made to depend on the resistance of the galvanometer coil and on the lengths of the active and inactive wire.

The total moment of inertia of the moving parts is (referring to the table of symbols)

$$P = mn\left(v + \frac{h}{3}\right)b^2 + P'$$

$$R_G = \frac{\rho\,(v+h)}{a} = \frac{\rho\delta\,(v+h)}{m}$$

$$\therefore b^2 = \frac{P - P'}{mn\left(v + \frac{h}{3}\right)} = \frac{(P - P')\,R_G}{n\rho\delta\,(v+h)\,\left(v + \frac{h}{3}\right)}$$

and

$$H = \frac{112{,}000}{l}\sqrt{\frac{n\rho\delta(v + h)\left(v + \frac{h}{3}\right)}{(P - P')\,R_G}}\sqrt{\frac{RP}{T_0}}$$

$$= \frac{437}{l}\sqrt{n(v + h)\left(v + \frac{h}{3}\right)\left(\frac{P}{P - P'}\right)}\sqrt{\frac{R}{R_G T_0}} \qquad (35)$$

$$\doteq A\sqrt{\frac{R}{R_G T_0}} \qquad (35a)$$

The value of A is generally between 600 and 800, depending mainly on the breadth of the coil. The quantity n depends to a certain extent on the size of the wire and will probably lie between 1.4 and 1.6.

As the resistance of the circuit is increased, the practical application of the results of this discussion become more and more difficult, especially if a quick-working (short period) instrument is desired.

Auxiliary Damping.—If the galvanometer is to be used in circuits of varied resistances, the damping will vary to correspond, rarely being such that the instrument may be read quickly; in such cases recourse is frequently had to auxiliary magnetic dampers which are in effect closed loops of wire attached to the coil and swinging in the same field. A familiar example of this is the damping device used on direct-current ammeters and voltmeters, the coil being wound on an aluminum frame or bobbin.

Possible Adjustments.—An instrument not specifically designed for a given piece of work may sometimes be made more effective by special adjustment. The things which may be varied are:

1. The total resistance of the circuit. This may be adjusted by resistances in series or in shunt with the galvanometer, as necessary.

2. The field strength. This may be decreased by using a magnetic shunt or increased by using a second set of magnets in parallel with the original one.

3. The torsion constant τ. The suspension may be changed.

4. The damping, by the use of an auxiliary damping loop.

5. The moment of inertia P. This may be increased by placing weights on the movable element.

As the relation $\left(\frac{C^2}{R}\right)^2 = 4P\tau$, which is necessary for critical damping, is to be preserved, *two* changes will be required, the second to compensate for the effect of the first on the damping.

For example, suppose that it is desirable to increase the total resistance, R, of the circuit N times. If this is done, the instrument will become under-damped and the voltage sensitivity will be reduced N times. To restore the damping a damping loop may be added. This will increase T_0 somewhat, due to the increased moment of inertia. As indicated by the equations, other compensating changes are possible.

The Einthoven String Galvanometer[9].—In this instrument the movable element is either a very fine silvered quartz fiber

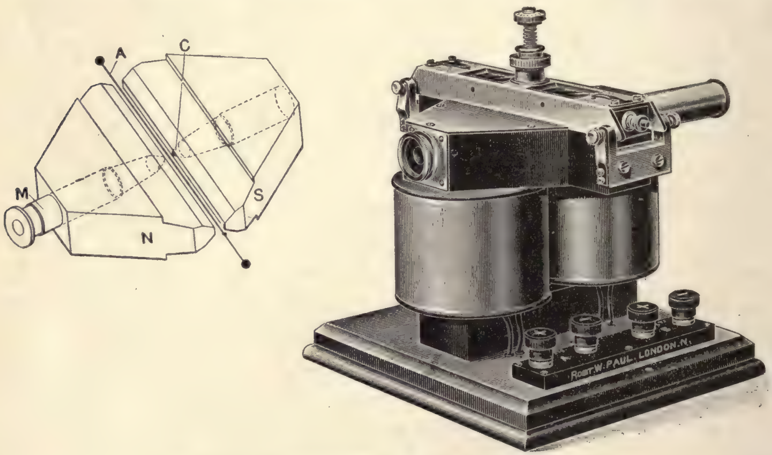


FIG. 17.—Einthoven string galvanometer.

about 8 or 10 cm. long, or a very fine wire. This is placed in the strong field due to an electromagnet and so mounted that the tension upon it may be varied.

When a current is sent through the fiber, it moves across the magnetic field. The motion is observed with a microscope having a micrometer eyepiece, a magnification of about 100 diameters being used. The deflections are proportional to the current.

Fig. 17 shows the essential features and one design of the complete instrument.

The resistance of the silvered quartz fiber, which is used in order to attain a sufficiently light "string," may vary between 2,000 and 10,000 ohms; with a silver wire about 0.02 mm. in diameter, the resistance is 4 or 5 ohms.

The electromagnet has a very massive core and the pole pieces are so shaped that they concentrate the field on the narrow air gap in which the "string" moves. The magnet is worked above saturation, consequently small variations of the exciting current have but little effect on the strength of the field in the air gap which is about 20,000 c.g.s. units.

The time of swing of the movable member of the Einthoven galvanometer is very short; it depends on the size and material of the "string" and the tension upon it. With a fine silvered quartz fiber having a diameter of from 0.002 to 0.003 mm., and under tension, it may be much less than 0.01 sec., while with a silver wire having a diameter of about 0.02 mm., it may approach 0.1 sec. when the tension is relaxed. If the tension on the silvered quartz fiber is very much relaxed, the instrument becomes unduly sluggish and as much as 10 sec. may elapse before the deflection is completed. The galvanometer is then exceedingly sensitive but the zero reading is likely to be unsteady and the fiber may move out of focus.

The advantages of this form of galvanometer are its extreme quickness of action and immunity from the effects of stray fields.

In a high-resistance circuit the damping is by air friction, but if the resistance be low and shunts are employed electromagnetic damping is also present. Einthoven has shown[10] that if a high resistance instrument is placed in series with an adjustable resistance and in parallel with a condenser and the whole combination shunted around another resistance, it is possible to adjust the combination so that the galvanometer is dead beat. The instrument thus becomes a low-period oscillograph, suitable for recording phenomena whose cycle is completed in a few tenths of a second.*

By using an arc lamp and the proper optical system, the deflection may be projected on a screen at about a meter's dis-

* The firm of Gans and Co. manufacture a regosular cillograph (without damping) based on the "string" principle.

tance. Cyclic phenomena are observed by the use of a revolving mirror, as is customary. Permanent records may be obtained photographically on a moving plate or film. A narrow slit is placed immediately in front of the photographic surface and behind a cylindrical lens so that the image of the fiber appears on the sensitized surface as a shadow, which prevents the exposure of the part of the surface on which it happens to fall.

This arrangement is used in physiological investigations.

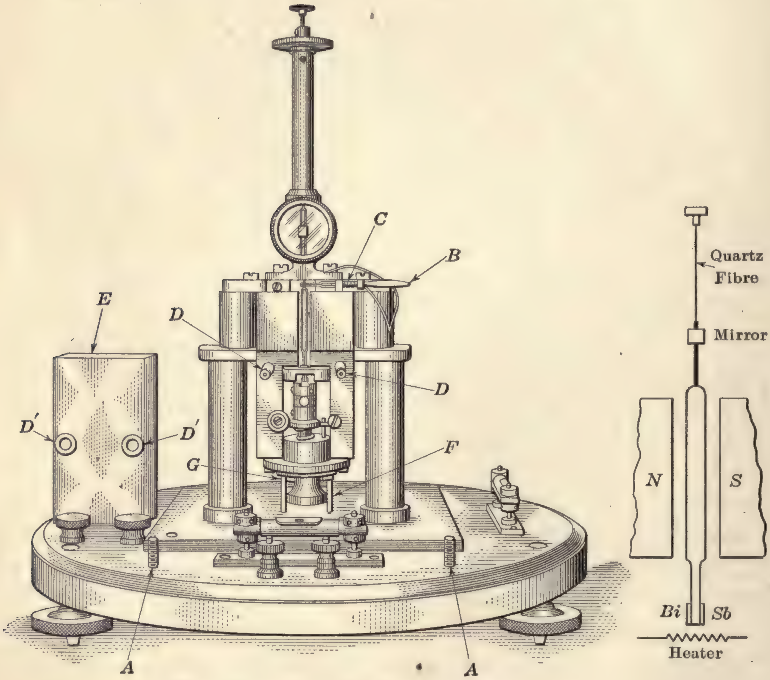


FIG. 18.—Duddell thermo-galvanometer.

The Duddell Thermo-galvanometer.—The essential features of this instrument, which is based on the radio-micrometer of C. Vernon Boys, are indicated in Fig. 18.

A single loop of silver wire having a high conductivity is suspended by a quartz fiber in the strong magnetic field due to a permanent magnet. At the lower end of the loop there is a bismuth-antimony thermo-couple and beneath this and as near

as possible without touching it, is a heater wire through which the current to be measured is sent. On the passage of the current the lower thermal junction is warmed by radiation and convection; consequently a direct current, due to the thermoelectric action, flows around the loop which is thus deflected. The heater filament is straight or else bent back and forth forming a grid. Its inductance is therefore very small and the instrument is consequently adapted for the measurement of alternating currents of high periodicity, the deflection, which is read by the mirror and scale method, being practically proportional to the mean square value of the current through the heater. The damping is due to currents induced in the loop as it moves in the field and the electrical constants are such that critical damping is attained. The period of the instrument is 3 or 4 seconds.

As with any thermo-electric device constancy of zero reading depends on uniformity of temperature. Sudden fluctuations in room temperature should be avoided. Slow variations which give time for the temperatures of the hot and cold junctions to equalize are not nearly as important. To assist in maintaining constant temperature conditions, the working parts of the instrument are enclosed in a heavy gun-metal case, the front of which, *E*, may be removed when it is necessary to inspect or adjust the instrument. The zero should be read after each observation.

Interchangeable heaters are used. They are of various resistances depending on the sensitivity required. Those having a resistance below 4 ohms are made of wire, while those above this value consist of a deposit of platinum on quartz, made into the form of a grid.

The sensitivity attained, as given by the makers of the instruments, the Cambridge Scientific Instrument Co., is shown in the following table.

The instrument may be calibrated with direct currents and then used on alternating-current circuits. It is more sensitive than the electrodynamometer, not subject to errors due to inductance or capacity, and at high frequencies does not disturb the circuit conditions as much as the dynamometer.

The sensitivity may be controlled to a certain extent by adjusting the proximity of the heater to the hot junction. This is done by turning the ebonite milled head, *F*. Great care is

necessary in manipulating the instrument not to injure the delicate loop or the thermo-couple.

TABLE OF APPROXIMATE SENSITIVITIES OF THERMO-GALVANOMETERS. SCALE DISTANCE 1,000 mm.

Resistance of heater, ohms	Current to give 250 mm. deflection, microamperes	Current to give 10 mm. deflection, microamperes	P. D. to give 250 mm. deflection, millivolts	P. D. to give 10 mm. deflection, millivolts	
About 1,000	110	22	110.0	22.0	Heater close to junction
About 100	350	70	35.0	7.0	
About 10	1,100	220	11.0	2.2	
About 4	1,750	350	7.0	1.4	
About 1	3,500	700	3.5	0.7	
About 1	10,000	2,000	10.0	2.0	Heater lowered away from junction

The same principle is applied in the Duddell thermo-ammeter (see page 60).

POINTS TO BE CONSIDERED WHEN SELECTING A GALVANOMETER

A galvanometer must be selected with special reference to the work to be done, for no instrument is equally useful under all sorts of conditions. Among the points to be considered are the following.

Sensitivity.—The sensitivity should be sufficient for the work in hand so that measurements may be made without undue fatigue and loss of time. On the other hand, a much higher sensitivity is not an advantage for it means a more delicate instrument and therefore one more liable to injury and more difficult to manipulate. Also, high sensitivity may mean an unduly long period of vibration and a subsequent loss of time in making measurements. Sensitivity, though important, should not be the only thing considered in estimating the utility of a galvanometer.

Period.—The time of vibration of the movable system should be short so that the instrument will respond quickly to the current.

Damping.—*When the instrument is in use,* the system should, if possible, be critically damped. This will economize time.

In many cases the damping is not an inherent property of the galvanometer but depends both on the instrument and on the circuit to which it is attached (see page 40).

Resistance.—The resistance should be appropriate for the measurement in hand so that the maximum sensibility of method may be obtained.

Freedom from Effects of Mechanical Disturbances.—Great care should be exercised in making and mounting the movable system so that symmetry about the axis of rotation is attained; this contributes much to the stability of the system when it is subjected to mechanical disturbances. Choice of location for the instrument and the method of setting up are important.

Freedom from Stray-field Effects.—It is essential that the indications be uninfluenced by the unavoidable variations of the local field.

Definiteness of Zero Reading.—The zero reading should be definite and the deflections should come promptly to their final values with no viscous action of the controlling spring.

Law of Deflection.—Throughout its useful range the deflection as read from the scale should be proportional to the current.

Visibility of Suspended Parts.—When the instrument is set up and ready for use, it should be possible to see the movable parts and to satisfy oneself that the clearances are properly adjusted.

Accessibility for Repairs.—It should be possible to take out easily the entire movable system with its suspension.

Temperature Effects.—The effect of temperature on the sensitivity should be small and inequalities of temperature should not set up thermo-electric currents in the galvanometer circuit.

Optical System.—The definition obtained by the optical system used in reading the deflections should be so perfect that readings to the limit of accuracy of the instrument may be obtained without undue fatigue.

THE JULIUS DEVICE FOR ELIMINATING THE EFFECTS OF MECHANICAL DISTURBANCES ON GALVANOMETERS[11]

The object to be attained is the suspension of the movable system of the instrument from a point which is practically stationary.

The dimensions of the apparatus which are given below have been found satisfactory.

In the arrangement there are two non-magnetic rings 13 in. in diameter, 1 in. wide and $\frac{1}{2}$ in. thick. These rings slide on three brass rods, $\frac{1}{2}$ in. in diameter and 30 in. long, and can be clamped by set screws at any desired position. Between the rings is a movable platform to which the galvanometer can be firmly attached.

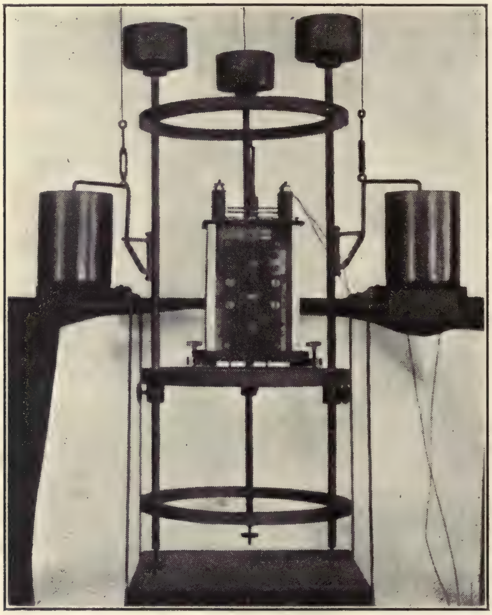


FIG. 19.—Julius suspension.

The arrangement is hung by three parallel steel wires, about No. 18 B. & S. gage, from a three-armed support, attached by a single lag screw to a bracket. The points of attachment of the three wires are stout hooks fastened to the brass rods so that the neck of the hook is $12\frac{3}{4}$ in. from the upper end of the rod. Attached to the hooks are damping vanes 3 by 4 in., dipped in jars filled with oil; the centers of these vanes are at the height of the necks of the hooks. Above the upper rings are three iron weights of 6 lb. each, which slide on the rods and can be clamped

at any desired height by means of set screws. The lower ends of these rods are provided with levelling screws. The weight of the arrangement is about 45 pounds.

To make the necessary adjustments the device is levelled and the galvanometer put in place, levelled and firmly clamped in position. The axis of the suspended system should be in the vertical axis of the arrangement, for symmetry is important.

The platform is now raised until the point of attachment of the suspension fiber to the frame of the instrument is in the plane passing through the necks of the hooks. It is then clamped in position.

The whole device, galvanometer and all, (it may be necessary to remove the suspended system) is now hung by one of the hooks and the weights adjusted until the rods which are normally in a vertical position are truly horizontal; the weights should be at equal distances from the upper ends of the rods. These adjustments insure that the point of attachment of the fiber and the center of gravity of the whole arrangement are at the same point and in the plane of support.

The device may now be put in position, the three suspension wires, of equal length, attached to the hooks, and levelling screws raised, leaving the arrangement freely suspended. It is well, for convenience of adjustment, to attach the wires to the frame at their lower ends by small turn-buckles. Complete shielding from draughts is essential. To prevent serious results arising from the breaking of the suspension wires a shelf should be placed immediately below the device.

SHUNTS

In using galvanometers, it is often found either that the instruments are too sensitive or that their carrying capacities are insufficient. In such cases, shunts placed between the terminals of the galvanometer and acting as bypasses for the current, are employed (see Fig. 20).

When zero methods are used, shunts are resorted to for the purpose of protecting the galvanometers during preliminary adjustments. Much time is thus saved, for the violence of the deflection and consequently the time necessary for the needle to

come to rest are reduced. A familiar example of the use of shunts to extend the range of galvanometers is found in direct current, moving coil ammeters.

Where it is desired to compute the total current in a circuit from the indication of a shunted galvanometer, an exact knowledge of the resistance of both shunt and galvanometer at the time of use is necessary. Attention must be given to possible sources of error, such as defective contacts and changes of resistance due to temperature.

Let I = line current.
I_G = galvanometer current.
I_S = shunt current.
R_G = resistance of galvanometer.
S = resistance of shunt.

Then

$$I = I_S + I_G = I_G \left(\frac{R_G + S}{S}\right).$$

$\left(\frac{R_G + S}{S}\right)$ is called the multiplying power of the shunt. The ordinary arrangement of a shunt box for use with a reflecting galvanometer is shown in Fig. 20. By changing the position of the plug, definite portions, usually $\frac{1}{10}$, $\frac{1}{100}$, or $\frac{1}{1{,}000}$ of the total current can be sent through the galvanometer, which may be short-circuited by placing the plug in the last hole to the right.

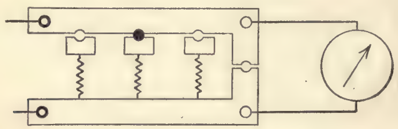


FIG. 20.—Diagram for ordinary shunt box.

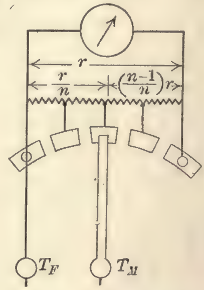


FIG. 21.—Diagram for universal shunt box.

The Ayrton-Mather Universal Shunt.[12]—The universal shunt is shown diagrammatically in Fig. 21. A high resistance, of r ohms, is permanently connected across the galvanometer terminals, one of which, T_F, is permanently connected to the external circuit. By the proper arrangements, the other lead

from the external circuit, T_M, may be connected at will to points on r which are commonly distant r, $\frac{r}{10}$, $\frac{r}{100}$, etc., from the fixed terminal. Referring to the figure, it will be seen that the line current is given by

$$I = I_G \frac{R_G + \left(\frac{n-1}{n}\right)r + \frac{r}{n}}{\frac{r}{n}} = I_G \frac{(R_G + r)n}{r}.$$

For any particular galvanometer and shunt box $\left(\frac{R_G + r}{r}\right)$ is constant. This factor is the multiplying power of the shunt when the movable terminal is at the extreme right of r.

It is seen that the *relative* multiplying power is n. The values of n depend on the locations of the taps by which the movable terminal, T_M is connected to r. They are independent of the relative magnitudes of the resistances of the galvanometer and the shunt. The box is graduated in terms of the relative multiplying powers; consequently it may be used with any galvanometer: hence the name universal. However, though the multiplying powers are not affected, the same shunt box cannot be applied indiscriminately to all galvanometers and satisfactory results attained. For the maximum current through the instrument is $I_G = I \frac{r}{R_G + r}$. Therefore, in order that practically the full sensitivity of the galvanometer may be realized, r must be much larger than R_G; if it is nine times the galvanometer resistance, 90 per cent. of the sensitivity may be realized. Again, if r is too small, and a moving coil galvanometer is employed, the instrument will be over-damped and, therefore, sluggish in its action.

The distinct advantage of the universal shunt box is that when it is used in *open-circuit work*, any damping due to currents set up by the motion of either the needle or the movable coil of the galvanometer is constant. This is especially important when capacities are being compared by means of the ballistic galvanometer.

Also, when a universal shunt is used with a moving-coil galvanometer in a circuit of very high resistance, it is possible, by properly choosing r, to render the galvanometer dead beat for all

values of the multiplying power. Such a case arises when insulation resistances are being measured.

AMMETERS

An ammeter, in distinction from a galvanometer, is an instrument so constructed that the current strength in amperes can be read directly.

Before referring to various designs of these instruments, it will be well to refer to certain considerations which have influenced the development of indicating electrical instruments, especially for direct-current work.

1. The resistance of all current-measuring instruments should be very low, while that of all instruments for measuring voltages should be as high as practicable, the reason in both cases being that the disturbance of the circuit conditions by the insertion of the instrument must be reduced to a minimum. Another way of stating the same thing is that the energy dissipated in the instrument must be a minimum.

2. The construction must be such that the instrument will maintain its reliability. The relative positions of the parts must be maintained in spite of rough handling and the strength of all magnets used must be insured by proper ageing.

3. There must be no "set" of the controlling springs due to standing under load and no indefiniteness of the zero reading due to magnetic impurities in the movable coils.

4. The indications of the instrument must be independent of stray fields. The importance of this in industrial testing cannot be over-emphasized.

5. The indications must be independent of room temperature and no errors must result from the heating due to the passage of the current.

6. All shunts must be free from errors due to thermo-electromotive forces.

7. Ammeter shunts must be so constructed that they will not be injured by abnormal currents of short duration and ample provision must be made for dissipating the heat due to continuous operation.

8. There must be no effects due to the retentiveness of any

soft iron parts, for this causes the indications of the direct-current instruments to depend on their previous history.

9. Pivot friction must be reduced to a minimum and the moving system properly balanced.

10. The instrument must be dead beat, that is, critically damped, in order that fluctuations of the load may be followed with certainty and that the time necessary for taking readings may be reduced to a minimum.

11. The graduation should be convenient. This reduces the liability to mistakes in readings which have to be taken hurriedly.

Moving-coil Ammeters.—In direct-current instruments, the fulfilment of the conditions stated above is most readily obtained by employing the moving-coil principle.

The first thoroughly practical instrument of this class was designed by Edward Weston in 1888. It will be described, in its present form, as a typical example of a moving-coil ammeter.

Weston Standard Portable Ammeter.—This instrument is essentially a shunted D'Arsonval galvanometer, so designed that it fulfils the requirements of portability and general reliability.

The magnet, which is of the horseshoe type, is made of tungsten steel and is artificially aged; the cross-section is about 1.25 by 0.3 in. Carefully shaped soft-iron pole pieces are attached to the magnet by screws so that the space between them is cylindrical. In this space is placed a soft iron cylinder supported from a brass yoke attached to the pole pieces. The air gap is about 0.04 in. wide; consequently, the coil moves in a radial field (see page 35). The movable coil, of copper, is wound on an aluminum frame which also serves as a damping device to make the instrument dead beat. The movable system is provided with steel pivots which turn in two jewelled (sapphire) bearings which are carried by non-magnetic yokes attached to the pole pieces in such a manner that the coil is truly centered. The directive force is given by two flat spiral springs, one above and one below the coil; they are made of non-magnetic material and also serve as leads to the movable coil. The inner ends of the springs are attached to brass collars which form the terminals of the coil, the outer ends to the extremities of two insulated crossarms which can be moved coaxially with the coil, to adjust the zero. In the more recent instruments this adjustment may

be made without opening the case, by turning a slotted head at the front of the base. The pointer is an aluminum tube flattened at the index end. The moving parts are balanced by three adjustable counterweights on short arms which project at right angles to the axis of rotation and are in the plane of motion of the pointer; adjustment is made by moving the weights along the arms. Parallax is eliminated by the use of a mirror beneath the pointer.

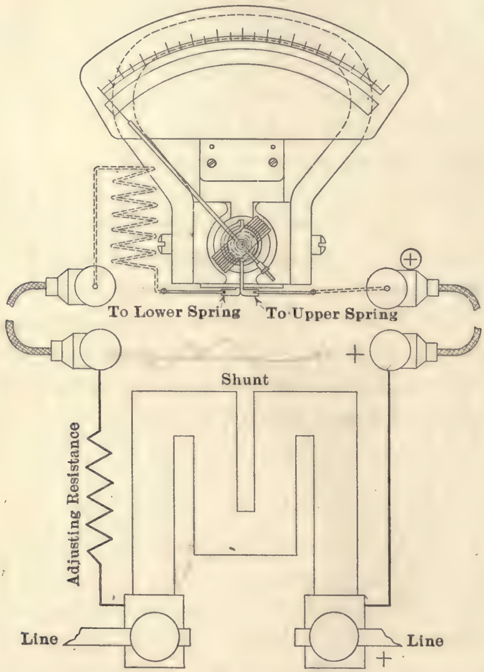


FIG. 22.—Diagram for Weston moving-coil ammeter.

The graduation of the scale is practically uniform, but no particular law of deflection is assumed. The principal points are determined by comparison with a standard instrument and the subdivision is done by a dividing engine.

A small resistance coil is included in the galvanometer circuit, and the final adjustment is made by altering its resistance.

For self-contained portable instruments, the present practice of the Weston Instrument Co. is to use in milliammeters, up to

1,500 milliamp., a drop of approximately 150 millivolts at full-scale deflection. In ammeters having ranges from 2 to 150 amp., the drop is about 50 millivolts; for ranges from 200 to 500 amp., it is 35 millivolts. In all external shunts of the new type the drop is approximately 100 millivolts and in the switchboard shunts it is about 50 millivolts.

In self-contained instruments the shunts are mounted in the base which, if the current capacity is considerable, is properly ventilated. For large currents the best procedure is to separate the millivoltmeter and the shunt and mount them in different cases; this allows the shunt to be designed so that the heat is readily dissipated. An external-shunt instrument gives a flexible arrangement very convenient for general testing, for shunts of different ranges may be used with the same millivoltmeter.

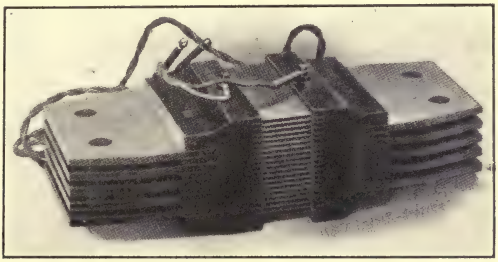


FIG. 23.—Switchboard shunt.

In using any form of shunt and millivoltmeter, it is necessary to calibrate and to use the instrument with the same set of leads connecting the shunt and the millivoltmeter and to avoid all extraneous resistances in the leads due to imperfect contacts at the terminals.

The moving-coil principle is now universally employed in the best makes of direct-current instruments. Separate millivoltmeters and shunts are universally used in direct-current switchboard work. The shunts are put at any convenient point in the busbars and small leads are run to the indicating part which is on the front of the switchboard. This greatly simplifies the construction of the board and reduces expense.

Fig. 23 shows a switchboard shunt. It will be noted that the resistance strips are very short and are soldered into massive

terminal blocks which can be interleaved with the busbar. The heat is thus disposed of by conduction as well as by convection.

Reference should be had to the introduction to the chapter on the "Calibration of Instruments" where various errors found in commercial ammeters, voltmeters, etc., are discussed.

Thermo-ammeters.—The rise of temperature of a wire carrying a current is a function of the current strength; it may be utilized for purposes of measurement. Various forms of indicators have been devised. They utilize the change of electrical resistance of the wire, its expansion, or its rise of temperature.

For alternating-current work the hot-wire or thermal principle possesses certain theoretical advantages, due to the fact that there are neither coils nor soft iron in the instruments; hence, inductance effects are reduced to a minimum and saturation effects eliminated altogether. With proper design such instruments should then be applicable to both direct- and alternating-current circuits and their indications should be independent of frequency, wave form, and stray-field effects.

The practical difficulties met with are due to the uncertainty of the zero reading, to the sluggishness of action due to the heat capacity of the various parts of the instrument and to the influence of room temperature. Also, as the carrying capacity of the hot wire is seriously taxed, there is the liability of burning out the instrument by a temporary overload or short-circuit, which in the ordinary type of instrument would result in nothing worse than a bent pointer. Usually the energy consumption in this class of instruments is large.

All things considered, hot-wire instruments are unsuited for switchboard work. Their particular field of usefulness is in high-frequency work, such as radio-telegraphy or in the laboratory where they are employed as "crossover" instruments between alternating and direct currents in calibration work.

At ordinary frequencies shunts may be employed, but for switchboard work no great advantage results from this, for in American practice all alternating-current instruments on circuits of above 500 volts are actuated through transformers, thus keeping the front of the switchboard free from high-voltage circuits which would be a source of danger.

The earliest commercial instrument based on the thermal or

hot-wire principle was the voltmeter invented by Major Cardew, R. E. In this instrument the expansion of a long platinum wire due to the passage of the current caused the pointer to move over the scale. The more recent form of hot-wire instrument made by Hartmann and Braun is shown in Fig. 24. Its distinguishing feature is the exceedingly ingenious method of multiplication by which the small expansion of a short wire is caused to produce a large deflection of the pointer, without the use of levers, knife edges, or gears.

A and *B* are lugs carried by a composite metal frame, the length of the iron and brass parts being such that the frame is designed

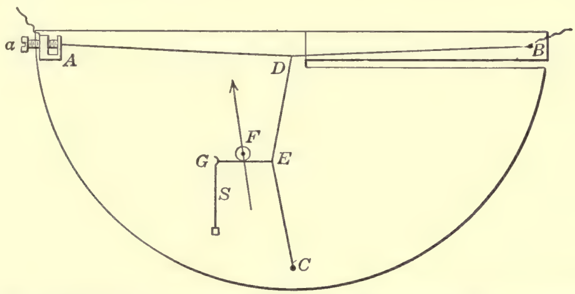


FIG. 24.—Diagram for Hartmann and Braun hot-wire ammeter.

to have the same coefficient of expansion as the wire *AB*. The lug *C* is carried by a portion of the frame having the same coefficient of expansion as the wire *DC*. An inextensible cord, *EG*, passes once around the drum, *F*, to which the pointer is attached, and is drawn taut by the spring *S*.

The current flows through the wire *AB*, which is heated and expands; the slack is taken up by the spring and the pointer is moved over the scale. The zero reading may be adjusted by the screw *a*.

The wire *AB* is of platinum-iridium. This has a smaller coefficient of expansion but a higher melting point than the platinum-silver wire formerly used. It may thus be worked at a higher temperature and gives less trouble from variations of room temperature.

For work at ordinary frequencies the range of the indicator is extended by the use of shunts. This is of importance in labora-

tory work, for the shunted instrument may be calibrated with direct and used with alternating currents.

To increase the current capacity of this form of indicator the wire AB may be sectionalized, as indicated in Fig. 25.

The connections to AB at 1, 2, 3, are made with thin strips of silver foil so that the motion is not impeded.

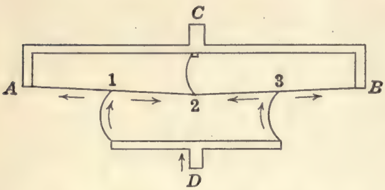


FIG. 25.—Sectionalized wire for hot-wire ammeter.

The current capacity of the indicator is about 5 amp., the resistance about 0.05 ohm, consequently the full-load drop is approximately 250 millivolts, or about three or four times that in the ordinary direct-current switchboard shunt.

As the resistance is so low, it is essential that all connections between the indicator and the shunt be carefully made and that the same set of leads connecting the indicator and the shunt be used during calibration and subsequent use of the ammeter.

The Duddell Thermo-ammeter.—The working parts of this modification of the thermo-galvanometer (see page 46) are

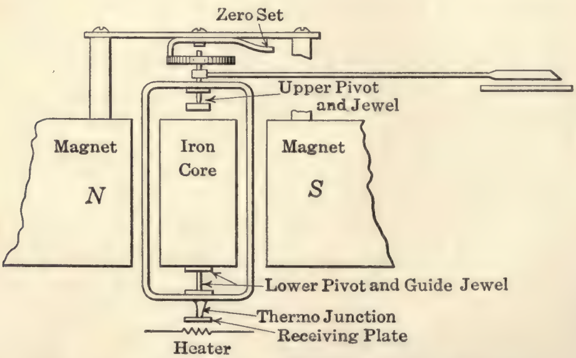


FIG. 26.—Diagram for Duddell thermo-ammeter.

shown in Fig. 26. The movable coil is so mounted that it is practically supported from the upper pivot, the lower pivot acting largely as a guide. Pivot friction is thus minimized.

The instrument is primarily designed for measurements at

high periodicity, such as occur in telephonic work or in wireless telegraphy.

The standard resistances of the heaters are 2 and 150 ohms, the latter for telephonic work. A full-scale deflection can be obtained with a current of about 10 milliamp. Consequently the power taken by the instrument is about 0.015 watts at full-scale reading.

To obtain a compact heater for use with currents below 20 milliamp. a deposit of platinum on mica is used, the platinum being scraped away to form a grid. A resistance of several hundred ohms may thus be obtained in a space of less than 0.2 sq. cm. For larger currents a wire heater is used. In either case the self-induction and capacity are so small as to be negligible in their effects.

High-frequency Ammeters.[13]—By proper design the thermo-ammeter may be adapted to the measurement of large high-frequency currents, such as occur in radio-telegraphy.

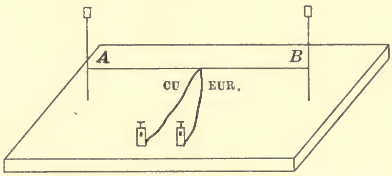


FIG. 27.—High-frequency ammeter for small currents.

An instrument for *small* high-frequency currents is shown in Fig. 27. The entire current flows through the wire AB, which for currents up to 0.3 amp. may be a Eureka wire 0.05 mm. in diameter. The skin effect in this wire at 1,000,000 cycles is less than 0.001 per cent. For currents up to 1.2 amp. a copper wire 0.08 mm. in diameter is used. In this the skin effect at 1,000,000 cycles is less than 0.3 per cent. The temperature of the wire is determined by a thermo-electric junction. The indicator may be a moving coil galvanometer connected between the binding posts.

Instruments for small currents present no difficulties. The trouble arises when it is necessary to have a large carrying capacity. In this case, two or more wires must be used in parallel and the difficulty comes from the fact that the current may not divide properly between the wires.

The heating in all the wires should contribute to the functioning of the indicator. Then the errors due to the improper distribution of the current are much decreased, for though the total heat

production changes with the distribution of the current, the change is less than the change in the current distribution itself. The reading of any ammeter which depends on the whole heat production will either remain constant or increase as the frequency is increased.

A two-wire instrument where the total heat production is utilized is shown diagrammatically in Fig. 28.

The high-frequency current is taken in through the leads *Aa* and *Bb* which are perpendicular to the active wires *ACB* and *ADB*, which are of copper and 0.1 mm. in diameter.

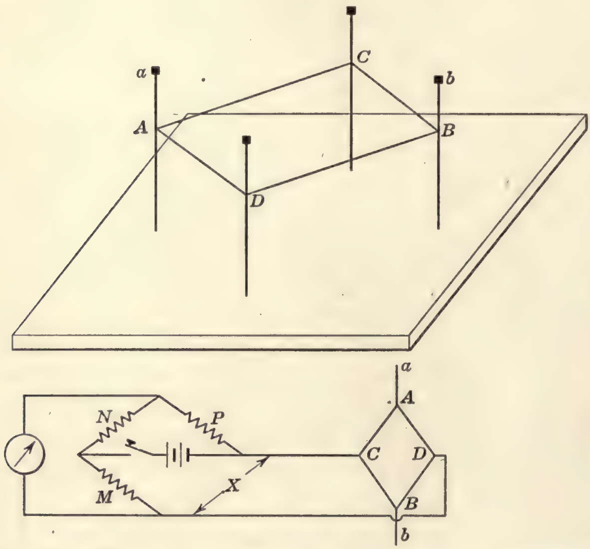


FIG. 28.—Two-wire ammeter for high-frequency currents.

The points *C* and *D* are located so that the arrangement is as nearly as possible a Wheatstone bridge. The high-frequency current is thus kept out of the auxiliary bridge which is used for measuring the resistance. As the location of *C* and *D* may not be exact, it is preferable to use a low-frequency alternating current instead of direct current in calibrating the arrangement. In order to eliminate the effects of thermal e.m.fs. the galvanometer is used on closed circuit. This arrangement when immersed in oil has a capacity of 10 amp.

The Parallel-wire Ammeter.—In this form of instrument the conductor is a group of several straight wires of the same length and diameter and so fine that changes of resistance due to skin effect are negligible; they are parallel in direction and usually are equally spaced.

A typical arrangement of this sort is shown in Fig. 29. The current is led in through the heavy terminals which are perpendicular to the active wires and therefore exercise no inductive effect on them. The distributing terminals at the ends of the parallel wires have a negligible impedance.

In a perfect instrument all the wires will have the same resistance and a direct or low-frequency current will divide equally between them, all inductance effects being negligible. At very high frequencies the distribution of current between the wires is determined by the inductances, self and mutual, rather than by the resistances, so that it may be very different from the direct-current distribution; that is, the current distribution may be a function of the frequency.

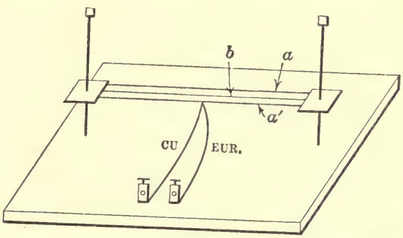


FIG. 29.—Parallel-wire high-frequency ammeter.

It is essential that the wires be of uniform resistance. Lack of uniformity may be due to variation in hardness or small variations in the cross-section. These things will not affect the inductances, so the wires may carry equal currents at high frequencies but very different currents at low frequencies where the resistance effects preponderate.

If the indicator is attached to one wire, as in Fig. 29, the magnitude of the error will depend on which wire is used. It will be decreased if the arrangement is such that all the wires contribute to the functioning of the indicator, for if the current in one wire is decreased, that in the others must correspondingly increase and the change in the total heat production is much less than that in any one wire. Dellinger cites the case of a seven-wire instrument, the indicator of which was operated by one of the wires somewhat distant from the others. At 100,000 cycles

it gave readings 10 per cent. high, and at 750,000 cycles, 46 per cent. high (see page 67).

The explanation of the distribution errors will be seen by examining the theory of the three-wire instrument shown in Fig. 29, where the arrangement is such that these errors are pronounced.

Symbols Used and the Data for a Particular Case

l = length of wires, 10.00 cm.
δ = diameter of wires, 0.008 cm.
d = distance between wires, 0.40 cm.
R_a = mean resistance of a and a', 0.347 ohm
R_b = resistance of b, 0.352 ohm.
L = coefficient of self-induction of one wire, 155.00 cm.
M_{ab}, $M_{a'b}$, $M_{aa'}$ = coefficients of mutual induction.
M_{ab} = $M_{a'b}$ by symmetry.
v = instantaneous potential difference between the ends of the wires.
i = total instantaneous current.
i_a, $i_{a'}$, i_b = instantaneous currents in the wires.

It will be necessary to calculate the self-inductance of the straight wires by the approximate formula

$$L = 2l\left\{\log_\epsilon \frac{4l}{\delta} - 0.75\right\}.$$

The mutual inductance will be given by

$$M = 2l\left\{\log_\epsilon \frac{2l}{d} - 1 + \frac{d}{l}\right\}.$$

$$L = 20\left\{\log_\epsilon \frac{40}{0.008} - 0.75\right\} = 155 \text{ cm.}$$

$$M_{ab} = 20\left\{\log_\epsilon \frac{20}{0.4} - 1 + \frac{0.4}{10}\right\} = 59.0$$

$$M_{aa'} = 20\left\{\log_\epsilon \frac{20}{0.8} - 1 + \frac{0.8}{10}\right\} = 46.0$$

The potential difference between the ends of the wires will be

$$v = R_a i_a + L\frac{di_a}{at} + M_{ab}\frac{di_b}{at} + M_{a'a}\frac{di_{a'}}{at}$$

$$= R_b i_b + L\frac{di_b}{dt} + M_{ab}\frac{di_a}{dt} + M_{a'b}\frac{di_{a'}}{dt}.$$

If $R_a = R_{a'}$, then by symmetry $i_a = i_{a'}$ and $i_b = i - 2i_a$.

Making this substitution and arranging terms

$$(R_a + 2R_b)i_a + (3L + M_{aa'} - 4M_{ab})\frac{di_a}{dt} = R_b i + (L - M_{ab})\frac{di}{dt}$$

Assuming sinusoidal currents

$$(R_a + 2R_b)\dot{I}_a + j\omega(3L + M_{aa'} - 4M_{ab})\dot{I}_a = R_b\dot{I} + j\omega(L - M_{ab})\dot{I} \qquad (36)$$

$$\therefore \{(R_a + 2R_b)^2 + \omega^2(3L + M_{aa'} - 4M_{ab})^2\}I_a^2 = \{R_b^2 + \omega^2(L - M_{ab})^2\}I^2$$

Solving for $I^2{}_b$

$$\{(R_a+2R_b)^2+\omega^2(3L+M_{aa'}-4M_{ab})^2\}I_b^2 = \{R_a^2+\omega^2(L+M_{aa'}-2M_{ab})^2\}I^2 \qquad (37)$$

At low frequencies, denoted by the subscript 0, where the inductance effects are negligible,

$$\frac{(I)_0}{(I_a)_0} = 2 + \frac{R_a}{R_b} = 2.986 \qquad (36a)$$

and

$$\frac{(I)_0}{(I_b)_0} = 1 + 2\frac{R_b}{R_a} = 3.029 \qquad (37a)$$

$$\frac{(I_a)_0}{(I_b)_0} = \frac{R_b}{R_a} = 1.014$$

At frequencies so high that the resistance effects are swamped by those of inductance, these ratios become

$$\frac{I}{I_a} = \frac{3L + M_{aa'} - 4M_{ab}}{L - M_{ab}} = 2.864 \qquad (38)$$

$$\frac{I}{I_b} = \frac{3L + M_{aa'} - 4M_{ab}}{L + M_{aa'} - 2M_{ab}} = 3.312 \qquad (39)$$

and

$$\frac{I_a}{I_b} = \frac{L - M_{ab}}{L + M_{aa'} - 2M_{ab}} = 1.156 \qquad (40)$$

From the data given, at any frequency, f, by (36) and (37)

$$\sqrt{\frac{I_a^2}{I^2}} = \sqrt{\frac{0.124 + 0.363 \times 10^{-12}f^2}{1.105 + 2.98 \times 10^{-12}f^2}} \qquad (41)$$

$$\sqrt{\frac{I_b^2}{I^2}} = \sqrt{\frac{0.120 + 0.272 \times 10^{-12}f^2}{1.105 + 2.98 \times 10^{-12}f^2}} \qquad (42)$$

Combining (41) and (36*a*) and (42) and (37*a*) and assuming that the same total current flows, $I_0 = I$,

$$\frac{I_a}{(I_a)_0} = 2.986\left(\sqrt{\frac{I_a^2}{I^2}}\right) \tag{43}$$

$$\frac{I_b}{(I_b)_0} = 3.029\left(\sqrt{\frac{I_b^2}{I^2}}\right) \tag{44}$$

The results obtained by using these formulæ are compared with the experimental results in the following table. The agreement is as good as the experimental accuracy warrants.

Table Showing Distribution in a Three-wire High-frequency Ammeter. All Three Wires in Same Plane

Frequency	Per cent. increase of current in (*a*)		Per cent. decrease of current in (*b*)	
	Calculated	Observed	Calculated	Observed
	Per cent.	Per cent.	Per cent.	Per cent.
150,000	0.3	0.4	0.6	0.2
500,000	1.8	1.3	3.4	3.0
1,000,000	3.1	2.8	6.1	5.3
1,500,000	3.9	3.6	7.4	6.0
∞	4.3	....	8.5	

From the table it is seen that the changes in distribution are practically confined to the frequencies between 100,000 and 1,500,000. That is, the range of frequencies in which the changes of distribution occur is that used in radio-telegraphy. Extreme effects of change in current distribution are shown in Fig. 30, which applies to the seven-wire arrangement there shown.

The Use of High Resistance Wires.—The distribution errors may be minimized by using wires of high resistivity, keeping their lengths and diameters, and therefore their inductances, the same. For example, suppose all the resistances in equation (43) were increased to 30 times their original values, then

$$\frac{I_a}{(I_a)_0} = \left(\sqrt{\frac{111.6 + 0.363 \times 10^{-12} \times f.^2}{994.5 + 2.98 \times 10^{-12} \times f.^2}}\right)(2.986).$$

Practically, the value of the radical is now determined by the

resistance terms, the variable or reactance term being almost negligible for the range of frequencies used in radio-telegraphy. At 1,000,000 cycles

$$\frac{I_a}{(I_a)_0} = 1.0005$$

instead of the value $\frac{I_a}{(I_a)_0} = 1.032$ which obtains when low resistance wires are used.

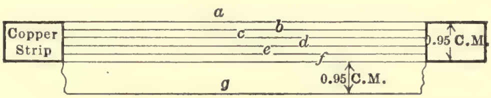


DIAGRAM FOR A SEVEN WIRE INSTRUMENT

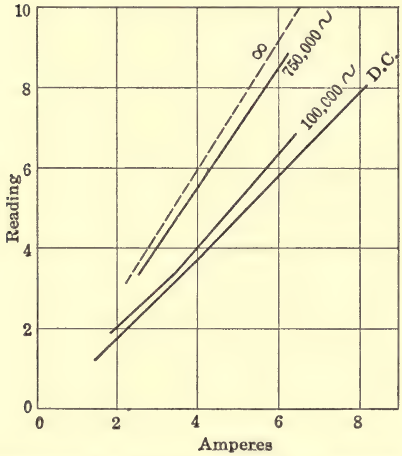


FIG. 30.—Plot showing distribution errors in a seven-wire high-frequency ammeter. All seven wires in the same plane.

The Utilization of the Whole Heat Production.—It has been stated that it is important that the whole heat production contribute to the operation of the indicator. This may be illustrated by reference to the three-wire instrument. Assume that with direct currents, 10 amp. flows in each of the wires; assuming them to be of equal resistance, the total heat production will be proportional to $10^2 + 10^2 + 10^2 = 300$. At 1,000,000 cycles the currents will be approximately 10.3, 9.4, 10.3, and the total heat

production will be proportional to $(10.3)^2 + (9.4)^2 + (10.3)^2 = 300.6$, a change of 0.2 per cent. But the change in heat production in wire a is from 100 to 106.1 or over 6 per cent. and that in wire b is from 100 to 88.4, a change of over 11 per cent. To take advantage of this total heat production the thermo-electric arrangement may be altered.

A fine Eureka wire may be soldered between a and b and a copper wire between b and a'. The lead from T_1 to a is of copper, that from T_2 to a' of Eureka wire.

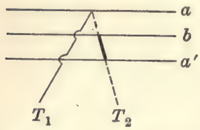


FIG. 31.—Thermo-function arrangement for utilizing whole heat production in a high-frequency ammeter.

Sectionalized Wires.—High-frequency ammeters in which the carrying capacity is increased by sectionalizing the wire, as shown in Fig. 25, may, contrary to general belief, show distribution changes due to the self and mutual inductions of the various parts. The location of the leads A and B plays an important part in these changes; they should be symmetrically placed. However, as all the sections of the wire contribute to the deflection, that is, as the indication depends on the total heat production, the resultant error is small though appreciable. Wires of high resistance will eliminate the trouble.

Use of Strips.—A favorite method of obtaining large current capacity is to employ a thin strip, of high resistivity, soldered

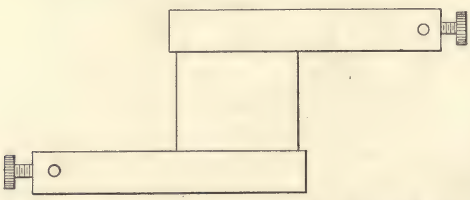


FIG. 32.—Roller's arrangement of terminals for high-frequency ammeter.

between massive terminal blocks. With a high-resistance strip the errors in this class of instruments are due to the effects of the terminal blocks on the current distribution, the inductances of the current paths being the determining factors. These effects are avoided by the arrangement suggested by F. W. Roller, shown in Fig. 32. Each part of the strip has about the same

amount of the terminal rod in series with it. Up to a frequency of 750,000 cycles no distribution error could be detected.

A theoretically perfect arrangement of the terminal blocks is that shown in Fig. 33. Everything is symmetrical about a central axis along which the current enters and leaves.

The conductors are thin strips or wires soldered between the terminal blocks; this makes the mutual inductance of each wire with respect to the others the same. Of course the self-inductances of all the wires will be equal.

This symmetry insures that high-frequency currents will divide equally among the wires. The practical difficulty is to

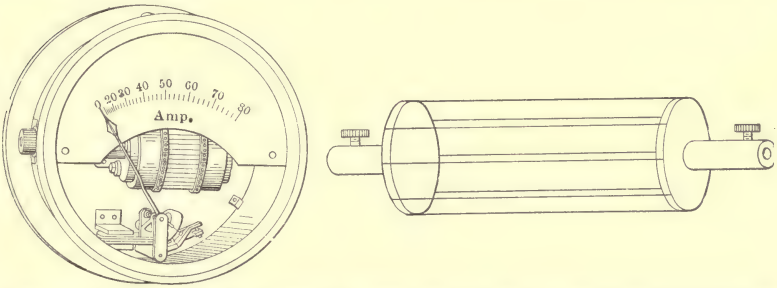


Fig. 33.—Arrangement of parallel-wire conductor for avoiding distribution errors in high-frequency ammeters.

get all the fine wires of the same resistance. If this is not done, there will be changes of distribution in passing from low to high frequencies. This construction is used by Hartmann and Braun.

SOFT-IRON INSTRUMENTS

The first ammeters and voltmeters used for commercial measurements on electric light and power circuits were of the soft-iron type, instruments in which an iron core is drawn into a solenoid, in opposition to the action of either gravity or a controlling spring.

As the induced pole reverses in sign with reversal of the current, a soft-iron instrument deflects in but one direction, irrespective of the direction of the current through it.

The law connecting the current and the pull of a solenoid on an iron core depends on the degree of saturation of the iron.

Suppose the core to remain fixed in position. If it is but weakly magnetized by the current, the strength of the induced pole will be roughly proportional to the current. This pole reacts with the field of the solenoid which is proportional to the current; so the attraction is approximately proportional to the square of the current. If the magnetizing field is so strong that the core is "saturated," any further increase of the current alters the strength of the induced pole but little and the attraction is approximately proportional to the current.

Thus, as the current is gradually increased from zero to a high value, the law of the pull changes from that of the square to that of the first power of the current, and the pull is also modified by the change in the position of the iron core due to the yielding

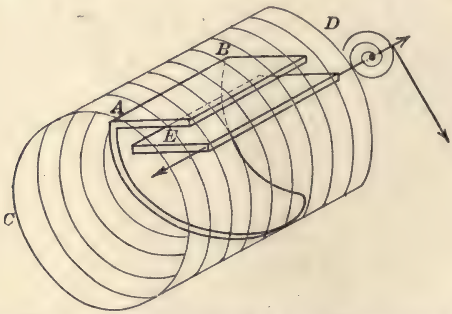


FIG. 34.—Illustrating principle of magnetic-vane instruments.

of the spring or gravity control. The net result is that the scales of direct current soft-iron instruments are not uniformly divided.

The quality of the iron is important. In order that the indications be independent of the previous magnetic history of the iron, it should be as free from hysteresis as possible. For direct-current work soft-iron instruments are now employed where cheapness and robustness of construction are essential and a moderate accuracy will suffice.

In alternating-current work this construction is employed in ammeters where its use avoids the necessity of taking large currents into the movable parts of the instrument.

Magnetic-vane Instruments.—The principle involved in the magnetic-vane instruments is sufficiently illustrated by Fig. 34.

As shown in the figure, *E* is a soft-iron vane fixed to the spindle

which carries the pointer and the inner end of the controlling spring.

On the passage of the current through the coil *CD*, in which the above arrangement is inserted, the iron is magnetized, the like poles repel each other and the needle is moved over the scale. An air damper is usually added.

There are many variations on this fundamental design, which is used for both ammeters and voltmeters.

Weston Soft-iron Instruments.—In the Weston soft-iron instruments the arrangement is as indicated by Fig. 35.

A thin piece of soft iron *abc* of the form shown is bent to conform to a cylinder; *de* is another thin piece of iron of rectangular form so bent that it is coaxial with *abc*. It is rigidly attached to

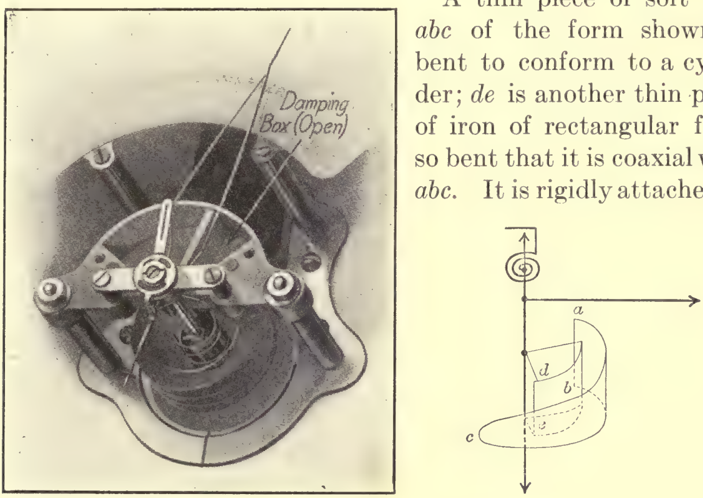


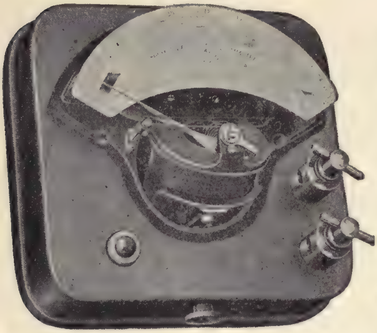
FIG. 35.—Weston soft-iron instrument.

the spindle. The coil has a large opening at the center in which this arrangement is placed with the spindle coinciding with the axis.

On the passage of the current the neighboring edges of the iron are similarly magnetized, the like poles repel each other and the index is forced over the scale. This construction is employed in both ammeters and voltmeters intended for use with alternating currents.

The General Electric Co.'s Inclined-coil Ammeter.—Referring to Fig. 36, the current flows through the coil which is inclined at an angle of about 45° to the spindle which carries the iron

vanes a, b, and the pointer. When no current is passing, the plane of the vanes makes a slight angle with that of the coil; on the passage of the current the vanes tend to place themselves along the lines of force, that is, perpendicular to the plane of the coil.



By adopting the inclined coil arrangement, a long scale is obtained, for in order to turn the plane of the soft iron from a position coincident with the plane of the coil, to one perpendicular to it, it is necessary to turn the pivot, and therefore the pointer, through 180°; the actual working range of deflection is about 100°. These instruments are now made with laminated magnetic shields and magnetic damping.

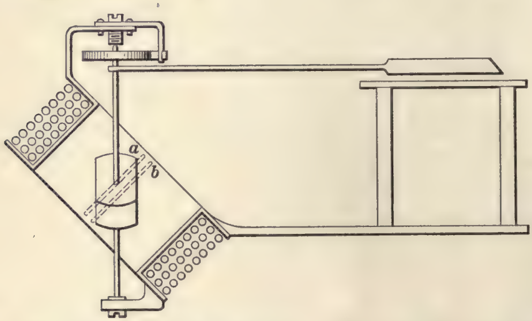


FIG. 36.—Inclined-coil ammeter. (General Electric Co.)

THE ELECTRODYNAMOMETER AND THE CURRENT BALANCE

The electrodynamometer is distinguished from the moving coil galvanometer by having the movable member suspended, not in the field of a permanent magnet, but in that due to a system of fixed coils which are traversed by the current.

Electrodynamometers may be either absolute or secondary instruments. The coils may be arranged in various ways but in

an absolute instrument the arrangement and proportions must be such that the constant of the instrument may be accurately calculated from the measured dimensions. The Helmholtz arrangement is sometimes adopted for both the fixed and movable members, but in the instrument shown in Fig. 37, which was especially designed for absolute measurements, the coils are wound on cylinders.

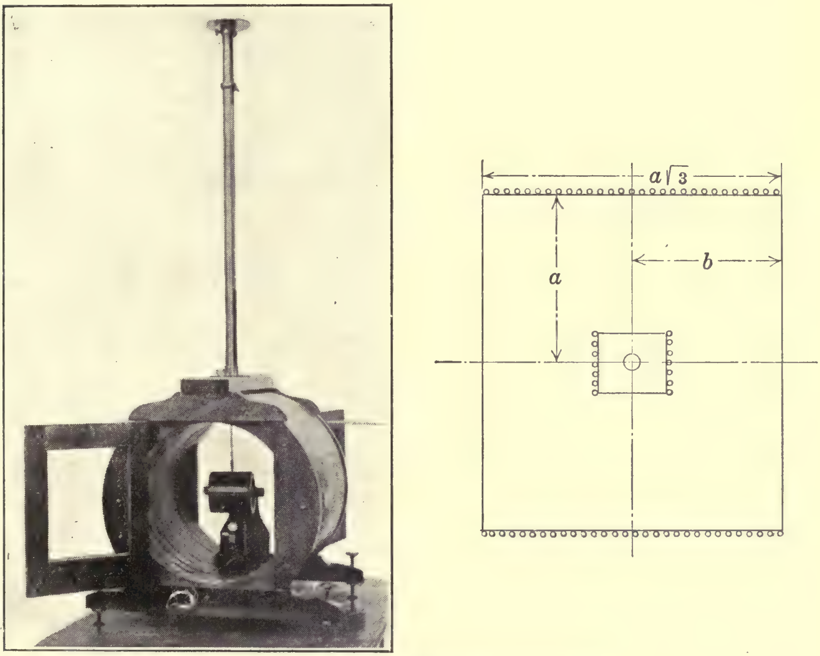


FIG. 37.—Absolute electrodynamometer.

The fixed coil of N_F turns consists of a single layer of insulated wire wound on a plaster of paris or marble cylinder, which is very accurately turned.

The movable coil of N_M turns is wound in a single layer on a carefully ground porcelain cylinder. The coils are placed concentrically. The diameter of the movable coil is about one-fifth that of the fixed coil.

The movable member is suspended by a torsion wire and the

current is taken to it by two mercury cups, the leads being as far as possible concentric, to avoid disturbing effects.

A torsion head is used in reading, so any effect due to the angular displacement of the axes of the coils from the perpendicular position is avoided. The suspension wire is the most troublesome feature of the instrument, for its torsional properties must be so definite that they can be determined to a high degree of accuracy. A well-aged phosphor-bronze wire is the most satisfactory.

The force, in any given direction, which acts on a circuit carrying a current I_M, when it is placed in a magnetic field, is equal to the product of the current and the space rate of change of flux through the circuit when the circuit is displaced in the given direction. If the flux is due to a second circuit, its value will be $I_F m$, where I_F is the current in the second circuit and m is the coefficient of mutual induction of the two circuits. Consequently, the force in the direction x is

$$F = I_F I_M \frac{dm}{dx} \tag{61}$$

Similarly, the turning moment acting between the coils of an electro-dynamometer and tending to change the angle θ between their axes is given by

$$M = I_F I_M \frac{dm}{d\theta} \tag{62}$$

To use these relations it is necessary to have expressions for m in terms of the numbers of turns in the coils, their dimensions, and their distance apart; in general, m must be expressed in the form of a series, examples of which may be found in Maxwell's "Treatise on Electricity and Magnetism," and in Gray's "Absolute Electrical Measurements."

In accordance with the above, if a plane circuit of area A, which is traversed by a current I_M, is suspended in a uniform magnetic field of strength, H', it will experience a turning moment $-M = AI_M H' \sin\theta$ where θ is the angle between the perpendicular to the coil and the direction of the field H'. This simple relation cannot be used in connection with the electro-dynamometer except as a first approximation, for the field due to the fixed coil is not uniform throughout the space occupied by the movable coil.

In electrodynamometers when used with direct currents, H' is the sum of the field due to the current circulating in the fixed coils which form a part of the instrument, and the local field.

An *approximate* expression for the turning moment acting on the movable coil, assuming it to be a plane circuit, may be obtained.

Referring to Fig. 37, the field at the point O, the center of the fixed coil, may be obtained as follows: Let the number of turns per centimeter of length of the coil be n and let x be the axial distance of a turn from the center. If I_F is the current in the wire, a belt of winding δx cm. long will produce at the center a field whose component along the axis is

$$\delta H = \frac{2\pi a n \delta x I_F}{(a^2 + x^2)} \cdot \frac{a}{\sqrt{a^2 + x^2}} = 2\pi a^2 n I_F \frac{\delta x}{(a^2 + x^2)^{3/2}}.$$

The effect of the whole coil will be

$$H = 2\pi a^2 n I_F \int_{-b}^{+b} \frac{dx}{(a^2+x^2)^{3/2}} = 2\pi n I_F \left. \frac{x}{\sqrt{a^2 + x^2}} \right]_{-b}^{+b} = \frac{4\pi n I_F b}{\sqrt{a^2 + b^2}}.$$

If N_F is the total number of turns

$$H = \frac{2\pi N_F I_F}{\sqrt{a^2 + b^2}}.$$

If the movable coil of N_M turns has a radius r, and the field in which it is placed is uniform and of strength H, the mutual inductance between the fixed and movable coils is

$$m = H\pi r^2 N_M \cos\theta = \frac{2\pi^2 r^2 N_M N_F \cos\theta}{\sqrt{a^2 + b^2}}$$

where θ is the angle between the axes of the coils.

The turning moment is

$$M = I_F I_M \frac{dm}{a\theta} = \frac{2\pi^2 r^2 N_M I_M N_F I_F \sin\theta}{\sqrt{a^2 + b^2}},$$

and when the axes are perpendicular, this becomes

$$M = \frac{2\pi^2 r^2 N_M I_M N_F I_F}{\sqrt{a^2 + b^2}}$$

As implied above, the turning moment due to the mutual ac-

tion of the two coils cannot be exactly calculated in this simple manner, for to be exact, m must be expressed in the form of a series. However, in this particular case, as pointed out by A. Gray,[14] if the ratio of the radius of each coil to its length is $\frac{1}{\sqrt{3}}$ all the terms in the series between the first and seventh drop out and all but the first term are so very small that they may be considered as corrections, to be calculated if the accuracy of the work demands it.

The result of careful analysis shows that the turning moment due to the mutual action of the two coils, if they are concentrically placed with their axes perpendicular, is given, to a very high degree of approximation, by

$$M = \frac{2\pi^2 r^2 N_M I_M N_F I_F}{\sqrt{a^2 + b^2}}$$

or if the two coils are in series, by

$$M = \frac{2\pi^2 r^2 N_F N_M I^2}{\sqrt{a^2 + b^2}}.$$

Of course the field in which the movable coil is placed is not uniform, but with coils proportioned as stated above, the instrument acts as if a plane circuit having the net area of the movable coil were suspended in a uniform field of strength,

$$H = \frac{2\pi N_F I_F}{\sqrt{a^2 + b^2}}.$$

Secondary Electrodynamometer.—Siemens Dynamometer.—A form of secondary electrodynamometer in common use is shown in Fig. 38. The fixed coils are firmly supported from a wooden pillar which carries at its top a torsion head provided with a pointer which can be moved over a uniformly graduated circle.

The wooden frame effectually prevents any errors which might be introduced in alternating-current work by currents induced in the supports for the coils.

The movable coil hangs freely from a pivot which rests in a jewel carried by a stirrup attached to the graduated plate. The torsion head is connected to the movable system by a loosely

coiled spiral spring. A pointer, which normally stands at zero, is attached to the movable coil; on the passage of the current, this pointer deflects against a stop and is brought back to its original position by turning the torsion head. The amount of twist which it is necessary to give the spring in order to return the coil to its zero position is read from the graduated circle. If the spring be perfect, the moment exercised by it will be proportional to this angle of twist. As springs cannot in general be relied upon throughout the whole range of twist, the instrument should be calibrated at a number of points and a calibration curve drawn.

The current is led into the movable coil by two stout wires which dip into mercury cups.

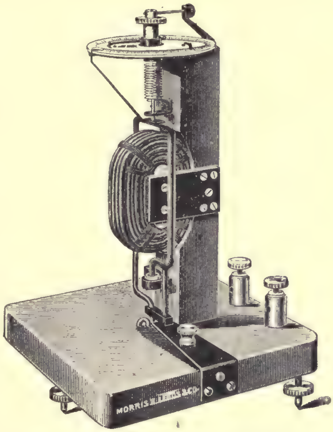


FIG. 38.—Siemens electrodynamometer.

Setting up the Siemens Electrodynamometer.—The instrument must be levelled and the same relative position of the coils maintained during calibration and subsequent use. Though the instrument is to be used with alternating currents, it is convenient to employ direct currents in the calibration. If this is done it is desirable to place the instrument so that the local field will have no influence. This may be accomplished by turning the dynamometer in azimuth until a position is found where the strongest current which is to be used produces no deflection when sent through the movable coil alone.

The Law of the Electrodynamometer.—The law of the electrodynamometer is dependent on the method of reading. Two cases will be considered:

1. When the movable coil is always brought back to its initial position by the use of a torsion head, as in the Siemens instrument.

2. When the movable coil is allowed to deflect, as in an ordinary reflecting galvanometer.

1. When an electrodynamometer, set up without regard to the local field, is used with direct currents, a part of the turning

moment is due to the action of that field on the movable system. This will be proportional to the current through the movable coil. The field due to the fixed coil is proportional to the current through it, so the total turning moment will be $M = K_1 I_F I_M \pm K_2 H I_M$. If the coils are in series, the moment corresponding to a current I, will be

$$M = K_1 I^2 \pm K_2 HI.$$

K_1 is a factor which depends on the dimensions and the numbers of turns of both coils, and on the angle between their axes. K_2 depends on the dimensions and number of turns of the movable coil and on the position of the coil in the local field, H.

To eliminate the effect of the local field two readings may be taken, the current being reversed. In each case the torsion head is adjusted until the original relative position of the coils is reproduced. Then, if τ is the torsion constant of the controlling spring and θ is the twist in the spring,

$$M_1 = K_1 I^2 + K_2 HI = \tau\theta_1$$
$$M_2 = K_1 I^2 - K_2 HI = \tau\theta_2$$

or

$$K_1 I^2 = \tau \frac{(\theta_1 + \theta_2)}{2}.$$

If an alternating or regularly pulsating current is employed, the turning moment passes through a cycle of values with each complete period. As the natural time of vibration of the movable system is much greater than the period of the current, the system will take up a position dependent upon the average turning moment, that is, the twist in the controlling spring will be given by

$$\theta = \frac{1}{\tau} \cdot \frac{1}{T}\int_o^T (K_1 i_F i_M + K_2 H i_M)dt.$$

The subscripts F and M refer to the fixed and movable coils respectively. T is the time of a complete cycle.

With alternating currents this becomes

$$\theta = \frac{K_1}{\tau}\frac{1}{T}\int_0^T i_F i_M dt$$

that is, the deflection when a torsion head is used, is proportional

to the mean product of the currents in the two coils. Therefore, if the coils are in series the deflection is proportional to the mean square of the current or to the square of the effective value.

The currents in the two coils may differ in wave form and in time phase as well as in magnitude. For instance, expressing both i_F and i_M in the form of a series, if the waves are non-sinusoidal,

$$i_F = A_1 \sin \omega t + A_2 \sin (2\omega t - \varphi_2) + A_3 \sin (3\omega t - \varphi_3) + \ . \ . \ .$$
$$i_M = B_1 \sin (\omega t - \varphi'_1) + B_2 \sin (2\omega t - \varphi'_2) + B_3 \sin (3\omega t - \varphi'_3) + \ . \ . \ .$$

It is well known that the mean product of two sine curves which differ in periodicity is zero, and that the mean product of two sine curves of the same periodicity is $\frac{I_1 I_2}{2} \cos \alpha$ where I_1 and I_2 are the maximum values and α is the angle of phase difference. Consequently,

$$\tau\theta = K_1 \frac{1}{T}\int_0^T i_F i_M \, dt = K_1\left[\frac{A_1B_1}{2} \cos \varphi'_1 + \frac{A_2B_2}{2} \cos (\varphi_2 - \varphi'_2) + \frac{A_3B_3}{2} \cos (\varphi_3 - \varphi'_3) + \ . \ . \ . \right] \quad (63)$$

If the fixed and movable coils are in series, as they are when currents are measured, $A_n = B_n = I_n$ and $\cos (\varphi_n - \varphi'_n) = 1$, so

$$\tau\theta = K_1 \frac{1}{T}\int_0^T i^2 dt = K_1\left[\frac{I_1^2}{2} + \frac{I_2^2}{2} + \frac{I_3^2}{2} + \ . \ . \ . \right] = K_1 I^2 \quad (64)$$

I_1, I_2, I_3, etc., are the maximum values of the various components. $\frac{I_1^2}{2}$ is the square of the effective value of the fundamental, $\frac{I_2^2}{2}$ is the square of the effective value of the second harmonic, etc., and I^2 is the square of the effective value of the current.

2. When the movable coil is allowed to deflect, the factor K_1 becomes a variable depending on the angle between the axes of the fixed and movable coils. If the field due to the fixed coil is uniform or, what practically amounts to the same thing, if the movable coil is very much smaller than the fixed coil, K_1 is proportional to the cosine of the angle of displacement of the axes of the

coils from a perpendicular position. However, Lord Rayleigh* has called attention to the fact that the mutual inductance of two concentric circles, the ratio of whose radii is $\sqrt{\frac{0.3}{1}} = 0.548$, is, over a considerable range, very nearly proportional to the angular displacement of the axes from the perpendicular position.

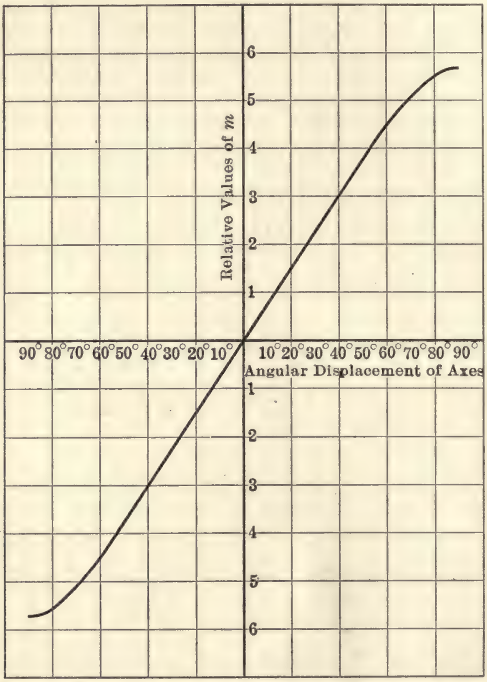


FIG. 39.—Showing relation between the mutual inductance of two circles and the angular displacement of their axes from the perpendicular position when the ratio of the radii is 0.548.

This is illustrated by Fig. 39 in which relative values of the mutual inductance are plotted against the angular displacements of the axes of the coils from the perpendicular position.

From the figure it is seen that over a wide range $\frac{dm}{d\theta}$ is practically constant. As the turning moment due to the mutual action

* "The Inductance and Resistance of Compound Conductors," (*Phil. Mag.*, December, 1886, p. 470).

of two coils is $I_F I_M \frac{dm}{d\theta}$ the deflection of an electrodynamometer with coils of small cross-section thus proportioned, neglecting the action of local fields, should be very nearly proportional to the square of the current. Also, if the coils of a deflectional wattmeter (page 306) be thus proportioned, the scale should be sensibly uniform throughout the working range of the instrument.

If the movable coil of a sensitive dynamometer, designed as just suggested, is allowed to deflect and the angular movement is read by the mirror and scale method, in general, when direct currents are used the conditions are as indicated by Fig. 40.

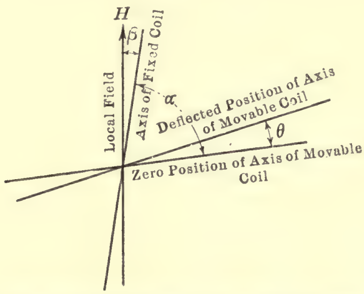


FIG. 40.—Pertaining to effect of local field on a deflectional electrodynamometer.

If A is the equivalent area of the movable coil, H the strength of the local field, and θ the deflection,

$$\tau\theta_1 = K_1 I_F I_M + I_M AH \sin(\alpha + \beta - \theta_1).$$

If both currents are reversed the sign of the term involving H will be changed,

$$\tau\theta_2 = K_1 I_F I_M - I_M AH \sin(\alpha + \beta - \theta_2).$$

In setting up the instrument β is made as near 0° and α as near 90° as possible, then

$$\tau\theta_1 = K_1 I_F I_M + I_M AH(\cos\theta_1)$$

and if the coils are in series

$$\tau_1\theta = K_1 I^2 + IAH \cos\theta_1.$$

If the instrument is read by telescope and scale the angle of

deflection is small and cos θ is practically unity. If D represents the scale reading then, nearly enough,

$$D_1 = K'_1I^2 + K'_2I$$

and on reversing the current

$$D_2 = K'_1I^2 - K'_2I$$

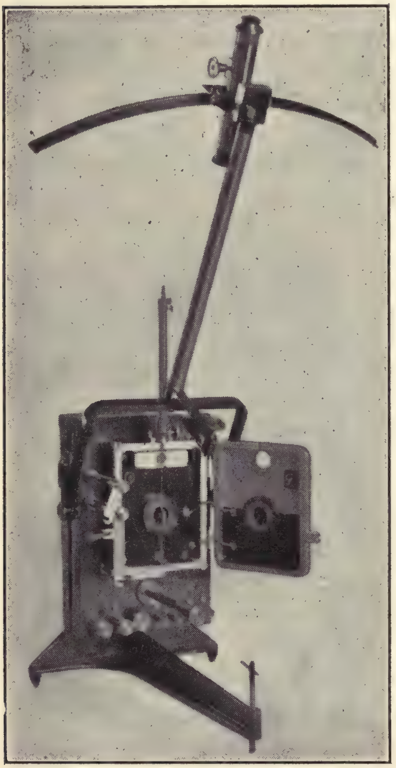


FIG. 41.—Sensitive electrodynamometer.

where K'_1 and K'_2 are constants. I is the numerical value of the current without regard to sign. The law of a sensitive electrodynamometer when used with direct currents is shown in Fig. 42. The instrument was adjusted so that initially the axes of the coils were perpendicular, with the axis of the fixed coil coinciding in direction with the local field.

To make this adjustment an alternating current may be sent through the fixed coil, the circuit of the movable coil being closed through a telephone. The desired position is attained when the mutual inductance is zero, that is, when the telephone is silent. This adjustment having been made, the axis of the fixed coil may be made to coincide with the local field if a direct current be sent through the movable coil alone and the instrument turned in azimuth until the deflections with reversed currents are equal.

Referring to Fig. 42 and employing 10^{-3} amp. as the unit of current, for this particular instrument,

$$D_1 = 25.8I^2 + 18.3I$$

$$D_2 = 25.8I^2 - 18.3I$$

When alternating currents are used, the term due to the local field is absent, consequently the calibration curve will be obtained by plotting a new curve, the abscissæ being $D = \frac{D_1 + D_2}{2}$. For this case $D = 25.8I^2$. The new curve is shown by the dotted line in Fig. 42.

It is to be noted that a high sensitivity is obtained by having many turns on the coils. At high periodicities, therefore, both the resistance and reactance of the coils conspire to alter the circuit conditions when the instrument is introduced.

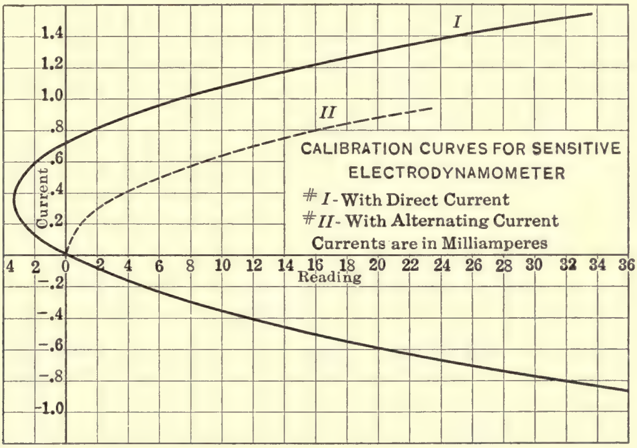


FIG. 42.—Calibration curves for sensitive electrodynamometer.

Astatic Electrodynamometers.—All troubles due to uniform local and stray fields may be obviated by making the instrument astatic. Fig. 43 shows a simple arrangement of this sort.

The current circulates in opposite directions in the two rigidly connected movable coils which are identical in dimensions and numbers of turns and are connected in series; hence, the movable system when traversed by direct currents will experience no turning moment due to the earth's field. Again, stray fields due to either direct or alternating currents have no effect *provided* they are sufficiently uniform so that the strength is the same at both the upper and lower coils. The two sets of fixed coils are so connected that they both tend to turn the movable member in the same direction.

An astatic instrument can be set up without regard to the earth's field and calibrated with direct currents. The curves so obtained are immediately applicable to either direct- or alternating-current measurements.

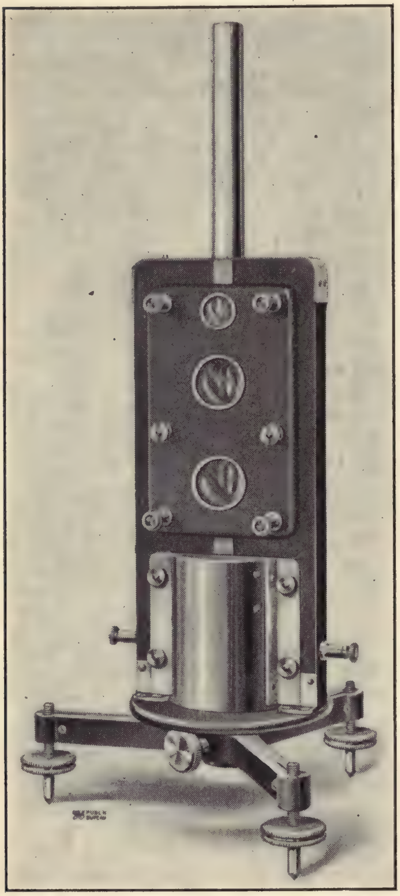


FIG. 43.—Astatic electrodynamometer.

In the instrument shown in Fig. 43, the damping is obtained by means of mica vanes moving in the closed chamber at the base of the instrument.

The Irwin Astatic Electrodynamometer.—The Irwin astatic instrument is intermediate between the electrodynamometer and the current balance. The fixed turns are in the form of two coaxial circular coils of the same diameter, through which the current circulates in opposite directions, thus producing between the faces of the coils a field which is directed radially outward. The movable member consists of two semicircular coils mounted on a thin disc of mica, the directions of the currents being as indicated. The straight sides are as nearly as possible in the axis of rotation, the curved sides move in the field between the two fixed coils, being attracted by one and repelled by the other. This construction brings the two movable coils very near together, which is advantageous as it reduces any effect which may be due to non-uniformity of the local or stray field.

Rewinding an Electrodynamometer to Obtain a Given Performance.—As the deflection of an electrodynamometer when used with alternating currents is proportional to the square of the current, the range of the instrument is limited. It is desirable, therefore, to have the fixed coil subdivided by taps so that the number of turns may be varied.

It is sometimes necessary to alter the sensitivity of secondary dynamometers so that definite deflections may be obtained with stated currents. If the corresponding dimensions of the coils of the original and the rewound instruments are the same, the calcu-

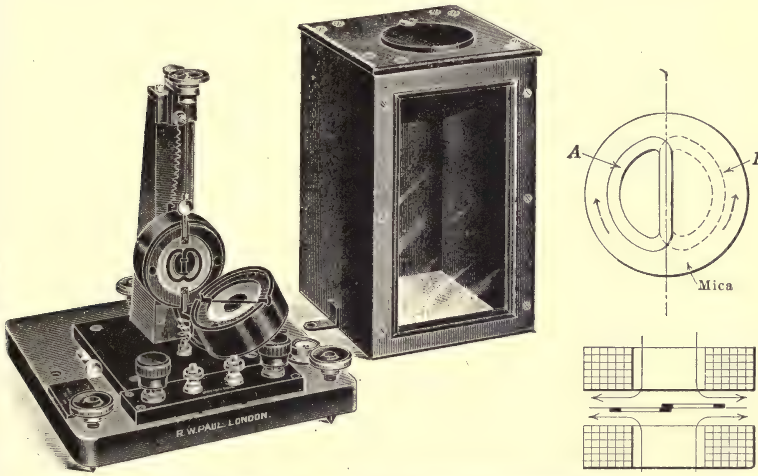


Fig. 44.—Irwin astatic electrodynamometer.

lation of the proper number of turns is simply a matter of proportion, provided one has data concerning the number of turns on the coils and the performance of the original instrument. If the instrument is properly set up,

$$\tau\theta = KFMI^2 \tag{65}$$

F and M are the numbers of turns on the fixed and movable coils respectively, and K is a constant depending upon the geometry of the coil system. This may be determined from a knowledge of the torsion constant of the spring, the number of fixed and moving turns on the original instrument and the deflection corresponding to a given current. After K has been found, the value

of FM, the required product of the fixed and moving turns for the new winding, is readily determined.

Electrodynamometers for Heavy Currents.—For several reasons it is difficult to apply the ordinary electrodynamometer to the measurement of very large currents. If the coils are used in series, trouble is experienced in getting the current into and out of the movable member. Where any considerable current is to be carried, the necessary flexible connections are made by means of mercury cups, and no dynamometer in which they are employed can be considered a portable instrument as that term is now generally understood. There are also the structural difficulties due to the necessity of supporting a heavy movable coil on pivots which are practically free from friction and are sufficiently strong so that the instrument will stand handling. These considerations preclude the use of the electrodynamometer with its coils in series as an alternating-current ammeter, and are the reasons for the survival of soft iron ammeters as alternating-current instruments.

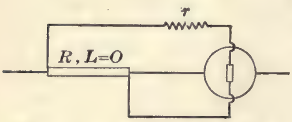


FIG. 45.—Wattmeter method for measuring large alternating currents.

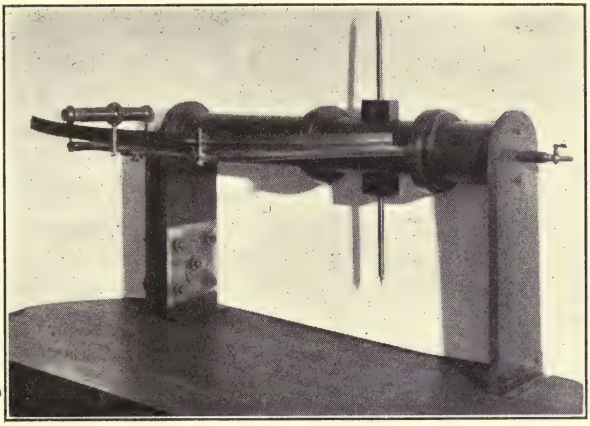
Again, it is essential that the field at the movable coil be the same with both direct and alternating currents, for direct currents will be used in calibrating and alternating currents in the subsequent use of the instrument. This equality of fields may be attained either by arranging the metal of the coil so that the current distribution will be the same in both cases, or by adopting an arrangement which from its symmetry is such that the change in distribution in passing from direct to alternating current does not affect the field in which the movable coil swings.

It is also necessary that there be no error due to eddy currents induced in the mass of the coils, or in the frames by which they are supported. For this reason, in instruments of moderate capacity recourse is had to stranding the conductors. This must be very carefully done so that all the strands will have the same effective inductance and resistance and the same increase of resistance due to heating.

Wattmeter Method for Measuring Large Alternating Currents.—To obviate the necessity of taking a large current into the

movable coil, recourse has been had in investigation and calibration work, to a wattmeter method as indicated in Fig. 45.

The current coil of the wattmeter and a known resistance R are inserted in the circuit; the connections are such that the I^2R



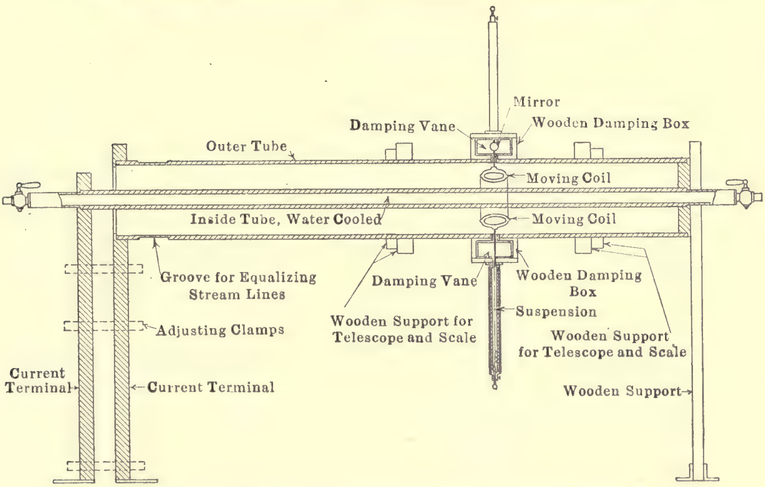


FIG. 46.—Agnew tubular electrodynamometer.

loss in the constant resistance, R, is measured, and thus the current is determined.

Agnew Tubular Electrodynamometer.[15]—This instrument is primarily designed for measuring very large alternating cur-

rents, up to 5,000 amp., by the wattmeter method. It can, of course, be used as an ordinary wattmeter for power measurements. Its distinctive feature is the means taken to avoid errors due to the skin effect in the very massive conductors which must be used for the current coil.

As seen from Fig. 46 the current "coil" is made in the form of two coaxial tubes. When they are traversed by the current a strong field will exist in the space between them, while the field external to the tubes will be *nil.* Stray-field effects due to the heavy current in the instrument are thus avoided.

The movable system consists of two rigidly connected coils one above, the other below the central tube. The working position of the movable system is approximately 90° from that shown in Fig. 46. As the movement takes place in a strong field, it is essential that the movable system be entirely free from magnetic impurities. If this is not the case, the zeroes with the current on and with the current off will not coincide.

The suspension strip is of phosphor-bronze and air damping is provided. Diaphragms between the tubes are necessary to prevent disturbance of the movable system by air currents.

Theory of the Tubular Electrodynamometer.—Suppose that two coaxial circular tubes are arranged as shown in Fig. 47, the directions of the current being as indicated. Consider any point, P, in the space between the tubes and distant r from the axis. A direct current will distribute itself uniformly over the cross-section of the tubes. The work done in taking a unit pole around the indicated path is

$$2\pi rH = 4\pi I$$

where I is the current encircled by the path, that is, the current which flows in the central tube, and H is the field strength at any and all points in the path. Therefore,

$$H = \frac{2I}{r}$$

It is obvious that this relation will hold as long as a symmetrical arrangement of the current around the central axis is preserved. It is thus evident that the generating lines of the surfaces of the coaxial tubes need not be straight but may have any form what-

soever. Symmetry is the all-important thing. It is also apparent that any symmetrical redistribution of the current in the conductors will not alter H.

If an alternating be substituted for a direct current, the distribution of the current over the cross-section of the tubes will be altered, due to the skin effect, but from the symmetry of the conductors the new distribution of the current will be symmetrical about the axis and therefore the field due to the changed distribution of the current is the same as before.

In the actual instrument, owing to the manner of taking the current into the large outside tube, the natural distribution will not be quite uniform. For this reason, a groove about 5 cm. wide is turned eccentrically in the tube. By filing, the groove may be so adjusted that the stream lines are uniformly distributed.

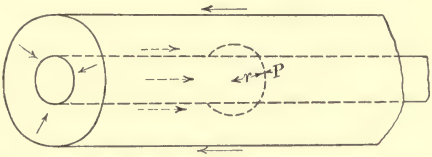


FIG. 47.—Pertaining to the Agnew tubular electrodynamometer.

It is essential that the symmetrical distribution of currents about the axis be maintained; therefore, any springing of the slender inner tube must be avoided. A test for distribution errors may be made by the method described on page 316.

The principal dimensions of the instrument, as used at the Bureau of Standards, are

Outer tube, length, 101 cm.; radii, 6.41 and 7.07 cm.

Inner tube, length, 125 cm.; radii, 0.50 and 1.27 cm.

Current capacity, air-cooled, 1,200 amp., (water-cooled) 5,000 amp.

Field at center of movable coils at full load, 300 gausses.

Movable coils, 116 turns of 0.2-mm. silver wire, diameters 2.5 and 5.0 cm., weight of each coil 7.3 gm.; total resistance of movable coil circuit 14.3 ohms; current capacity 0.06 amp.; inductance 1.4 millihenries.

Sensitiveness, 100-cm. deflection at 86-cm. scale distance requires 100 amp. in tubes and 0.06 amp. in movable coils.

The Current Balance.—In this class of instruments the current is measured by weighing, with a gravity balance, the pull exerted

by a coil upon another placed in a parallel plane, their axes being coincident.

An absolute current balance of this sort was used by Lord Rayleigh and Mrs. Sidgwick, 1884, in their determination of the electrochemical equivalent of silver. Since that time the instrument has been brought to a very high degree of perfection, par-

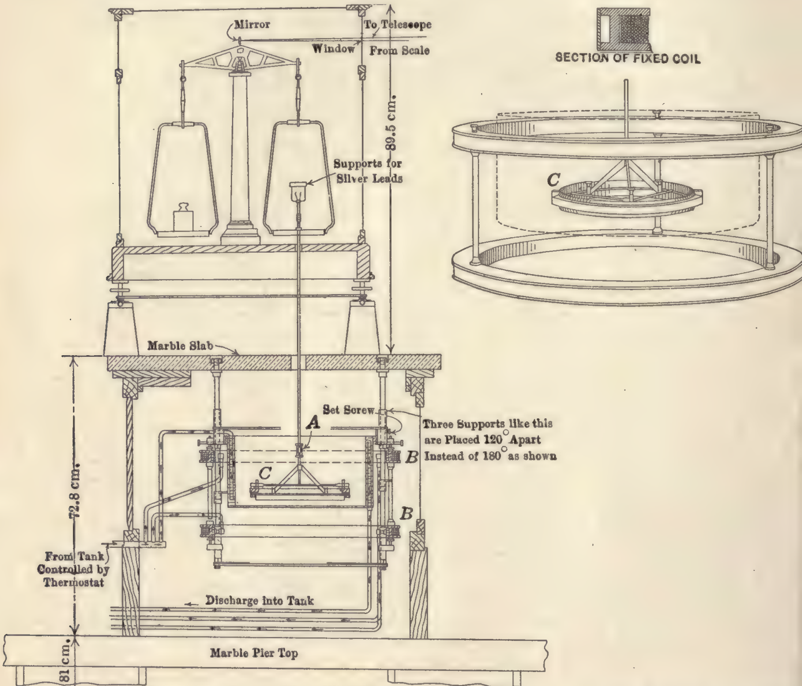


FIG. 48.—Absolute current balance used at Bureau of Standards by Rosa, Dorsey and Miller.

ticularly through the work of Ayrton, Mather and Smith at the National Physical Laboratory in England and of Rosa, Dorsey and Miller at the Bureau of Standards at Washington, D. C.

Rayleigh, and following him, Rosa, Dorsey and Miller used a balance with two equal fixed coils, the smaller movable coil being placed midway between them, all three coils being coaxial.

The Rayleigh current balance is not intended for general use as a current-measuring device, but for the absolute measurement

of currents in special investigations such as are necessary in the determination of electrical standards it is of great service, and is generally considered to be the most accurate device which has been developed for the purpose. Its advantages are:

1. The constant of the instrument depends principally upon the ratio of the effective radii of the coils. This number can be determined experimentally to a high degree of accuracy, by a method originally due to Bosscha; the difficulties met with in determining the mean radii of multiple-layer coils from mechanical measurements are thus avoided. This is the peculiar advantage of the Rayleigh form of balance.

2. The measurements are independent of the local field and its variations.

3. The determination of torsion constants is entirely avoided.

4. Analysis shows that when the distance between the fixed and movable coils is equal to one-half the radius of the larger coil, slight inaccuracies in the placing of the movable coil produce very small errors in the calculated constant of the instrument.

Fig. 48 shows the current balance used by Rosa, Dorsey and Miller, together with an enlarged view of the coil system. The two fixed coils have a radius of 50 cm. and are placed 25 cm. apart. They are wound on brass frames, the material for which was carefully selected, for even good brass is slightly magnetic. Enamelled wire was used. The movable coil, 25 cm. in diameter, is hung from a precision balance and the force determined by the change in weight necessary to restore the balance to equilibrium when the current in the fixed coil is reversed. This change was 6 gm. To prevent disturbance of the balance by air currents set up by the heating of the coils, the fixed coils are water-cooled and the movable coil is hung in a water-jacketed chamber which is kept at a constant temperature.

When the balance is used in the manner indicated,

$$I^2K = \frac{Mg}{2}.$$

M is the change in the weights, corrected for buoyancy of the air, which is necessary to restore the balance to equilibrium when the current in the fixed coil is reversed, and g is the acceleration due to gravity.

The factor K depends principally on the ratio of the radii of the fixed and movable coils. It is equal to $\frac{dm}{dx}$ where m is the mutual inductance of the coils and dx refers to an axial displacement of the movable coil. m can be calculated, in the form of a series, from the numbers of turns and the dimensions of the coils.

Secondary Current Balances.—The Kelvin Balance.—In the Kelvin balance there are four fixed and two movable coils, the latter being carried at the ends of a balance arm which turns about a horizontal axis, see Fig. 49. Each movable coil is situated between two fixed coils and all six coils are connected in series. In place of a knife edge, the beam is suspended by a large number of filaments of copper wire, forming a sort of stranded ribbon, which provides a ready means for making the electrical connection to the movable coil. Absence of friction, large radiating surface, and cooling by conduction to the frame are the advantages attained by this means.

The center of gravity of the movable system may be adjusted as in an ordinary balance, by means of a weight on a short vertical rod immediately above the point of support.

The bobbins and the base plate are made of slate, thus insuring rigidity and absence of eddy currents. Means for levelling the instrument and for removing the weight of the movable parts from the suspension wires when the instrument is not in use are provided.

A uniformly graduated bar is attached to the movable system and is supplied with a sliding carriage upon which the weights can be placed in a definite position, small conical pins on the carriage fitting into corresponding holes in the weights. The carriage can be manipulated from without the case by means of cords attached to a self-releasing pendent, which slides on guides attached to the base of the instrument.

The zero of the graduated bar is at the extreme left hand. If, when no current is flowing in the balance, the sliding weight is set at zero and the corresponding counterpoise placed in the V-shaped trough attached to the right-hand movable coil, then the pointer attached to the beam should stand at a reference mark upon the short vertical scale attached to the base of the instrument. If this is not so, a means of adjustment is provided in a

small metal flag attached to the movable system which can be manipulated by means of a forked lever operated from outside the case.

Suppose that the movable weight is set at zero and that the balance is in equilibrium when no current is passing. On the passage of the current the equilibrium will be upset and the right-

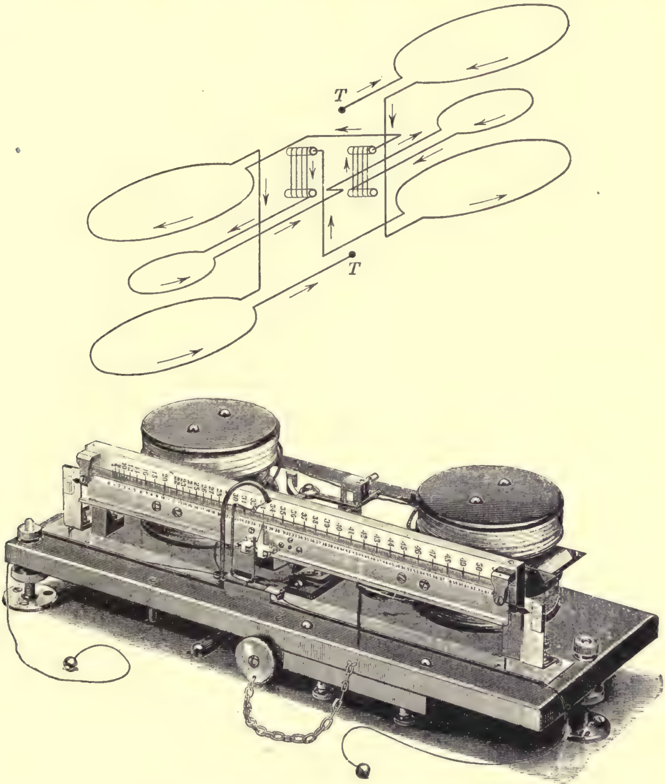


FIG. 49.—Kelvin current balance.

hand end of the beam will rise. To restore the equilibrium it is necessary to increase the turning moment due to the weights. This is accomplished by displacing the movable weight toward the right. The change in the total moment due to the weights is proportional to the displacement of the movable weight from its zero position, and this must be equal to the moment due to the

action of the coils, which is proportional to the square of the current.

The device of using the weight in two portions is adopted because the beam can then be made twice as long as would otherwise be the case.

The current is calculated by the formula $I = K\,2\sqrt{R}$, where R is the displacement of the weight as read on the uniformly graduated scale. A table of doubled square roots, supplied by the maker, is of service in the calculation.

For rough work, the position of the sliding weight may be referred to the fixed inspectional scale, placed immediately behind the scale beam. It is graduated to give the quantity $2\sqrt{R}$ directly. For the purpose of extending the range four sets of weights are supplied with each instrument.

The Kelvin balances are made in a variety of ranges up to 2,500 amp. Their particular field of usefulness is as secondary standards in the calibration of alternating-current ammeters. They are serviceable in laboratories where the circuit conditions can be controlled, but are not adapted to the measurement of fluctuating currents.

MEASUREMENT OF CURRENTS IN PERMANENTLY CLOSED CIRCUITS[17]

Occasionally it is necessary to measure the current in a conductor which cannot be broken to allow the introduction of an ammeter or shunt. For example, such cases occur in the investigation of the electrolytic deterioration of underground pipes for water or gas, due, for instance, to the stray currents caused by the use of a ground return, or imperfect bonding, in a traction system. The damage occurs where the current leaves the pipe, and may cause such a menace to health and property that large expenditures of time and money are justifiable in locating the source of the trouble and in its elimination.

In this or similar cases, if the resistance between two potential points on the pipe or other conductor can be determined, the current may be measured by using this resistance as a shunt, a millivoltmeter being used to determine the P.D. between the points. The problem thus resolves itself into the determination

of the resistance of a portion of a conductor which may be carrying a current and which must be measured *in situ* without being opened.

Suppose that, as a preliminary to measuring the stray current in, for instance, a water main, it is necessary to determine the resistance between two plugs which have been screwed into the pipe to serve as potential terminals.

The pipe is supposed to be traversed by stray currents of unknown strength. The necessary connections are shown in Fig. 50. At V_1 and V_2 are 2 millivoltmeters, with, perhaps, 10 millivolt scales; they are connected to potential points at b, c and e, f. These points are obtained by drilling into the pipe and firmly inserting brass plugs to which the leads may be soldered. It is

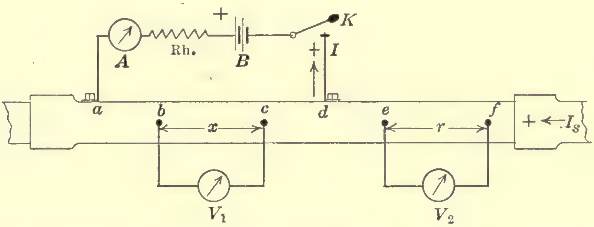


FIG. 50.—Connection for measuring a direct current without opening the circuit.

essential that the instruments be calibrated with the leads which are to be used in the test. At B is a storage battery capable of yielding enough current to give a good reading on V_1 (100 or 200 amperes for a 15-in. iron pipe), at A is the ammeter by which the current from B is measured, and K is the switch by which the current is controlled.

The spacing of the points a, b, c, d is important, for the four points b, c, e, f should be on four equipotential planes through the pipe. Therefore the distance between a and b and between c and d should be great enough so that the current from B may spread out and the stream lines become uniformly distributed before the points c and b are reached; the lengths of ab and cd should be about twice the diameter of the pipe, the points a and d being on the top and b and c on the side.

The positive direction of the currents may be assumed as indicated; it is essential that such an assumption be made at the

start, otherwise confusion may arise and the results be of no value.

The procedure is as follows: Observe, if necessary, the temperature of the conductor. With K open, *simultaneous* readings of the millivoltmeters are taken. Denote them by V_1 and V_2, then

$$\frac{x}{r} = \frac{V_1}{V_2}.$$

K is then closed and *simultaneous* readings of the ammeter and of the two millivoltmeters are taken. Call the readings I, V'_1, V'_2, and denote the stray current in the pipe at the instant of reading by I_s. Then the current across x, at that instant, will be

$$I_x = -I + I_s$$

also

$$I_x x = V'_1$$

$$I_s r = V'_2$$

From these

$$I_x = \frac{V'_1}{x} = -I + I_s$$

$$V'_1 = -Ix + I_s x = -Ix + V'_2\frac{x}{r}.$$

$$\therefore x = -\frac{V_1'}{I} + \frac{V'_2 V_1}{IV_2}$$

and

$$r = -\frac{V_2 V_1'}{V_1 I} + \frac{V'_2}{I}.$$

The test should be repeated with the battery reversed; I is then $-$. Throughout, care must be taken as to the algebraic signs of all the deflections.

The final result is independent of the stray current in the pipe, I_s; its elimination is possible, even though it be varying rapidly, because all three instruments are read simultaneously. The periods and the damping of the three instruments must be such that they keep pace with one another when the current changes.

It will be noted that if the current I is so adjusted that the reading of V'_1 becomes zero,

$$I = I_s$$

and the reading of the ammeter gives the strength of the stray current. This method of measuring the current requires that the observer remain continuously at his station. It is frequently desirable to obtain records extending over a considerable time, in which case the resistance of a section of pipe may be determined as above and a registering millivoltmeter used.

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CHAPTER II

THE BALLISTIC GALVANOMETER

In the comparison of the electrostatic capacities of condensers and cables, and in the examination of the magnetic properties of iron, an instrument is necessary which will measure the quantity of electricity displaced in a circuit by a transient current. For this purpose the ballistic galvanometer is employed, and to avoid disturbances due to local magnetic fields an instrument of the D'Arsonval type is now generally used.

For reading the instrument, a telescope and a uniformly divided circular scale, having its center at the axis of the movable system, should be employed; with this arrangement the deflection as read from the scale is directly proportional to the angle turned through by the movable system.

Observations are made as follows: The movable system is brought to rest in its proper zero position; the discharge is then passed through the instrument, giving an impulse to the movable system, which slowly deflects; the reading is taken at the first turning point, or elongation, just as the movable system is about to begin its swing back towards zero. If this maximum angular deflection of the coil from its original position be called θ_1 and Q be the quantity of electricity in the discharge, then when a D'Arsonval instrument is employed,

$$Q = K'\theta_1$$

For any particular instrument used in a definite manner K' is a constant. As will be shown later, its value depends on the current sensitivity of the instrument, on the time of vibration of the movable system, on the amount of damping and on the manner in which the discharge is sent through the galvanometer.

The ballistic instrument differs from the ordinary current galvanometer in one essential particular, in that the moment of inertia of the movable system is made very large compared with

the restoring moment due to the suspension strip. In other words, the instrument is one having a long time of vibration. This is necessary in order that the response to the impulse caused

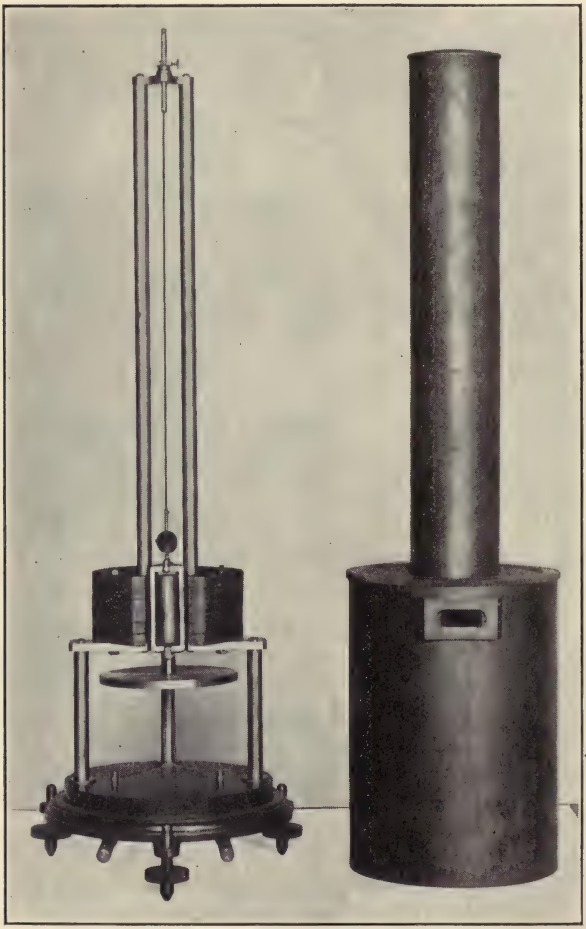

FIG. 51.—Long-period ballistic galvanometer.

by the passage of the transient current may be rendered so sluggish that the entire quantity due to the discharge of the condenser or the change of magnetic flux may have time to pass through the instrument before the system has deflected appreciably. In the

ordinary discussion of the ballistic galvanometer this is assumed to be true, but in certain cases the assumption is not tenable.

Some of the devices for obtaining a large moment of inertia are indicated in Fig. 52.

No. 1 is for the Kelvin galvanometer, while the others are for the D'Arsonval type. In 1 and 2 the crossbar is of aluminum with a screw-thread on it; the little non-magnetic weights are thus made adjustable. In 3 the weights may be removed from the pans and others substituted as desired. Instruments of very long period may have the moment of inertia increased as shown in Fig. 51. The rim of the disc, seen just below the movable coil, is made of brass, the web of aluminum.

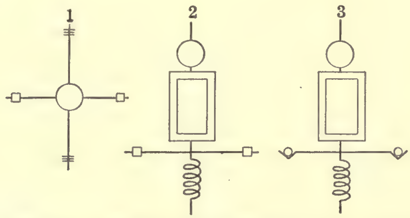

FIG. 52.—Suspended systems for ballistic galvanometer.

The time of vibration which it is necessary to give the movable system in order to obtain accurate results depends entirely on the use to which the instrument is to be put. In comparing condensers, when the resistance of the circuit is low, the discharge is practically instantaneous and an instrument with a period of about 20 sec. is convenient; so long a period is not necessary in this case for the fulfilment of the assumption that the entire discharge has passed before the movable system has been deflected appreciably, but it renders the reading of the instrument much easier. For magnetic work with the usual small-sized specimens such a period would be adequate, but for investigation of the behavior of massive electromagnets, an instrument with so short a period would be of no value whatsoever, since in this case the change of flux is very slow. When a solid iron core is tested as much as 30 sec. may elapse before the change in flux is practically complete; for such work a galvanometer with a period as great as 600 sec. is sometimes employed.

If the motion of the movable system results from a series of

impulses given as the coil swings from its zero position, or if it be due to a prolonged discharge, the magnitude of the deflection will be affected by an amount which will depend on the manner in which the galvanometer current varies.

It is frequently stated that one of the essential characteristics of a ballistic galvanometer is absence of damping, or, as damping must of necessity be present to a certain degree, that it must be reduced to a minimum. This does not mean, however, that a dampe mnstrument cannot be used ballistically; in fact, a critically dadiped ballistic galvanometer is frequently most convenient, it being a great timesaver. The damping should be electromagnetic; the law governing it is then definite and capable of a simple mathematical expression. If air damping be present, the proviso that it be small is a safe one, for its law is not exactly known. In the analytical theory it is assumed to be the same as for electro-magnetic damping.

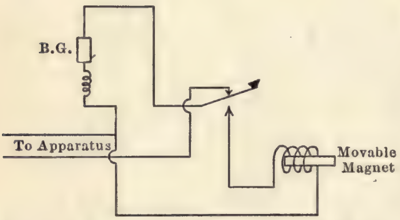

FIG. 53.—Checking device for ballistic galvanometer.

Checking Devices.—If there be little damping, it is necessary in order to economize time to have some form of checking device by which the moving system may be brought promptly to rest. For general purposes, that shown in Fig. 53 is convenient.

By a little practice the motion of the magnet and the manipulation of the key may be so timed that the zero is promptly attained. A key in series with a resistance placed across the galvanometer terminals is frequently convenient; the resistance should be of such a value that the instrument may be critically damped. Thermo-electric currents are frequently present and somewhat complicate the action of these devices.

Precautions in Reading[1].—Trouble is likely to be experienced with galvanometers of the D'Arsonval form, due to changes in direction of the very weak magnetism of the supposedly non-magnetic coil, and also possibly to "set" in the suspension. Both of these must be reduced to a minimum in the manufacture of the instrument, the first by the use of a radial

field and extreme care in the preparation of the materials used, and in the winding of the coil. The "set" may be minimized by the proper choice of suspension strip and by care in mounting it.

When readings are made they should all be taken toward the same end of the scale and the coil should not be allowed to swing very much beyond zero on its return; proper damping or use of the checking device will insure this. Before taking any readings, a deflection in the proper direction and as large as any which are to be used should be given the system; after this there will be no appreciable change of zero. This precaution should be taken each time the instrument is used.

If it is necessary to take a reading when the coil is not absolutely at rest, but swinging so that the amplitude as read on the scale is only a small fraction of a centimeter, the impulse shoud be given to the system when the swing is at its maximum and the elongation θ should be calculated from the true mechanical zero, not from the scale reading when the discharge was passed. This applies when the swinging on either side of the mechanical zero is not more than about 3 per cent. of the first elongation, θ_1.

Thermo-electromotive forces in any part of the circuit are troublesome; those arising in the galvanometer itself should be minimized by shielding from draughts or anything which could cause irregularities of temperature. In the best instruments the binding posts, connections to the movable coil and the coil itself are all of copper. In specially designed instruments the current is not taken in through the suspension, which may be of steel, but through spiral connections made of very thin copper strip. This strip may be made by rolling out a fine wire, about No. 40. The spirals may be made so delicate that they contribute practically nothing to the restoring moment.

The Calibration of a Ballistic Galvanometer.—It will be shown that the quantity of electricity which is instantaneously discharged through a ballistic galvanometer is given by

$$Q = \left(\frac{T}{2\pi}\right)\left(\frac{I_G}{\theta}\right)\left\{1 + \frac{\lambda}{2}\right\}\theta_1'$$

where

T = time of a complete swing.

θ = steady deflection caused by a steady current of strength I_G.

θ_1' = first elongation, or throw.

λ = logarithmic decrement, or natural logarithm of the ratio of two successive elongations.

In the following discussion, if the displacement of electricity is "instantaneous," the corresponding value of θ_1 will be primed; if the displacement is not instantaneous the prime will be omitted.

With any definite arrangement of the apparatus

$$Q = K'\theta_1'.$$

A determination of the time of vibration and current sensitivity, together with λ, enables the constant of the instrument to be calculated. For most purposes, however, it is preferable to calibrate by discharging a known quantity of electricity through the galvanometer and reading the corresponding deflection θ_1'. If λ at calibration differs from its value during the subsequent work, it must be determined and allowed for.

To obtain a definite quantity of electricity an earth inductor, a mutual inductance, or a standard condenser charged to a known voltage may be employed.

The earth inductor is a large coil of many turns mounted on a vertical or horizontal axis so that it can be quickly turned through 90° or 180°. The total area of the turns is known. The coil is included in the galvanometer circuit. If the plane of the coil is originally in the magnetic meridian and the rotation is through 90° about a vertical axis, the quantity of electricity displaced in the circuit is $Q = \frac{AH}{r}$. A is the total area of the turns, H the horizontal intensity of the local field and r the resistance of the galvanometer circuit. Owing to erratic variations of the local field, this method of calibration has ceased to be of importance.

Duddell has developed the idea embodied in the earth inductor so that an instrument of practical value has resulted. In his magnetic standard two movable coils arranged astatically are used in series with the galvanometer and the local field is replaced by the fields due to two oppositely wound fixed coils, one acting on each movable coil. On releasing a catch the movable system

is rotated through 180° by a spring, thus cutting the lines due to the fixed coils.

This is a secondary instrument. The number of lines cut by the movable coils is

$$n = KI$$

where I is the current in the fixed coils and K an experimentally determined constant the value of which is furnished by the instrument maker. It is seen that the number of lines cut may be varied by altering the current through the fixed coils.

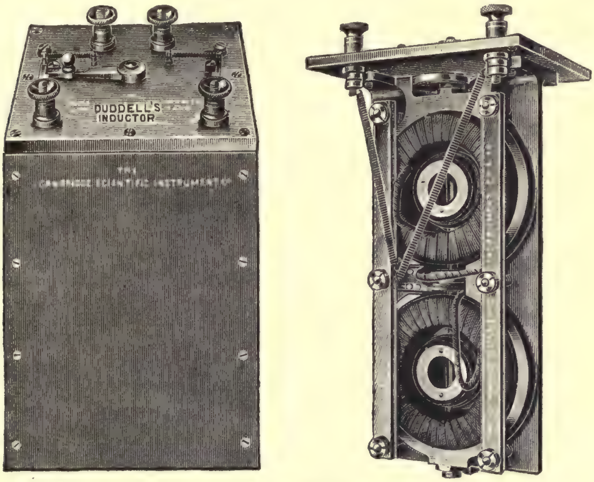

FIG. 53*a*.—Duddell inductor.

When a mutual inductance is employed, the galvanometer is placed in series with the secondary winding. The primary circuit may be arranged so that a measured value of the current may be suddenly *reversed*. In this case

$$Q = \frac{2mI}{r} \tag{1}$$

where m is the mutual inductance, I the primary current and r the resistance of the secondary or galvanometer circuit.

A common form of mutual inductance used for this purpose consists of a long, straight primary coil of one layer wound on a non-magnetic core, and a short secondary coil wound outside the

primary. The arrangement, commonly called a solenoidal inductor, is indicated in Fig. 54.

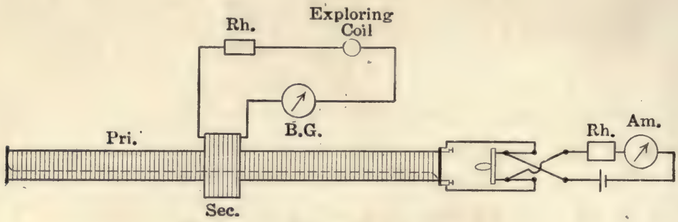

FIG. 54.—Solenoidal inductor.

Let l be the length of the primary coil, a its radius, n_P the number of primary turns per unit length, N_S the *total* number of secondary turns and A the cross-section of the primary. Then if l is many times a the mutual inductance is approximately

$$m = 4\pi n_P N_S A$$

and

$$Q = \frac{8\pi n_P N_S A I}{r},$$

all quantities being in the c.g.s. system,

or

$$Q = \frac{8\pi n_P N_S A I}{r 10^9}, \tag{2}$$

Q, I and r in the practical system.

Ordinarily when this method of calibration is employed, the instrument is to be used in a closed circuit. The damping will therefore be dependent on r and the constant of the galvanometer becomes a function of the resistance of the circuit. For this reason, it is common in magnetic work to arrange the apparatus so that the secondary of the calibrating solenoid is continuously kept in circuit with the galvanometer and the exploring coil. A substitution method is customarily used. First, a known change of flux is produced in the circuit by the mutual inductance and the change of flux per scale division of the galvanometer thus determined. After this, an observation of θ_1', corresponding to any change of flux in the specimen, enables the change in linkages to be calculated. If r is altered, recalibration is necessary.

Concerning the use of standard condensers, see page 357.

Theory of the Undamped Ballistic Galvanometer.—A D'Arsonval galvanometer with a uniform radial field will be assumed. With such an instrument when it is traversed by a steady current of strength I_G,

$$I_G C = \tau\theta$$

C is the coil constant or factor which when multiplied by the galvanometer current gives the turning moment acting on the movable system. Its value depends on the strength of field, the length of active wire and the breadth of the coil. τ is the torsion constant of the suspension, or the restoring moment per unit angular deflection; $\tau\theta$ is then the restoring moment due to twisting the strip through an angle θ.

It will be necessary to recall that when a body having a moment of inertia, P, is rotating about a fixed axis with an angular velocity, $\frac{d\theta}{dt}$, its kinetic energy is given by $E = \frac{1}{2}P\left(\frac{d\theta}{dt}\right)^2$; that when a body so rotating has its angular velocity changed, the moment of the forces producing the change is $M = P\frac{d^2\theta}{dt^2}$, where $\frac{d^2\theta}{dt^2}$ is the angular acceleration.

Suppose the coil to be at rest in its zero position and a transient current whose intensity at any instant is i_G to be sent through the instrument. Its electromagnetic action gives rise to a force which lasts for the very short time during which the current flows. This imparts a certain amount of energy to the movable system, which swings to its extreme deflection in opposition to the restoring force due to the suspension. At any instant the total energy of the system is in part kinetic and in part the potential energy stored in the twisted suspension. When the coil swings through its zero position all the energy is kinetic, while at the end of the swing it is all potential. These two amounts of energy must be equal, for by supposition there is no damping and therefore no dissipation of energy as the coil swings.

The turning moment acting on the coil at any instant is

$$i_G C - \tau\theta$$

$$\therefore P\frac{d^2\theta}{dt^2} = i_G C - \tau\theta.$$

It is assumed that the time occupied by the passage of the current is so short that the coil has not moved appreciably from its zero position; in other words, during the discharge $\tau\theta$ is zero.

$$\therefore C\int i_G dt = P\int \frac{d^2\theta}{dt^2}dt.$$

Therefore, if Q is the total quantity in the discharge,

$$CQ = P\left(\frac{d\theta}{dt}\right)_{\theta=0}$$

$\left(\frac{d\theta}{dt}\right)_{\theta=0}$ is the angular velocity at the zero position of the movable system, that is, at the time when all the energy is kinetic. The energy imparted to the system is then

$$E = \frac{1}{2}P\left(\frac{d\theta}{dt}\right)^2_{\theta=0} = \frac{1}{2}\frac{C^2Q^2}{P}.$$

The coil deflects and this amount of energy is expended in twisting the suspension through an angle θ_1'. The coil then swings back through its zero position and continues to oscillate.

The work done in twisting the suspension through the angle θ_1' is

$$W = \tau\int_0^{\theta_1'}\theta d\theta = \frac{\tau}{2}(\theta_1')^2.$$

As by supposition there is no dissipation of energy during the swing from $\theta = 0$ to $\theta = \theta_1'$,

$$\frac{C^2Q^2}{P} = \tau(\theta_1')^2$$

$$\therefore Q = \left(\frac{\sqrt{P\tau}}{C}\right)\theta_1'$$

θ_1' is the first throw or elongation. The quantities P, τ and C are not easily determined and the formula may be put in a more useful shape by introducing the time of vibration of the movable system considered as a torsion pendulum. If T_0 be the time of vibration when there is no damping,

$$T_0 = 2\pi\sqrt{\frac{P}{\tau}}.$$

Hence

$$Q = \left(\frac{T_0}{2\pi}\right)\left(\frac{\tau}{C}\right)\theta_1'.$$

There still remains the factor $\frac{\tau}{C}\cdot$ This may be evaluated as follows: If a current of constant intensity, I_G, be sent through the instrument, a deflection of constant magnitude θ will result and

$$\frac{\tau}{C} = \frac{I_G}{\theta}$$

$$\therefore\ Q = \left(\frac{T_0}{2\pi}\right)\left(\frac{I_G}{\theta}\right)\theta_1' \qquad (3)$$

Obviously, the ratio of the ballistic to the current sensitivity is

$$\frac{2\pi}{T_0}.$$

Formula for the Kelvin Galvanometer.—With a Kelvin instrument the work done in turning the suspended system through an angle θ'_1, is $MH(1 - \cos\theta'_1) = 2MH\sin^2\frac{\theta'_1}{2}$, where M is the magnetic moment of the movable system and H the strength of the controlling field.

The moment of the force due to the current in the coils is at any instant $GiM\cos\theta$, where G is the galvanometer constant or strength of field at the needle, due to a unit current in the coils. The time of vibration of the needle system considered as a magnetic pendulum is

$$T_0 = 2\pi\sqrt{\frac{P}{MH}}.$$

Therefore, in this case, the expression for Q is

$$Q = \frac{T_0 H}{\pi G}\sin\frac{\theta'_1}{2}.$$

Theory of the Damped Ballistic Galvanometer.—In the practical case a certain amount of damping is always present. It may be due to:

1. Induced currents set up in the metallic parts of the movable system by their motion through the field of the instrument.

2. Modification of the current through the instrument by the electromotive force induced in the movable coil by its motion.

3. Air friction.

4. Internal friction in the suspension wire.

In the D'Arsonval type of instrument, 4 is entirely negligible and 3 is small.

As the ballistic galvanometer is ordinarily used, the time during which the current flows is very short. But cases arise where the displacement of electricity through the instrument is not instantaneous, and such cases must be included in the general discussion.[3]

The ballistic galvanometer is commonly employed:

(*a*) In the determination of magnetic fluxes.

(*b*) In the comparison of capacities.

In case (*a*) the instrument is used in a closed circuit, consisting of the galvanometer and of the exploring coil wound around the specimen.

In any case where the instrument is used in a closed circuit of resistance, r, the equation governing the motion is (see page 40)

$$P\frac{d^2\theta}{dt^2} + \left(k' + \frac{C^2}{r}\right)\frac{d\theta}{dt} + \tau\theta = \frac{C}{r}\left(e - L\frac{di}{dt}\right) \qquad (4)$$

k' is the damping coefficient when the instrument is on open circuit and $\frac{C^2}{r}$ is the damping coefficient due to the e.m.f. set up in the main circuit by the motion of the coil. In this discussion let

$$k = k' + \frac{C^2}{r}$$

e is the instantaneous value of the e.m.f. impressed on the circuit; it is a function of t, and usually becomes zero in a comparatively short time. $-L\frac{di}{dt}$ is the back e.m.f. due to the inductance of the galvanometer circuit. In the following discussion it will be assumed to be negligible.

If the conditions are such that the current through the instrument is not modified by the e.m.f. due to the motion of the coil, then in (4) the term $\frac{C^2}{r}$ is absent and the second member

becomes Ci. The process of solution for $\int_0^t idt$ is the same as that employed below for $\int_0^t edt$.

Assuming that $L\frac{di}{dt}$ is negligible, the solution of (4) is*

$$\theta = C_1\epsilon^{m_1t} + C_2\epsilon^{m_2t} + \frac{C}{rP(m_1 - m_2)}\left[\epsilon^{m_1t}\int e\epsilon^{-m_1t}\,dt - \epsilon^{m_2t}\int e\epsilon^{-m_2t}\,dt\right] \qquad (5)$$

The values of m_1 and m_2 are given on page 26.

When the e.m.f. is first applied to the circuit the movable system is supposed to be at rest in its zero position; that is, when

$$t = 0, \qquad \theta = 0. \quad \text{and} \quad \frac{d\theta}{dt} = 0$$

Since e is a function of t these conditions are imposed if

$$C_1 = -\frac{C}{rP(m_1 - m_2)}\left[\int e\epsilon^{-m_1t}\,dt\right]_{t=0} \qquad (6)$$

and

$$C_2 = \frac{C}{rP(m_1 - m_2)}\left[\int e\epsilon^{-m_2t}dt\right]_{t=0} \qquad (7)$$

With these values of C_1 and C_2 substituted in (5) the deflection at any definite time, t, is

$$\theta = \frac{C}{rP(m_1 - m_2)}\left[\epsilon^{m_1t}\int_0^t e\epsilon^{-m_1t}dt - \epsilon^{m_2t}\int_0^t e\epsilon^{-m_2t}dt\right] \qquad (8)$$

Equations (5) and (8) apply in all cases. A difficulty is encountered in using them, since in comparatively few instances is it possible to express e as an algebraic function of t. This precludes the taking of the integrals by purely analytical methods.

In the preliminary study of a proposed investigation, if it is found that the displacement of electricity through the ballistic galvanometer will not be "instantaneous," it is necessary to inquire how much the first elongation will be influenced by the manner in which e varies. For, see pages 105 and 106, the galva-

* COHEN "Differential equations," p. 103.

nometer is calibrated by methods in which e is applied to the circuit "instantaneously."

To obtain the necessary data, the logarithmic decrement and the time of vibration must be found and a preliminary test made which will experimentally determine the curve connecting e and t. This curve must be one that fairly represents the conditions which will exist in the subsequent work, and is to be used as described below. If the computations show that the first elongation will be greatly influenced by the manner in which the discharge is sent through the galvanometer, it will be necessary to modify the instrument by giving it a longer period of vibration.

Suppose that the experimentally determined graph connecting e and t shows that e has become sensibly equal to zero before the galvanometer deflection has reached its first elongation. If the time which elapses before e becomes sensibly equal to zero be denoted by t', the values of the integrals in (8) taken for times greater than t' are practically constant. In the case where the galvanometer is over-damped, that is, where m_1 and m_2 are real, let M and N be two quantities defined as follows:

$$M = \frac{\int_0^{t \gtreqless t'} e\epsilon^{-m_1 t}dt}{\int_0^{t \gtreqless t'} edt}, \text{ a constant} \tag{9}$$

$$N = \frac{\int_0^{t \gtreqless t'} e\epsilon^{-m_2 t}dt}{\int_0^{t \gtreqless t'} edt}, \text{ a constant} \tag{10}$$

Then

$$\Big[\theta\Big]_{t \gtreqless t'} = \frac{C}{rP(m_1 - m_2)}\left[\int_0^{t \gtreqless t'} edt\right]\Big[M\epsilon^{m_1 t} - N\epsilon^{m_2 t}\Big]_{t \gtreqless t'} \tag{11}$$

θ_1, which is the observed reading of the instrument, occurs at a time t_1, when $\frac{d\theta}{dt} = 0$. Neglecting the constant coefficient in (11)

$$\frac{d\theta}{dt} = Mm_1\epsilon^{m_1 t} - Nm_2\epsilon^{m_2 t} = 0$$

or

$$\epsilon^{(m_1 - m_2)t} = \frac{Nm_2}{Mm_1}$$

$$\therefore t_1 = \frac{1}{m_1 - m_2} \log_\epsilon \frac{Nm_2}{Mm_1}.$$

Substituting this value of t_1 in (11) gives for the first elongation

$$\theta_1 = \frac{C}{rP(m_1 - m_2)} \left[\int_0^{t \gtreqless t'} edt\right] \left\{ M \left(\frac{Nm_2}{Mm_1}\right)^{\frac{m_1}{m_1 - m_2}} - N \left(\frac{Nm_2}{Mm_1}\right)^{\frac{m_2}{m_1 - m_2}} \right\}$$

or

$$\theta_1 = \frac{C}{rP(m_1 - m_2)} \left[\int_0^{t \gtreqless t'} edt\right] \left\{ \left(\frac{m_2}{m_1}\right)^{\frac{m_1}{m_1 - m_2}} - \left(\frac{m_2}{m_1}\right)^{\frac{m_2}{m_1 - m_2}} \right\} N^{\frac{m_1}{m_1 - m_2}} M^{\frac{m_2}{m_2 - m_1}} \qquad (12)$$

The quantity in the brackets $\{\}$ depends only on m_1 and m_2 and is therefore independent of the manner in which e varies. If the displacement of electricity through the galvanometer is "instantaneous," that is, if e goes through its cycle of values "instantaneously," then by (9) and (10)

$$M = 1 \qquad N = 1$$

Therefore when the motion of the movable system is non-periodic, the ratio of the actual elongation to that which would have occurred had the same *integral* change in e been made instantaneously is

$$\frac{\theta_1}{\theta'_1} = N^{\frac{m_1}{m_1 - m_2}} M^{\frac{m_2}{m_2 - m_1}} \qquad (13)$$

It is seen that (13) gives a measure of the error produced in the deflection by the prolongation of the discharge through the galvanometer.

To obtain the numerical values of M and N, the ordinates of the graph connecting e and t are multiplied by ϵ^{-m_1t} and ϵ^{-m_2t} and the two curves thus obtained are plotted to the original scale. All three curves are then integrated by a planimeter giving the necessary data for calculating both M and N.

When the motion of the movable system is periodic, that is, when $\frac{k^2}{4P^2} < \frac{\tau}{P}$, m_1 and m_2 are complex.

$$m_1 = -a + jb, \quad m_2 = -a - jb$$

where

$$a = \frac{k}{2P} = \frac{2\lambda}{T} \text{ and } b = \frac{2\pi}{T}.$$

These values of m_1 and m_2 when substituted in (8) give

$$\theta = \frac{C}{rP} \cdot \frac{\epsilon^{-at}}{2jb}\left[\epsilon^{jbt} \int_0^t e\epsilon^{(a-j)bt} dt - \epsilon^{-jbt} \int_0^t e\epsilon^{(a+jb)t} dt\right] \tag{14}$$

but

$$\epsilon^{jbt} = \cos bt + j \sin bt$$

and

$$\epsilon^{-jbt} = \cos bt - j \sin bt.$$

Hence

$$\theta = \frac{C\epsilon^{-at}}{rPb}[(\sin\ bt)\int_0^t e\epsilon^{at}\cos\ btdt - (\cos\ bt)\int_0^t e\epsilon^{at}\sin\ btdt] \tag{15}$$

The method of dealing with this equation when e falls sensibly to zero before the first elongation is reached is similar to that just employed in the nonperiodic case. See page 112.

Let R and S be defined as follows:

$$R = \frac{\int_0^{t \gtreqless t'} e\epsilon^{at} \cos btdt}{\int_0^{t \gtreqless t'} edt}, \text{ a constant} \tag{16}$$

$$S = \frac{\int_0^{t \gtreqless t'} e\epsilon^{at} \sin btdt}{\int_0^{t \gtreqless t'} edt}, \text{ a constant} \tag{17}$$

Then when $t \gtreqless t'$

$$\theta = \frac{C\epsilon^{-at}}{rPb}\left[\int_0^{t \gtreqless t'} edt\right]\left\{R \sin bt - S \cos bt\right\} \tag{18}$$

To find θ_1 determine t_1 by placing $\frac{d\theta}{dt} = 0$ and substitute the result in (18). Neglecting the constant coefficient

$$\frac{d\theta}{dt} = \epsilon^{-at}[(-aR + bS) \sin bt + (aS + bR) \cos bt] = 0$$

$$\therefore \frac{\sin bt_1}{\cos bt_1} = \tan bt_1 = \frac{aS + bR}{aR - bS} = \frac{\lambda S + \pi R}{\lambda R - \pi S}.$$

Substituting t_1 in (18) gives

$$\theta_1 = \frac{CT}{2rP}\epsilon^{-\frac{\lambda}{\pi}\tan^{-1}\frac{\lambda S + \pi R}{\lambda R - \pi S}}\left[\int_0^{t \gtreqless t'} edt\right]\frac{\sqrt{R^2 + S^2}}{\sqrt{\pi^2 + \lambda^2}} \tag{19}$$

If the same integral change in e had been made instantaneously, then by (16) and (17)

$$R = 1 \qquad S = 0$$

and the deflection would have been

$$\theta_1' = \frac{CT}{2rP}\epsilon^{-\frac{\lambda}{\pi}\tan^{-1}\frac{\pi}{\lambda}}\left[\int_0^{t \gtreqless t'} edt\right]\frac{1}{\sqrt{\pi^2 + \lambda^2}} \tag{20}$$

It follows in this case that

$$\frac{\theta_1}{\theta'_1} = \frac{\sqrt{R^2 + S^2}}{\epsilon^{\frac{\lambda}{\pi}\tan^{-1}\frac{S}{R}}} \tag{21}$$

This quantity is a measure of the error produced in the deflection by the prolongation of the discharge through the galvanometer.

The equations 19 and 20 are more conveniently expressed if C and P are replaced by quantities which are more easily determined.

Since

$$T = \frac{2\pi}{\sqrt{\frac{\tau}{P} - \frac{4\lambda^2}{T^2}}}$$

and

$$\tau\theta = I_G C,$$

substituting in (19) and solving for

$$\left[\int_0^{t \gtreqless t'} edt\right]$$

gives

$$\left[\int_0^{t \gtreqless t'} edt\right] = \left(\frac{T}{2\pi}\right)\left(\frac{I_G}{\theta}\right)\left(\frac{\pi r}{\sqrt{\pi^2 + \lambda^2}}\right)\frac{\epsilon^{\frac{\lambda}{\pi}\tan^{-1}\frac{\lambda S + \pi R}{\lambda R - \pi S}}}{\sqrt{R^2 + S^2}}\theta_1 \tag{22}$$

Discussion.—When the Displacement of Electricity is Instantaneous.—The circuit in which e acts contains the inductance

of the galvanometer. If e goes through its cycle "instantaneously," that is, in a time so short that the cycle is over before the coil of the galvanometer moves appreciably, then as the current is zero both at the start and at the finish,

$$r\int_0^t idt = \int_0^{t \gtreqless t'} edt = rQ.$$

In the case of an instantaneous displacement of electricity through the instrument the quantity, Q, is given by

$$Q = \left(\frac{T}{2\pi}\right)\left(\frac{I_G}{\theta}\right)\left\{\frac{\pi}{\sqrt{\pi^2 + \lambda^2}}\epsilon^{\frac{\lambda}{\pi}\tan^{-1}\frac{\pi}{\lambda}}\right\}\theta'_1 \tag{23}$$

This is the formula commonly used for the ballistic galvanometer. The term involving λ gives the correction for damping.

The quantity in the brackets { } may be expanded by Maclaurin's theorem, giving

$$\frac{\pi}{\sqrt{\pi^2 + \lambda^2}}\epsilon^{\frac{\lambda}{\pi}\tan^{-1}\frac{\pi}{\lambda}} = 1 + 0.5\lambda - 0.026\lambda^2 - 0.055\lambda^3 \ . \ . \ .$$

and if λ is small,

$$Q = \left(\frac{T}{2\pi}\right)\left(\frac{I_G}{\theta}\right)\left\{1 + \frac{\lambda}{2}\right\}\theta'_1 \tag{24}$$

For a secondary instrument, assuming that T is not sensibly affected by any change in damping which is likely to be encountered,

$$Q = K\left\{1 + \frac{\lambda}{2}\right\}\theta'_1 \tag{25}$$

where K is a constant.

Another approximation sometimes used in making the correction for damping, results from assuming λ to be very small, in which case

$$\frac{\pi}{\sqrt{\pi^2 + \lambda^2}}\epsilon^{\frac{\lambda}{\pi}\tan^{-1}\frac{\pi}{\lambda}} = \epsilon^{\frac{\lambda}{\pi}\cdot\frac{\pi}{2}} = \epsilon^{\frac{\lambda}{2}} \text{ approximately.}$$

But, from the law of damped oscillations (page 30),

$$\frac{\theta_1}{\theta_2} = \epsilon^{\lambda} \qquad \frac{\theta_1}{\theta_3} = \epsilon^{2\lambda}.$$

Consequently

$$\epsilon^{\frac{\lambda}{2}} = \left(\frac{\theta_1}{\theta_2}\right)^{1/2} = \left(\frac{\theta_1}{\theta_3}\right)^{1/4}$$

and Q is given approximately by

$$Q = \left(\frac{T}{2\pi}\right)\left(\frac{I_G}{\theta}\right)\left(\frac{\theta_1}{\theta_2}\right)^{\frac{1}{2}}\theta'_1 = \left(\frac{T}{2\pi}\right)\left(\frac{I_G}{\theta}\right)\left(\frac{\theta_1}{\theta_3}\right)^{\frac{1}{4}}\theta'_1 \tag{26}$$

Discussion.—When the Discharge through the Instrument is Prolonged.—If the equation connecting e and t is known, the integrations indicated in (5), (6) and (7) may be performed. Suppose for example that

$$e = E_0\epsilon^{-pt}.$$

In this case, which is of practical importance[4], the total quantity of electricity displaced in this circuit is

$$Q = \frac{1}{r}\int_0^{\alpha} edt = \frac{E_0}{rp}$$

Substituting the value of e in (5),

$$\theta = \frac{CE_0}{2rPjb}\left\{\frac{\epsilon^{m_1t}}{p+m_1} - \frac{\epsilon^{m_2t}}{p+m_2}\right\} + \frac{CE_0}{rP}\cdot\frac{\epsilon^{-pt}}{(p+m_2)(p+m_1)} \tag{27}$$

To find t_1, the time of the first elongation or the turning point, this value of θ is differentiated and the result placed equal to zero. Combining the equation so formed with (27) gives

$$\theta_1 = \frac{CE_0}{2rPjbp}\left\{\epsilon^{m_1t_1} - \epsilon^{m_2t_1}\right\}$$

or after the values of m_1 and m_2 are substituted,

$$\theta_1 = \left(\frac{CT}{2\pi P}\right)\left(\frac{E_0}{rp}\right)\epsilon^{-\frac{2\lambda}{T}t_1}\sin\left(\frac{2\pi}{T}\right)t_1 \tag{28}$$

Equating $\frac{d\theta}{dt}$ to zero gives t_1, the time of the first elongation, as a solution of the equation

$$\epsilon^{-\left(p-\frac{2\lambda}{T}\right)t_1} = \cos\left(\frac{2\pi}{T}\right)t_1 + \left(\frac{-p\lambda T + 2\lambda^2 + 2\pi^2}{\pi pT}\right)\sin\left(\frac{2\pi}{T}\right)t_1. \tag{29}$$

The value of t_1 is obtained by successive approximation.

Referring to (20) and (28) it will be seen that the ratio of the deflection to that which would have been obtained had the same quantity been discharged through the galvanometer instantaneously is

$$\frac{\theta_1}{\theta'_1} = \left(\frac{\sqrt{\pi^2 + \lambda^2}}{\pi}\right)\left(\frac{\epsilon^{-\frac{2\lambda}{T}t_1}}{\epsilon^{-\frac{\lambda}{\pi}\tan^{-1}\frac{\pi}{\lambda}}}\right)\sin\left(\frac{2\pi}{T}\right)t_1 \tag{30}$$

This ratio gives a measure of the effect produced on the first elongation by the prolongation of the discharge.

As another example, take one which may be realized by the manipulation of the apparatus used for calibrating a ballistic galvanometer by means of a mutual inductance.

Suppose the primary circuit to be traversed by a current. At a time $t = 0$ the circuit is broken by the reversing switch. This *instantaneously* removes $\frac{n}{2}$ magnetic linkages from the secondary. After an interval of 10 sec., the circuit is made in the reverse direction and a second *instantaneous* change of $\frac{n}{2}$ linkages is made.

It is desired to know how the deflection so obtained will compare with that which would have been obtained had the change of n linkages been made instantaneously.

In this case using (16) and (17)

$$R = \frac{\int_o^{t \gtreqless 10} e\epsilon^{at}\cos bt\,dt}{\int^{t \gtreqless 10} e\,dt} = \frac{\frac{n}{2} + \frac{n}{2}\epsilon^{10a}\cos 10b}{n}$$

$$S = \frac{\int^{t \gtreqless 10} e\epsilon^{at}\sin bt\,dt}{\int^{t \gtreqless 10} e\,dt} = \frac{0 + \frac{n}{2}\epsilon^{10a}\sin 10b}{n}$$

Suppose the following data apply to the galvanometer in question:

Time of a complete vibration $= T = 149$ sec.

Ratio of two successive swings $= \frac{\theta_1}{\theta_2} = 1.063.$

Logarithmic decrement $= \lambda = \log_\epsilon 1.063 = 0.0611.$

$$b = \frac{2\pi}{T} = 0.0422$$

$$a = \frac{b\lambda}{\pi} = \frac{2\lambda}{T} = 0.00082$$

Consequently

$$R = \tfrac{1}{2}[1 + \epsilon^{0.0082} \cos 0.422] = 0.9597$$
$$S = \tfrac{1}{2}[\epsilon^{0.0082} \sin 0.422] = 0.2064$$
$$\sqrt{R^2 + S^2} = 0.982$$
$$\epsilon^{\frac{\lambda}{\pi}\tan^{-1}\frac{S}{R}} = \epsilon^{0.0041} = 1.004.$$

By (21)

$$\frac{\theta_1}{\theta'_1} = \frac{0.982}{1.004} = 0.978.$$

The error is therefore 2.2 per cent. An actual test under the above conditions gave

$$\frac{\theta_1}{\theta'_1} = 0.978.$$

When the ballistic galvanometer is used in series with a test coil for determining magnetic fluxes through iron cores it is not possible to apply the above purely analytical method, for the law connecting time and the e.m.f. induced in the test coil is not known. In this case the integration must be made with the aid of a planimeter.

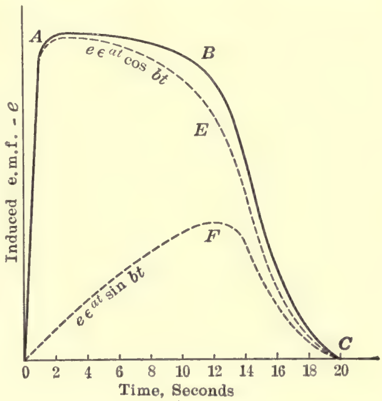

FIG. 55.—Pertaining to effect of prolonged discharge through a ballistic galvanometer.

The graph connecting e and t may be obtained by an oscillograph, an exploring coil being wound for that purpose on the specimen under test. In general, the oscillograph is not inserted in the galvanometer circuit for the current in it is modified by the e.m.f. set up by the movable coil. In cases where this e.m.f. is insignificant compared with that due to the change of flux through the circuit, the separate exploring coil is not necessary. Suppose the e.m.f.-time curve $OABC$, shown in Fig. 55, has been obtained.

The curves OEC and OFC are obtained by multiplying the ordinates of $OABC$ by the corresponding values of $\epsilon^{at} \cos bt$ and $\epsilon^{at} \sin bt$.

From the figure it will be seen that t' is 20 sec. This means that e is practically zero after this time; in reality, a very small e.m.f. may persist for a considerably longer period but the quantity of electricity displaced by it may be neglected.

$\int_0^{t'} edt$ is obtained from the area under the curve $OABC$. Integrating the curves by a planimeter, and using (16) and (17).

$$R = 0.920$$
$$S = 0.320$$

therefore

$$\sqrt{R^2 + S^2} = 0.973$$

if $\lambda = 0.0611$

$$\epsilon^{\frac{\lambda}{\pi}\tan^{-1}\frac{S}{R}} = \epsilon^{0.0065} = 1.007.$$

By (21)

$$\frac{\theta_1}{\theta'_1} = \frac{\sqrt{R^2 + S^2}}{\epsilon^{\frac{\lambda}{\pi}\tan^{-1}\frac{S}{R}}} = \frac{0.973}{1.007} = 0.966.$$

The error due to the prolongation of the discharge is 3.4 per cent. for this particular form of e.m.f. curve.

The Critically Damped Ballistic Galvanometer.—Mathematically, critical damping occurs when the roots of the equation $Pm^2 + km + \tau = 0$ are equal, or when $m_1 = m_2 = -\frac{k}{2P}$ corresponding to

$$\frac{k^2}{4P^2} = \frac{\tau}{P}.$$

In this case, the solution of (4) becomes

$$\theta = (C_1 + C_2t)\epsilon^{-\frac{kt}{2P}} + \frac{C}{rP}\left[t\epsilon^{-\frac{kt}{2P}}\int e\epsilon^{\frac{kt}{2P}}\,dt - \epsilon^{-\frac{kt}{2P}}\int te\epsilon^{\frac{kt}{2P}}\,dt\right] \qquad (31)$$

The movable system is supposed to be at rest in its zero position, when e is applied to the circuit, that is, when

$$t = 0 \quad \theta = 0.$$

$$t = 0 \quad \frac{d\theta}{dt} = 0.$$

These conditions will be fulfilled if

$$C_1 = \frac{C}{rP}\left[\int te\epsilon^{\frac{kt}{2P}}\,dt\right]_{t=0} \tag{32}$$

$$C_2 = -\frac{C}{rP}\left[\int e\epsilon^{\frac{kt}{2P}}\,dt\right]_{t=0} \tag{33}$$

so

$$\theta = \frac{C}{rP}\epsilon^{-\frac{kt}{2P}}\left[t\int_0^t e\epsilon^{\frac{kt}{2P}}\,dt - \int_0^t te\epsilon^{\frac{kt}{2P}}dt\right] \tag{34}$$

If the integral change in e is "instantaneous"

$$\theta = \frac{C}{rP}\left[\int_0^t edt\right] t\epsilon^{-\frac{kt}{2P}} \tag{35}$$

The first elongation occurs when

$$\frac{d\theta}{dt} = 0 = 1 - \frac{kt}{2P}$$

$$t_1 = \frac{2P}{k} = \sqrt{\frac{P}{\tau}} = \frac{T_0}{2\pi}$$

Substituting this value of t_1 gives

$$\left[\int_0^t edt\right] = \frac{2\pi}{T_0}\cdot\frac{rP}{C}\epsilon\theta'_1 = \left(\frac{T_0}{2\pi}\right)\left(\frac{I_G}{\theta}\right)r\epsilon\theta'_1 = K\theta_1' \tag{36}$$

where θ is the deflection due to a steady current, I_G.

It is seen that the elongation is proportional to the quantity of electricity in the discharge, and that the galvanometer factor is ϵ times that of the undamped instrument. Consequently, the quantity sensitivity is $\frac{1}{\epsilon}$, or 37 per cent that of the undamped instrument. Also, the time necessary for arriving at the elongation θ_1 is $\frac{2}{\pi}$, or 64 per cent that of the undamped instrument.

Equation (36) applies when the ballistic galvanometer is used in a series circuit; the resistance, r, being such that the galvanometer is critically damped. When the instrumental constants used in equation (4) are introduced, (36) becomes

$$\left[\int_0^t edt\right] = \left(\frac{\epsilon C\sqrt{\tau P}}{2\sqrt{\tau P} - k'}\right)\theta'_1 \tag{37}$$

If the resistance of the apparatus to which the galvanometer is attached is so high that the instrument is underdamped, critical damping may be obtained by shunting the galvanometer. In this case if R is the resistance of the apparatus to which the galvanometer is attached and R_G that of the galvanometer,

$$\left[\int_0^t edt\right] = \left(\frac{\epsilon RC\sqrt{\tau P}}{C^2 - 2R_G\sqrt{\tau P} + R_G k'}\right)\theta_1' \tag{38}$$

As no time is lost in bringing the movable system to rest, the critically damped ballistic galvanometer is frequently a most convenient instrument.

The Use of Shunts with the Ballistic Galvanometer.—At first sight it would seem that if a ballistic galvanometer were shunted with a non-inductive resistance, the total quantity of electricity discharged from a condenser would not, on account of the inductance of the galvanometer, divide inversely as the resistance of the galvanometer and of the shunt. However, the quantity does so divide as will be seen from the following:

Let R_G = galvanometer resistance.
L_G = galvanometer inductance.
i_G = galvanometer current.
Q_G = quantity discharged through galvanometer.
S = shunt resistance.
L_S = shunt inductance.
i_S = shunt current.
Q_S = quantity discharged through shunt.

Then

$$R_G i_G + L_G \frac{di_G}{dt} = Si_S + L_S \frac{di_S}{dt}.$$

Consequently

$$R_G Q_G + L_G \int di_G = SQ_S + L_S \int di_S$$

Both currents are zero at the beginning and zero at the end of the discharge, so

$$\frac{Q_G}{Q_S} = \frac{S}{R_G}.$$

It is seen that any error caused by the shunt must be due to the variation of the damping when the multiplying power is changed.

The total quantity of electricity discharged through a shunted instrument which is slightly damped is

$$Q = Q_G\left(\frac{R_G + S}{S}\right) = K\left\{1 + \frac{\lambda}{2}\right\}\left(\frac{R_G + S}{S}\right)\theta'_1 \text{ nearly enough.}$$

Considering the damping to be electromagnetic and that the shunted galvanometer is used on an open circuit,

$$\lambda = \frac{kT}{4P} = \frac{C^2T}{(R_G + S)\,4P}$$

$$\therefore Q = K\left\{\frac{R_G + \dfrac{C^2T}{8P} + S}{S}\right\}\theta'_1.$$

But $\dfrac{C^2T}{8P}$ is practically constant. Therefore when the damping is not large, the multiplying power to be used with ballistic throws may be obtained by considering the galvanometer resistance to be increased by a certain constant amount which depends upon the construction of the instrument. The effective multiplying power of the shunt is

$$m = \frac{(R_G + A) + S}{S}.$$

This is Latimer Clark's method of correction. The quantity A may be determined experimentally. Suppose a condenser to be charged to V_1 volts and discharged through the shunted instrument; then

$$Q_1 = K\left\{\frac{(R_G + A) + S}{S}\right\}\theta'_1 = CV_1.$$

When the condenser is charged to V_2 volts and discharged through the unshunted galvanometer,

$$Q_2 = K\left(\theta'_1\right)_2 = CV_2$$

$$\therefore A = S\left(\frac{V_1}{V_2}\right)\frac{(\theta'_1)_2}{(\theta'_1)} - (R_G + S) \qquad (39)$$

A practical difficulty met with when applying shunts of the ordinary sort to a moving-coil ballistic galvanometer is that the range of the instrument cannot be greatly extended before the damping becomes excessive.

In open-circuit work, that is, in measurements upon condensers

or cables, the universal shunt (page 52) should be used. The advantage of this arrangement is that the resistance through which the damping current, set up by the motion of the coil, must flow is always the same. Consequently, λ does not vary, even though the multiplying power of the shunt be changed. Another advantage is that the total shunt resistance, r, may be made so great that the instrument is not over-damped, even though it is heavily shunted.

Obviously the universal shunt loses its advantages, if the condenser be replaced by a closed circuit, as an exploring coil for magnetic work.

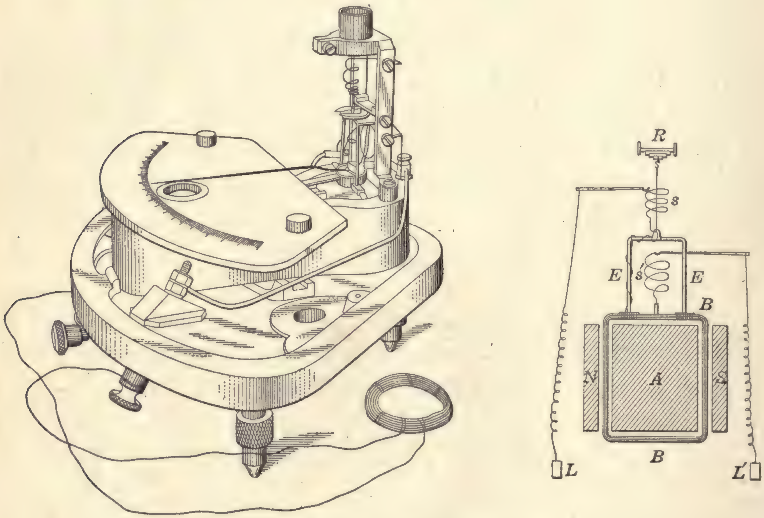

FIG. 56.—Flux meter.

The Flux Meter.—The flux meter,[6] as its name indicates, is designed for measuring the flux in magnetic circuits. This instrument is essentially a moving coil ballistic galvanometer in which the restoring moment is reduced to a minimum by the removal of the controlling spring. In a perfect instrument the restoring couple would be zero; practically it is difficult to reduce the controlling action of the necessary leads to the movable coil to a negligible amount and usually after the movable system has been deflected it very gradually sinks back towards zero.

Fig. 56 shows the instrument and the suspended system.

The moving coil is hung by a silk fiber from the spring support R; the current is led to the coil through the delicate spirals S, S, which are of annealed silver and supposed to exercise no controlling effect on the movable system. A mechanical device is employed by which the system may be quickly reset to its zero position.

The instrument is used in series with an exploring coil and therefore in a closed circuit. When the change of flux through the test coil is completed, the movable system comes to rest in its deflected position and the change in linkages is given by

$$n = C\theta_1 \tag{40}$$

which is true, irrespective of the manner in which the flux is changed and of the time occupied in making the change. This connection between the deflection and the change of flux linkages may be seen from equation (12).

When the torsional control, τ, is indefinitely reduced, m_1 and m_2 which are the roots of the equation $Pm^2 + km + \tau = 0$ approach the values

$$m_1 = 0$$
$$m_2 = -\frac{k}{P}.$$

Under these conditions (see page 112).

$$M = 1,$$
$$N = \text{some definite numerical value},$$

and the factor $N^{\frac{m_1}{m_1-m_2}} M^{\frac{m_2}{m_2-m_1}}$ in (12), which depends on the manner in which the discharge takes place, always becomes unity. The factor in (12) which is within the brackets also approaches unity, so if n is the total change of flux linkages

$$\theta_1 = \frac{Cn}{rk}.$$

As

$$k = \frac{C^2}{r} + k', \text{ see page 110.}$$

$$n = \left(C + \frac{k'r}{C}\right)\theta_1.$$

The air damping is very small so $n = C\theta_1$, approximately. This relation may be proved directly as follows.

Let n = number of linkages of flux with exploring coil.
θ_1 = final value of deflection.
C = coil constant.
ω = angular velocity of movable coil.
$\left(\frac{dn}{dt}\right)$ = rate of change of linkages through exploring coil.
L = inductance of exploring coil circuit.
P = moment of inertia of movable system.
k' = damping constant for air damping.
r = resistance of exploring coil circuit.
i = current at any instant.
N' = number of turns in exploring coil.
φ = flux through exploring coil.

As soon as the coil begins to move, it sets up a back e.m.f. whose value is ωC.

The current at any instant is

$$i = \frac{\frac{dn}{dt} - C\omega - L\frac{di}{dt}}{r}$$

and

$$P\frac{d\omega}{dt} = Ci - k'\omega = \frac{C}{r}\left(\frac{dn}{dt}\right) - \frac{C^2\omega}{r} - \frac{CL}{r}\frac{di}{dt} - k'\omega.$$

At the beginning and at the end of the deflection, both ω and i are zero, so on integrating,

$$0 = \frac{Cn}{r} - \left(\frac{C^2}{r} + k'\right)\int \omega dt$$

$$\therefore n = \left(C + \frac{k'r}{C}\right)\theta_1.$$

The air damping is small. Very closely then the change of flux linkages is given by

$$n = C\theta_1$$

and

$$\varphi = \frac{C\theta_1}{N'}.$$

hat is, the deflection is proportional to the change in the flux hreading the exploring coil. In the actual instrument the scale is calibrated experimentally. The advantages of the flux meter over the ballistic galvanometer are portability, ease of reading, and independence of the time required for the flux changes and of the manner in which they take place. It is therefore a convenient workshop instrument. As ordinarily constructed, its accuracy is inferior to that of the ballistic galvanometer.[7]

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CHAPTER III

RESISTANCE DEVICES

The resistance devices used in electrical measurements may be divided into two groups, resistance boxes and rheostats.

A resistance box is a device by which the resistance of a circuit may be altered by accurately known amounts. It contains an aggregation of coils, each coil having a definite and known resistance, and the construction is such that the coils may be connected in various combinations so that any required resistance, up to the full capacity of the box, may be obtained.

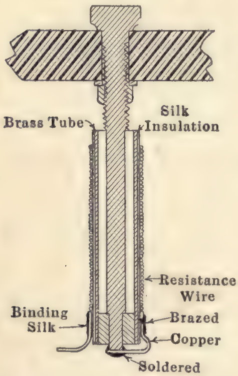

FIG. 57.—Section of resistance coil.

The term rheostat is applied to the devices commonly used for varying the resistance of the circuit where regulation of currents and absorption of power are concerned. The circumstances under which rheostats are used render it unnecessary that the magnitudes of the variations in resistance be known.

Resistance Coils.—Formerly it was the universal practice to wind resistance coils on wooden bobbins, but, in the better class of work, these bobbins have been replaced by metal spools (see Fig. 57). A layer of shellaced silk which is dried out by baking before the coil is wound serves to thoroughly insulate the wire from the metal spool.

Non-inductive windings are always employed. The wire is arranged in a bight before it is wound upon the bobbin and the two wires are kept side by side in the coil.

If possible, the winding is concentrated in a single layer, for as all the heat must be dissipated through the surfaces of the coil, one wound several layers deep with a large wire is not superior to one wound with but a single layer of small wire.

After it is wound, the coil is shellaced and then baked for 10 hours or more at 140°C.; this frees the entire coil of moisture and alcohol and at the same time anneals the wire. After baking, the coil should be given a protective coating of paraffin.

The resistance wires are hard-soldered to copper terminals which in turn are soft-soldered to the working terminals.

The prime requisite of the resistance material used for winding coils is permanence. In addition, its temperature coefficient should be small, its thermal e.m.f. when opposed to copper, low, and to obtain compactness, its resistivity should be high. Alloys rather than the pure metals are used for they have smaller temperature coefficients and higher resistivities. To settle the all-important question of permanence prolonged investigation is of course necessary. Up to the present time, the alloy which has most commended itself is that known as manganin.[2] Other alloys are used for certain kinds of work but manganin has been under critical examination longer than the others and its properties are more definitely known.

Edward Weston discovered in 1889, that alloys of copper and nickel containing some manganese have very small temperature coefficients and high resistivities. Investigation has shown that the particular alloy known as manganin is, when properly employed, sufficiently permanent for resistance coils and resistance standards.

The composition of manganin is given as 84 per cent. copper, 12 per cent. manganese, and 4 per cent. nickel. Its resistivity at 20°C. is about 44.5 microhms (cm.) and its thermo-electromotive force when opposed to copper is only 0.000002 volt per degree C. To insure permanence this material must be protected by a well-dried coating of shellac.

The curious effect of a rise of temperature on a manganin resistance coil is shown in Fig. 58. The point at which the temperature coefficient changes sign varies with different samples of wire.

It will be noted that the temperature coefficient is very small, the average value between 15°C. and 20°C. being only 0.0005 per cent. For engineering work, consequently, temperature corrections may be neglected. In work of the highest precision (a few parts in 100,000) temperature corrections must be made,

in which case the coefficient applying to the particular coil in hand must be employed.

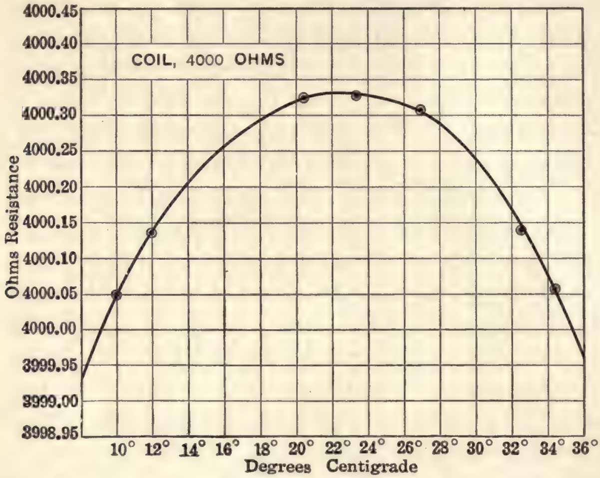

FIG. 58.—Showing effect of temperature on a manganin resistance coil.

The possible effects of a rise of temperature on a resistance coil are:

1. If the rise of temperature is small there will be a temporary change in the resistance; that is, one which disappears when the normal temperature is regained.

2. If the rise of temperature is great there will be a permanent alteration of the resistance.

3. At a still higher temperature the insulation will be impaired.

It is evident that the ability of a resistance coil to dissipate the heat due to the passage of the current is most important, and as the precisions which may be obtained with various methods of measurement are proportional to the currents employed, it is advisable, when selecting a resistance box for general use, to choose one having coils of a high watt capacity.

In any case the safe working current is that which will heat the coil to a temperature just below that at which a permanent alteration, or "set," in the resistance will take place. A coil adjusted to 0.01 per cent., which is expected to maintain this degree of reliability, must be more carefully treated than one certified to 0.05 per cent. which is a degree of adjustment fre-

quently used in the better class of bridges and resistance boxes for general use.

No definite statement can be made as to the safe carrying, or watt capacity, of the resistance coils used in boxes for general laboratory work, for it depends on the construction of the coils and of the box in which they are mounted and on the accuracy of the initial adjustment of the coils. If the coils are of good construction and wound on wooden bobbins 0.5 watt per coil may be allowed. If metal spools in metallic connection with the connecting blocks on the top of the box are used, 3 watts per coil is a safe allowance. These figures are for coils enclosed in wooden boxes as is ordinarily the case. If many coils in the same box are simultaneously used lower figures must be employed.

By immersion in oil, which is well stirred, the watt capacity of a coil on a metal bobbin is increased to about 7 watts.

Standard Resistances[1].—Standard resistances are used for two purposes:

1. As standards of reference with which other resistances are compared.

2. As current carrying standards for use in potentiometer methods.

With the first class of coils, permanence is all important. The watt capacity is not so important provided it is great enough to allow comparisons to be made with the desired precision.

Experience has shown that lack of permanence in the finished coil may be due to corrosion, to stresses in the coil owing to the fact that the wire is wound on a small spool, to stresses due to the absorption of moisture by the insulating materials, and to the use of soft solder at the terminals which may crack and alter the effective length of the wire. These factors are now generally recognized and the coils prepared accordingly.

For coils of the second class, a high carrying capacity is absolutely necessary and one can tolerate a less degree of permanence provided comparisons are made from time to time with carefully preserved resistance standards; as a matter of convenience permanence is highly desirable.

The designs which are commonly employed for resistance standards are those developed at the Reichsanstalt, Charlottenburg,

and at the Bureau of Standards, Washington. Fig. 59 shows a group of these coils.

For standards having a resistance of $\frac{1}{10}$ ohm and greater, the resistance material is used in the form of wire; below $\frac{1}{10}$ ohm, strips are employed.

The coils are kept cool by immersion in oil. Great care must be taken that there are no acids or sulphur in the oil and that it

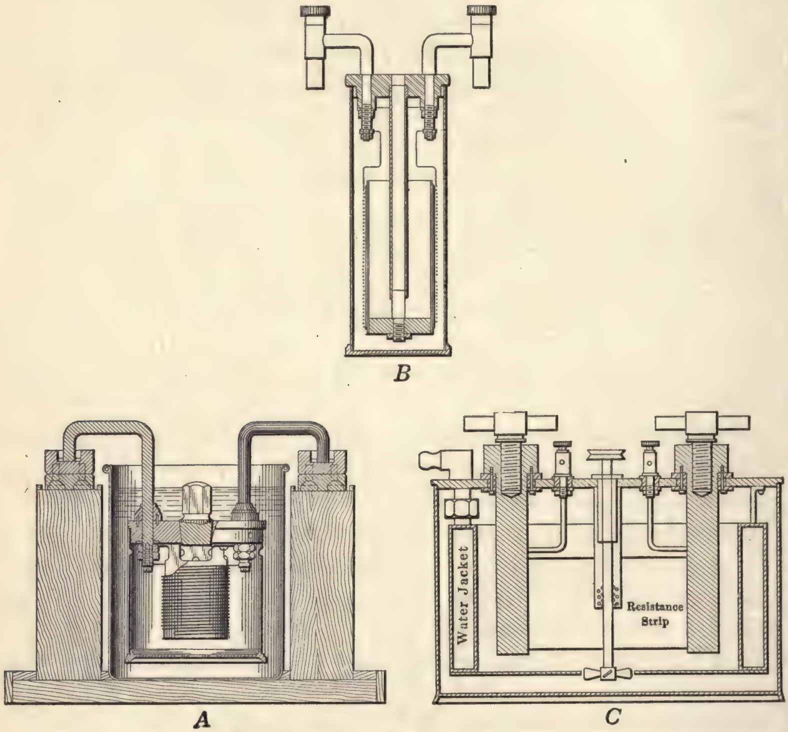

FIG. 59.—Standard resistances.

is kept free from water. Otherwise the resistance material may be attacked and the accuracy of the standards impaired. The oil should be renewed from time to time.

As a result of careful experiments at the Bureau of Standards it has been found that coils constructed like that shown at A in Fig. 59 are subject to slight variations due to the absorption of moisture by the shellac used in insulating the windings. This

swells and stresses the wire. The effect is most noticeable when fine wires are used; that is, in high resistance coils. It follows the seasonal variations of atmospheric humidity, and may be as great as 0.04 per cent. in a 1,000-ohm coil.

Immersion of the coil in oil which is freely exposed to the air, though retarding the effect, is not an absolute preventive, for the oil absorbs moisture and imparts it to the shellac. To overcome this source of error, Rosa has developed the form of sealed resistance standard[3] shown at *B* in Fig. 59.

The coil itself is prepared as specified by the Reichsanstalt, being insulated with silk and shellac and thoroughly dried by baking; the bobbin is supported from the cover by the thermometer tube. The case is nearly filled with a pure oil which has been carefully freed from moisture. The cover is then screwed into the protective case and the joint sealed with shellac. Potential terminals are used with coils of 1 ohm or less. Sealed standard coils show no seasonal variations in resistance.

Fig. 59*C* shows a current-carrying standard. Such resistances are immersed in oil, which is stirred and kept cool by a water jacket through which there is a brisk circulation. It is essential that the strips of resistance material be protected by a coating of shellac.

Resistance Coils for Alternating-current Work.—Though the bifilar winding usually employed in resistance coils reduces the inductance to a minimum, it increases the possibility of capacity effects, since at the terminals of the coil the two wires are separated by only the double thickness of insulation and have full voltage between them; the P.D. between the wires decreases with the increase of the distance from the terminals, becoming zero at the end of the bight.

It is clear that with alternating currents, especially at high periodicities, the behavior of the coil will be modified by its distributed capacity and inductance. The high resistivity of the wires eliminates trouble from skin effect, which at 3,000 cycles with a manganin wire 2 mm. in diameter is only about 1 part in 100,000.

Assuming as an approximation that the formulæ for two parallel wires apply in this case, Curtis and Grover have deduced the following relations[7]:

Effective resistance, $R' = R\,[1 + \omega^2 C(\tfrac{1}{3}L - \tfrac{2}{15}CR^2)]$.

Effective inductance, $L' = L - \tfrac{1}{3}CR^2$.

Phase displacement, $\tan^{-1}\dfrac{\omega\,(L - \tfrac{1}{3}CR^2)}{R}$.

R and L are the total resistance and inductance of the coil and C is the capacity between the wires if they be separated at the end of the bight.

It is seen that capacity and inductance tend to neutralize each other. In coils of low resistance with only one layer of bifilar winding, the inductance will preponderate, but when the resistance of the coil is increased, the effect of capacity increases more rapidly than that of inductance; consequently, there will be a point where the two effects will be balanced. The balance point is practically independent of the frequency.

From the formulæ it is evident that the phase displacement and the change of resistance cannot *both* be made zero, but if $L = \tfrac{1}{3}CR^2$ the phase displacement is zero and the change in resistance negligible.

After having found a construction which gives a practically non-reactive coil, higher resistances may be built up by simply placing coils in series. The several sections of a high-resistance coil may be wound side by side on the same spool which should be of porcelain. The use of a non-conducting spool avoids troublesome capacity effects between the sections.

Curtis and Grover[7] recommend the following constructions: The coils are designed for oil immersion and have approximately 50 sq. cm. surface. With an expenditure of 1 watt per coil the rise of temperature above the surrounding oil is about 1°, the bobbin being of poor thermal conductivity.

Tenth-ohm Coils.—Only the inductance need be considered. The coil is to be made of manganin strip $\tfrac{1}{10}$ mm. thick, width about 3 mm., length about 10 cm. The strip is to be folded back on itself at the middle of its length and the two halves bound together with insulation between them. The effective inductance is about +0.005 microhenry.

One-ohm Coils.—Only the inductance need be considered. The coil is to be of manganin strip $\tfrac{1}{10}$ mm. thick and 3 mm. wide, folded back on itself at the middle of its length, and with

proper insulation between the halves. The effective inductance is about $+0.05$ microhenry.

Ten-ohm Coils.—Only the inductance need be considered. Three 30-ohm coils are used in parallel. They are bifilar wound on spools 2.5 cm. in diameter, a single layer of double silk-covered manganin wire 0.24 mm. in diameter being employed. The effective inductance is approximately 0.3 microhenry.

One-hundred ohm Coils.—The coils are bifilar wound in one section, a single layer of double silk-covered manganin wire 0.24 mm. in diameter being used. The capacity effect preponderates, resulting in an effective inductance of -1.6 microhenrys; the phase angle at 2,000 cycles per second is about $35''$.

One-thousand-ohm Coils.—Five sections, each of two hundred ohms, are used in series. Each section consists of a single layer of double silk covered manganin wire 0.10 mm. in diameter. The five sections are bifilar wound on a spool 2.5 cm. in diameter, a space of 2 or 3 mm. being left between the sections. The effective inductance is about $-16.$ microhenrys.

Five-thousand-ohm Coils.—Five thousand-ohm coils may be built up of five of the above 1,000-ohm coils in series or the bifilar winding may be replaced by that illustrated in Fig. 60.

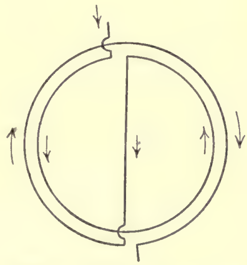

FIG. 60.—Special non-reactive winding for high-resistance coils.

The coil is wound on a porcelain cylinder which is slit along a diameter for about two-thirds of its length. Double silk-covered manganin wire is used. The wire goes once around the bobbin, then passes through the slit and around the bobbin in the opposite direction back through the slit. This cycle is repeated until the whole coil is wound. The capacity effect is very small as there is only a small P.D. between adjacent wires. The effective inductance is about $+30.$ microhenrys. The disadvantage is the difficulty of winding.

Ten-thousand-ohm Coils.—These are constructed by using in series two of the 5,000-ohm units just described. The effective inductance is about $+100.$ microhenrys.

In the following table are shown comparative results given by

the above coils and by coils as ordinarily supplied by representative American and German instrument makers.

TABLE SHOWING INDUCTANCE EFFECTS IN RESISTANCE COILS

Nominal resistance of coil, ohms	Effective inductance in microhenrys at 1,200 cycles per second			Change in resistance, 0 to 1,200 cycles per second		
	New coil	American	German	New, per cent,	American, per cent,	German, per cent,
0.1	0.005	0.14	0.18			
1.0	0.05	0.4	0.5			
10.0	0.3	0.9	1.0			
100.0	−1.6	−5.0	−2.0			
1,000.0	−16.0	−400.0	−100.0	<.001	−0.08	−0.05
5,000.0	30.0	....	−27,500.0	<.001	....	−0.2
10,000.0	100.0	....	−100,000.0	<.001	....	−1.0

Micanite cards wound with resistance wire, such as are used for series resistances in the potential circuits of alternating-current

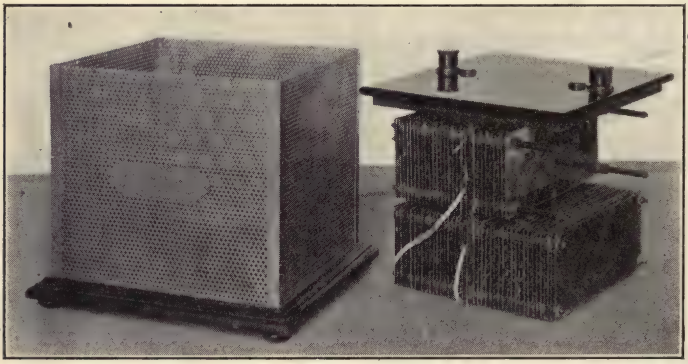

FIG. 61.—Showing mounted resistance cards.

instruments are highly satisfactory for general laboratory purposes as they have considerable surface. Capacity effects should be minimized by mounting the cards so that they are at least a centimeter apart. The time constant of one of these cards is 10^{-6} to 10^{-7} second. Duddell and Mather's plan of weaving a fine wire as the woof in a silk warp gives practically the same results as the cards.

Low-resistance Shunts for Use in Alternating-current Measurements.—In many of the modern methods for alternating-current measurements, for instance, those for determining the ratios and phase angles of current transformers and for measuring power by the electrostatic wattmeter, "non-inductive" shunts having large carrying capacities are employed.

It is necessary that the inductance be reduced to a minimum, for the phase displacement between the current and the P.D. at the shunt terminals must be as small as possible and the impedance must be sensibly the same as the direct-current resistance. Also, when the inductance is reduced to a minimum the stray field and therefore the disturbing effect on neighboring instruments is correspondingly decreased; this may be of importance when large currents are dealt with.

If low resistances are employed, the amount of inductance which can be tolerated is exceedingly small. For instance, at 60 cycles per second a phase displacement of 0°.04 will be produced in a resistance of 0.001 ohm by an inductance of 0.000000002 henry or 0.002 microhenry or 2 cm. c.g.s. At low power factors even this inductance is of importance when power measurements are made with the electrostatic wattmeter. These small inductances, having a magnitude of only a few centimeters, can be attained only by special care in design.

As such low resistances are for use with large currents and the voltage drop in them is likely to be considerable, special means must be provided for dissipating the heat generated. If air cooling is relied upon the shunt becomes bulky and expensive, so it is generally immersed in oil which is violently stirred and is kept cool by a water jacket, as in the Reichsanstalt form,[5] or the shunt is made in the form of a tube through which water is briskly circulated, as in that designed at the National Physical Laboratory, London.[4]

Two sizes of the Reichsanstalt shunts are shown in Fig. 62. They are made according to a suggestion originally due to Ayrton. A single thin strip of manganin is doubled back on itself at the middle of its length, and the two parts separated by a very thin layer of mica insulation. By this means the area and consequently the flux included by the circuit are reduced to a minimum. To obtain a small inductance and absence of skin

effect, as well as good cooling, the strip is made very thin. The potential leads are brought out so that they do not include any flux and the current leads are strips of copper with very thin insulation between them so that they set up no appreciable stray field.

When the shunts are immersed in oil, 1 watt per square centimeter of cooling surface is allowed. Of course only one side of

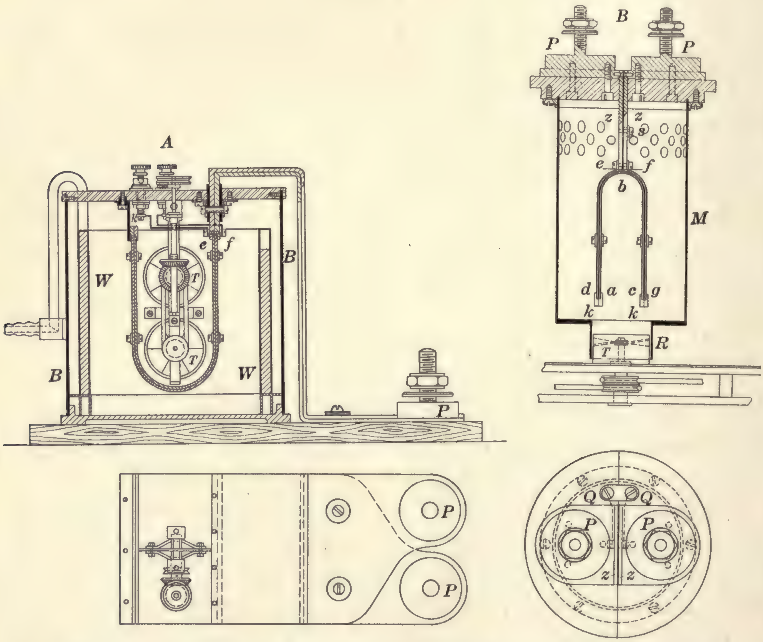

Fig. 62.—Low-resistance shunts for alternating-current measurements. Designed at the Reichsanstalt.

the sheet is effective in the cooling. The thickness of the mica insulation between the leaves is from 0.1 to 0.3 mm.; the distance between the resistance strips is from 0.2 to 0.5 mm. In the 0.001-ohm shunts the resistance strip is 69 cm. long, 14.5 cm. wide and 0.02 cm. thick; the self-inductance is 5.1 cm. As this resistance is intended for currents up to 1,000 amp. the loss at full load is 1 kw.

If the resistance is above 0.003 ohms, the construction is that shown at B in Fig. 62; ze and zf are the copper current leads.

The path of the current is down the outside strip from e to d, across the copper connection strip k, then up from a to b and down to c across k to g and up to the terminal b. The potential leads are attached at e and f. Mica insulation is used between the leaves of resistance material.

TABLE OF DATA CONCERNING OIL-COOLED MANGANIN RESISTANCES REICHSANSTALT DESIGN WITH LEAVES 0.5 MM. APART

Resistance, ohms.	Breadth of strip, cm.	Length of strip, cm.	Thickness of strip, mm.	Normal current, amp.	Volts drop at normal current	Kw. normal current	L, in cm.	Time constant $\frac{L}{R}$	Phase displacement at 60 cycles
0.03	1.42	51	0.5	40	1.2	0.048	17.0	5.7×10^{-7}	0°.012
0.01	3.6	42	0.5	100	1.0	0.100	5.8	5.8×10^{-7}	0°.013
0.003	6.85	49	1.0	333	1.0	0.333	5.0	17.0×10^{-7}	0°.036
0.001	14.5	69	2.0	1,000	1.0	1.000	5.3	53.0×10^{-7}	0°.114

The effects of the self-induction of a shunt intended for use in potentiometer methods may be compensated by mutual induction between the shunt and the potential leads.

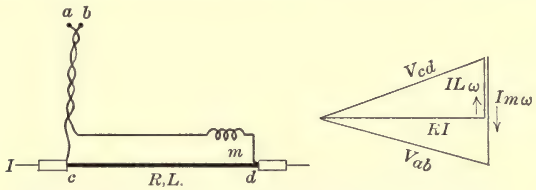

FIG. 63.—Compensation for shunt inductance.

Referring to Fig. 63, at balance no current flows in the voltage leads to a and b. It is desired to make the potential difference between the terminals a and b equal to and in phase with the ohmic drop in R. The potential difference between c and d is shown in the vector diagram by V_{cd}. If the potential leads are arranged so that there is mutual induction between them and the main part of the shunt, the e.m.f. induced in the potential circuit will be proportional to and in quadrature with the current I through the shunt.

The mutual inductance, m, may be arranged so that its action either aids or opposes the reactive drop in the shunt. If it opposes, then the P.D. between a and b is given on the vector diagram by V_{ab}, and if the mutual inductance be adjusted so that

$m = L$, then V_{ab} will coincide in both magnitude and direction with IR; that is, the shunt acts as if it were non-reactive.

This method of obtaining balance of inductances, proposed by A. Campbell,[4] is indicated in Fig. 64.

The shunt resistance, R, is in the form of a straight strip (or tube). The potential leads are copper strips of the same width as the main resistance. They are carried along the body of the shunt to about the middle of its length, only a very thin layer of insulating material being interposed, and then bent perpendicularly and attached to the potential terminals, a and b. By this means, the mutual induction may be made practically to balance the self-induction. The balance would be perfect if the potential leads could be made coincident with R. In reality, in the shunts described below, about 90 per cent. of the self-induction effect is eliminated.

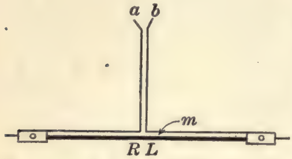

FIG. 64.—A. Campbell's design for non-inductive shunt.

In carrying out this scheme of construction at the National

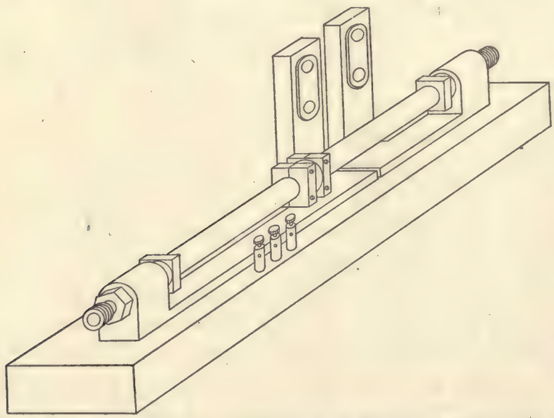
FIG. 65.—Non-inductive shunt. Designed at National Physical Laboratory.

Physical Laboratory, the strip has been replaced by a manganin tube enamelled on the inside.

The tube is hard-soldered to hollow copper terminals and these in turn are soft-soldered to the current leads, which are carried back parallel to the tube, to about the middle of its length, where

they are terminated by binding posts. This cuts down the stray field of the shunt itself and brings the current leads so close together, that their field has little effect on the neighboring apparatus. A thin copper ring is hard-soldered to the tube at the middle of its length. When the shunt is used in connection with the electrostatic wattmeter, this ring is used as a terminal (see Fig. 188).

The tube is covered with a layer of varnished cambric about 0.2 mm. thick and outside this are the potential leads, in the form of thin sheaths of copper foil about 0.04 mm. thick extending from the ends of the tube to near its middle, where they are terminated in potential posts.

The fluxes which produce inductive effects are those in the insulating medium between the tube and the potential leads and in the main tube itself. The insualtion therefore should be as thin as practicable and the potential leads should very closely surround it. The resistance tubes should be very thin and of large diameter. As the resistivity is high, the skin effect is negligible, less than 1 part in 10,000 at ordinary frequencies. This construction reduces the effective inductance to 3 or 4 cm. or to 0.003 or 0.004 microhenry.

Data Concerning Water-cooled Manganin Resistances Designed at the National Physical Laboratory

Resistance, ohms	Outside diam., mm.	Thickness of wall, mm.	Length, cm.	Normal current, amp.	Max. current, amp.	Volts drop at normal current	Kw. at max. current	L in cm.	Time constant $\frac{L}{R}$	Phase displacement at 60 cycles
0.04	6	0.25	35½	50	115	2	0.53	6.5	1.6×10^{-7}	0°.003
0.02	10	0.30	40	100	260	2	1.35	5.4	2.7×10^{-7}	0°.006
0.01	15	0.40	39	200	450	2	2.00	3.4	3.4×10^{-7}	0°.007
0.002	30	1.00	48	1,000	1,300	2	3.40	3.7	18.5×10^{-7}	0°.040
0.001	40	1.5	42½	2,000	2,500	2	6.25	3.0	30.0×10^{-7}	0°.060

To obtain a high carrying capacity, water from the city mains is briskly circulated through the resistance tube at a rate of about 15 liters per minute. The formation of a layer of hot water in contact with the resistance material is prevented by a centrally located glass rod which nearly fills the tube. For the same change in resistance due to heating, approximately three times as much energy may be dissipated as when simple air cooling is

relied upon. As much as 10 kw. may be dissipated by a current of 3,000 amp. in a tube 1½ in. in diameter and 18 in. long. The current density may be as great as 16,000 amp. per square inch. For manganin tubes, up to 1.5 mm. thick, 10 watts per square centimeter may be allowed as the working load.

To minimize the possibility of accidents, this form of resistance is used in a vertical position, the water inlet being at the bottom so that the tube is always filled.

Resistance Boxes.—Generally speaking, resistance boxes are constructed so that the resistance between their terminals may be varied from zero up to the full capacity of the box by 0.1 ohm or by 1-ohm steps. This must be accomplished by the use of a

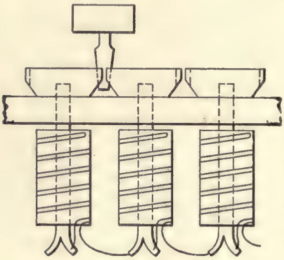

FIG. 66.—Diagram showing series arrangement of resistance coils.

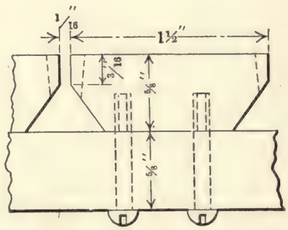

FIG. 67.—Connection block for resistance box.

moderate number of coils. A common arrangement is to employ coils of 1, 2, 2, 5 ohms, with similar sets for the tenths and the tens, hundreds and thousands. Another possible arrangement is based on coils having the denominations 1, 2, 3, 4. The coils are so mounted that any desired resistance is obtained by placing coils in series, as shown in Fig. 66. The drawing of a plug removes the short-circuit on the corresponding coil. The current must then pass from the terminal block through the coil, and on to the next block.

It is essential that the top of a resistance box be very rigid in construction. Consequently, the blocks must be firmly screwed to the vulcanite top. The taper of the plug should be such that a good contact may be obtained without the plug becoming wedged in place. The blocks should be undercut, as shown in Fig. 67, so that each may be thoroughly insulated from its

neighbors. Dimensions which have been found satisfactory under exceptionally severe service are shown in Fig. 67. Each block is held in place by four screws; the taper of the plugs is $\frac{1}{10}$ in. per 1 in. of length.

Dial and Decade Arrangements.—A common dial arrangement of coils employs nine 1-ohm coils, nine 10-ohm coils, nine 100-ohm coils and so on. The connections are such that any number of coils in any set may be put in series with any other set or sets. One method of accomplishing this is shown in Fig. 68*A*. A single plug is used in each dial.

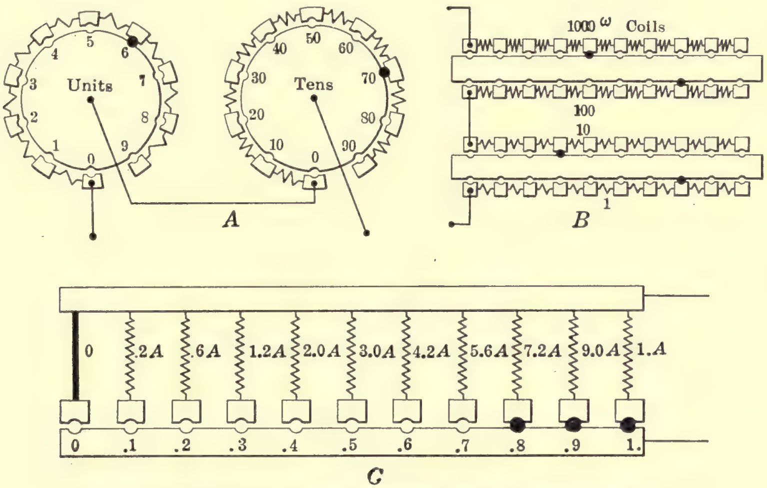

Fig. 68.—Dial and decade connections for resistance coils.

A great disadvantage of this particular arrangement, when plugs are used, is that it is kept clean with difficulty; dust and dirt collect on the hard rubber between the central block and the coil terminals and may partially short-circuit the coils, especially those of high resistance.

Obviously this arrangement lends itself readily to the employment of a rotative switch in place of the plug for connecting the central terminal to the coil terminals. It is now very commonly used and when the switch is well made is satisfactory (see Fig. 93).

Fig. 68*B* shows a decade set, which is the equivalent of a dial

arrangement, but with the coils in straight lines. This greatly facilitates keeping the top of the box clean.

Multiple Decade Arrangements.—Instead of arranging the coils so that they are in series, they may be put in multiple if given the proper values. Fig. 68*C* shows such an arrangement.

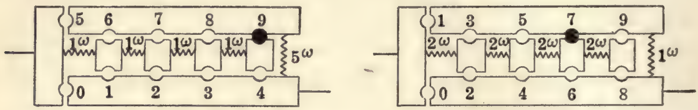

FIG. 69.—Feussner's and Smith's decade arrangements of resistance coils.

Let the highest resistance to be obtained in the decade be represented by A, or ${}^{10}\!/_{10}A$, the next lower step is ${}^{9}\!/_{10}A$; this is to be obtained by placing another coil in parallel with the first; by figuring the parallel circuits it is seen that the second coil must have a resistance of $9A$. Similarly the remaining coils must have resistances of $7.2A$, $5.6A$, $4.2A$, $3.0A$, $2.0A$, $1.2A$, $0.6A$, $0.2A$ and 0. Ten plugs must be used. This arrangement is most advantageous when the total resistance is low, for the component coils have higher resistances and are therefore more readily adjusted than when the series arrangement is employed.

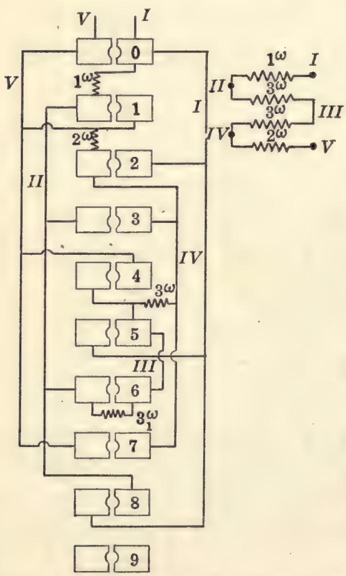

FIG. 70.—Northrup's decade arrangement for resistance coils.

Arrangements for Reducing the Number of Coils in a Decade.—The economical disadvantage of the original decade arrangement lies in the large number of coils which must be made and adjusted; several alternative arrangements are shown above.

In Feussner's decade arrangement, which gives resistances from 0 to 9 units, the first four coils are arranged as in the ordinary decade system; the fifth value is obtained by using a single

coil of 5 units and the succeeding values by employing this coil in series with the four-step decade. Only one plug is required.

The difference between Smith's and Feussner's arrangements is obvious.

In Northrup's arrangement, four coils having denominations 1, 3, 3, 2, units are used. In Fig. 70, I, II, III, IV, V, are terminal posts and taps which may be connected as desired. If all the coils are used in series the resistance is 9 units. The other values are obtained as shown below.

Points to be connected	Resistance between terminals I and V
	Units
I—V	0
II—V	1
IV—I	2
II—IV	3
III—V	4
I—III	5
II—III	6
IV—V	7
I—II	8

The construction necessary for carrying out this scheme by the use of a single plug is shown in Fig. 70.

RHEOSTATS

Water Rheostats.—To control a small current and to be able to give it any value between zero and a maximum, the arrangement shown in Fig. 71 may be used.

The compensating cell renders it possible to bring the current in the derived circuit smoothly down to zero. If the cell is not used, there will be a sudden change in the current when the electrodes *a* and *b* are brought into contact.

Water rheostats are commonly used for absorbing energy during tests of electrical machinery. The "water barrel," shown in Fig. 72, is convenient when small amounts of power are to be dealt with.

A stout wooden barrel is used. An ordinary cast-iron stove

grate about 15 in. in diameter is placed at the bottom of the barrel and provided with a terminal of insulated wire. A second grate, S', is screwed to an iron rod and suspended by a rope which passes over pulleys to a counterweight. Short wooden pegs prevent the two grates from being brought into contact. Fresh water is used and the required conductivity obtained by adding a salt, such as sodium carbonate.

Such a "water barrel" will take about 25 amp. at 2300 volts, absorbing about 100 kw.; the water will boil violently when the rheostat is forced to this extent. An adequate water supply must be provided and arrangements made by which the barrel may be kept full without danger to the operator. On account of the steam and gases, such rheostats should be used out-of-doors.

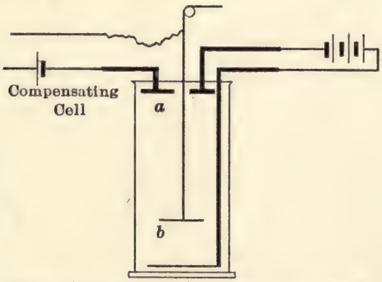

FIG. 71.—Water rheostat for small currents.

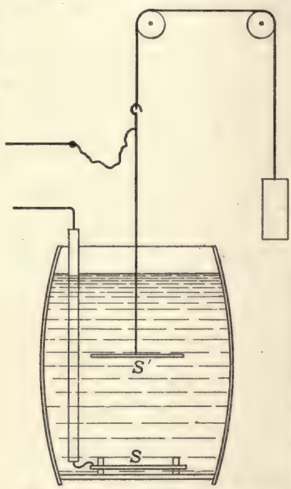

FIG. 72.—Water-barrel rheostat.

When rheostats of this general form are used, there is always more or less slopping over of the water. The ground and surrounding objects often become thoroughly saturated and the greatest care must be exercised by the attendants that severe or perhaps fatal shocks are not experienced through inadvertantly touching some of the wiring. One must not relax his vigilance because the voltage is low, for with sufficiently good contacts, shocks from 110-volt circuits have proved fatal. The station for the operator should be properly raised from the ground, so that the platform will be dry and the ropes by which the electrodes are manipulated should be rendered safe by the introduction of strain insulators.

Water Rheostats with Plate and Cylindrical Electrodes.—In case there is a running stream or open canal of fresh water near the station and the voltage is high, the forms of rheostat shown in Figs. 73 and 74 are convenient.

That shown in Fig. 73 was used for absorbing power, up to 700 kw., in a 2,300-volt three-phase circuit. There are three terminal and four neutral plates spaced 4 in. apart on centers, all of iron, the dimensions being 60 in. by 24 in. by 1/8 in. The frame is

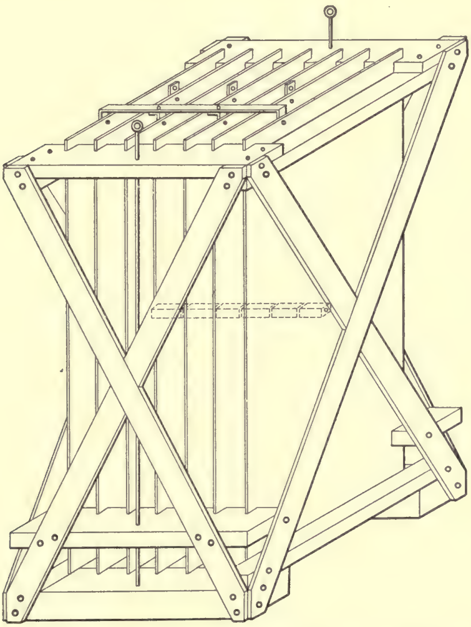

FIG. 73.—Three-phase power-absorbing rheostat with plate electrodes.

hung by a tackle so that the amount of power may be regulated by varying the immersed area. For these immersion rheostats the allowable current density at the electrodes is about 3.5 amperes per square inch.

In the rheostat shown in Fig. 74, which is also designed for three-phase loading, a wooden frame made in the form of an equilateral triangle is provided. The three vertical electrodes are of thin metal pipe and are connected by flexible cables to the

leads so that the frame may be raised or lowered by means of a tackle and the immersion of the electrodes varied.

If the conductivity of the water is known, the rheostat may be designed to absorb a given amount of power. The arrangement of electrodes is shown in Fig. 75.

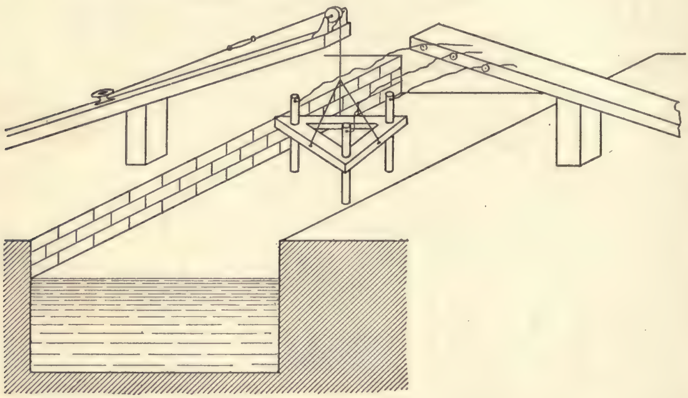

FIG. 74.—Three-phase power-absorbing rheostat with cylindrical electrodes.

The electrostatic capacity of two parallel cylinders in air, diameter D cm., spaced a cm. on centers, length l cm. is *

$$C = \frac{l}{4 \log_\epsilon \left\{ \dfrac{a + \sqrt{a^2 - D^2}}{D} \right\}}.$$

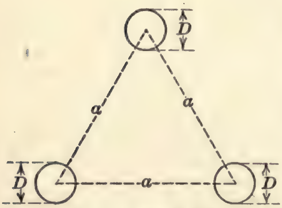

FIG. 75.—Arrangement of electrodes in three-phase water rheostat.

Hence, the conductance between these cylinders when immersed in an infinite medium of conductivity ρ' will be

$$g = \frac{\pi \rho' l}{\log_\epsilon \left\{ \dfrac{a + \sqrt{a^2 - D^2}}{D} \right\}}.$$

Let

$$\frac{a}{D} = K$$

then

$$g = \frac{1.36 \rho' l}{\log_{10} \left\{ K + \sqrt{K^2 - 1} \right\}}.$$

* RUSSELL, "Alternating Currents," vol. 1, p. 102.

The current which will flow between two parallel cylindrical electrodes when they are immersed in a great body of liquid is

$$I' = \frac{1.36\rho' lE}{\log_{10}\left\{K + \sqrt{K^2 - 1}\right\}} \qquad (1)$$

and the power absorbed is

$$P = \frac{1.36\rho' lE^2}{\log_{10}\left\{K + \sqrt{K^2 - 1}\right\}}.$$

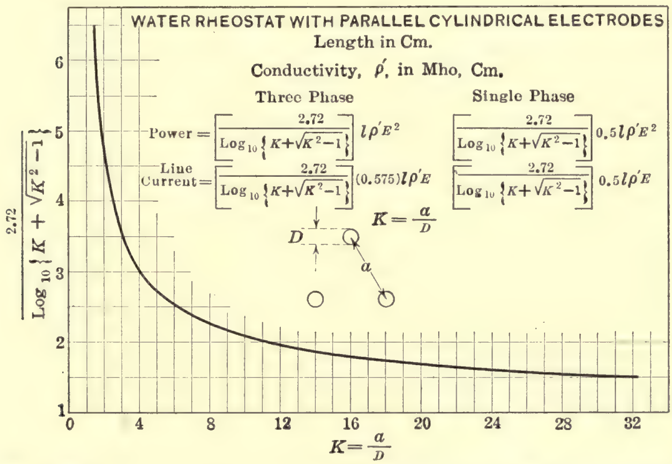

FIG. 76.—Showing constants of water rheostat with different spacings of cylindrical electrodes.

In the case of a three-phase rheostat the line current will be

$$I_L = \frac{2}{\sqrt{3}} I' = 1.15I'$$

and the power absorbed will be given by

$$P = EI_L\sqrt{3} = \frac{2.72\rho' lE^2}{\log_{10}\left\{K + \sqrt{K^2 - 1}\right\}}.$$

The conductivity, ρ', which is greatly influenced by local conditions and by temperature, must be found for the water which is to be used. To determine it two conducting cylinders of a known diameter may be fixed at a known distance apart and the arrangement immersed in the running water. A measured

alternating-current voltage is then applied between the two cylinders and the resulting current determined. ρ' is calculated by aid of (1).

Wire-wound Rheostats.—For general laboratory purposes, a very convenient rheostat adapted to low voltages is shown in Fig. 77.

The upright frame, 6 ft. high and 3 ft. wide, is strung with about 550 ft. of bare *IaIa* wire, contained in 100 sections. When 110 volts is applied at the terminals of the frame, one can, by means of spring clips, tap off voltages or small currents. By means of four clips and flexible connection wires, the arrangement may be divided into sections and these connected in parallel.

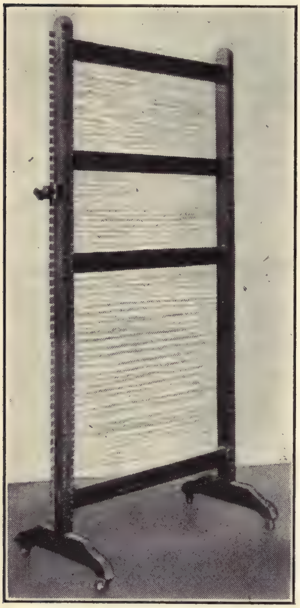

FIG. 77.—Resistance frame.

A cheap and convenient form of rheostat, which has proved very useful for loading the small generators used for purposes of instruction in electrical engineering laboratories is shown in Fig. 78.

The wire is wound in screw-threads on moulded porcelain cylinders. These cylinders are loosely held in place on the angle-iron frame in such a manner that they may be readily removed. The base and the top, which carries the switches, are of "ebony asbestos wood." The following sizes of *IaIa* wire have been employed:

Size of wire	Full load capacity
No. 17	104 amp. at 110 volts
No. 20	52 amp. at 110 volts
No. 26	20 amp. at 220 volts
No. 23	40 amp. at 220 volts

Immersed-wire Rheostats.—The carrying capacities of wires may be greatly increased by immersing them in water, as will be seen from the following table giving approximate data concerning galvanized-iron wire. Immersed rheostats are useful in tempo-

rary arrangements of apparatus. Fine wires may be corroded off after a short time. Provision must be made for safely replacing the water lost by boiling.

TABLE OF APPROXIMATE DATA CONCERNING CARRYING CAPACITY OF GALVANIZED-IRON WIRE WHEN IMMERSED IN WATER

No. B. & S.	In air			In water			
	Circular mils	Amperes	Ft. per 110 volts	Amperes	Ft. per 110 volts	Ft. per 550 volts	Ft. per lb.
20	1,018	2.5	594	36	25	125	369.0
19	1,253	2.9	626	42	27	135	293.0
18	1,624	3.5	673	50	29	145	232.0
17	2,048	4.2	710	60	30	150	184.0
16	2,583	5.0	750	71	32	160	246.0
15	3,257	6.0	790	88	34	170	107.0
14	4,107	7.1	840	103	36	180	91.9
13	5,178	8.5	886	122	38	190	72.1
12	6,530	10.1	941	145	40	200	57.8
11	8,234	12.0	990	173	42	210	45.8
10	10,380	14.3	1,054	205	45	225	36.4
9	13,090	17.1	1,103	245	47	235	33.3
8	16,510	20.3	1,354	293	49	290	25.0

The heating of these wires when immersed is so great that there must be no obstruction to a free circulation of the cooling water. Strong strings or fairly sharp edges of wooden sticks will make reliable supports. The water used must be clean, to prevent rapid destruction of the wires by electrolysis.

Drop Wires.—A very useful form of drop wire for controlling the voltages applied to the potential coils of instruments may be made by winding a single layer of double cotton-covered resistance wire on a piece of brass tube about a meter long and 5 cm. in diameter, which has been slit lengthwise and covered with stout paper. The insulation is sandpapered off where the slider makes contact. By use of this device, the voltage in a derived circuit may be adjusted from zero to a maximum. It should not be forgotten that the arrangement is a long solenoid and will have a considerable stray field.

Rheostats similar to those shown in Fig. 79 are regularly on the market, and are very convenient for general laboratory pur-

poses. In the G.R. type the constantin resistance wire is wound on slate blocks.

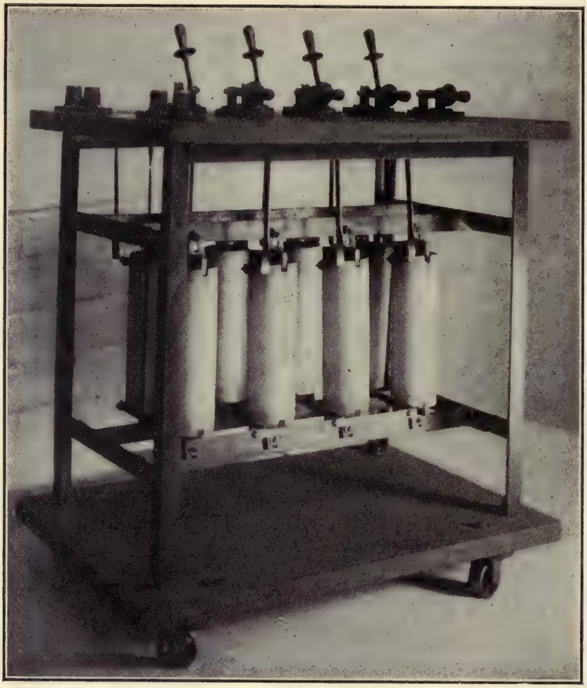

FIG. 78.—Laboratory rheostat for loading small generators.

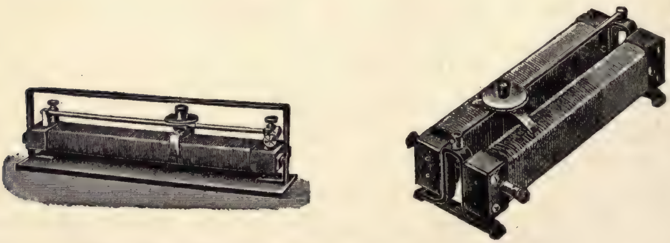

FIG. 79.—Slide-wire rheostats for small currents.

There are numerous stock forms of rheostats which may be obtained from the electrical manufacturing companies and which are useful in particular cases.

Carbon Compression Rheostats.—Carbon compression rheostats are exceedingly useful as laboratory appliances where low-voltage currents are to be controlled—as, for example, in calibration work.

The essential feature is a series of specially moulded carbon plates, which can be forced into more or less intimate contact by a screw.

A convenient form of carbon compression rheostat is shown in Fig. 80. It contains 90 plates each $1\frac{1}{2}$ in. by $1\frac{1}{2}$ in. by $\frac{1}{8}$ in.

A 4-volt current can be controlled between the limits 1 and 28 amp., the resistance of the circuit outside the rheostat being 0.1 ohm.

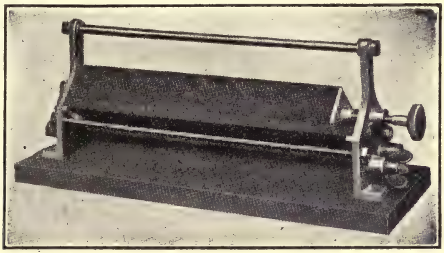

FIG. 80.—Carbon compression rheostat.

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CHAPTER IV

THE MEASUREMENT OF RESISTANCE

Volt and Ammeter Method.—The most obvious method of determining an electrical resistance is by the direct application of Ohm's law. The potential difference between the terminals of the resistor and the current flowing through it are measured by appropriate instruments, which have previously been calibrated.

It is important not to lose sight of the possible influence of the measuring instruments on the results. With the terminal at 1 (Fig. 81) the voltmeter gives the proper potential difference, but the ammeter measures the current through the unknown resistance plus that through the voltmeter. If the resistance, X, be at all comparable with that of the voltmeter, the error may be great unless allowance be made for the voltmeter current. In this

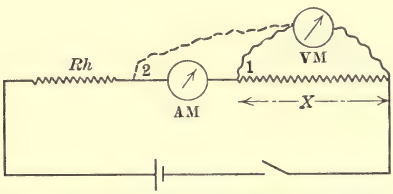


FIG. 81.—Volt and ammeter method for measuring resistance.

case the voltmeter resistance must be known. If the terminal be at 2, the ammeter gives the proper current but the measured potential difference includes the drop in the ammeter and its connections; if this be an appreciable fraction of that in X, the error will be large unless this drop is subtracted from the voltmeter reading. These considerations should be given weight when making connections for any particular test.

If the voltmeter be of low resistance, care must be taken that the resistances of the leads and contacts are negligible. The volt-ammeter method finds frequent employment in emergency work where comparatively rough measurements will suffice; for instance, in determining armature resistance.

Substitution Method.—This method is based on the assumption that the e.m.f. and resistance of the battery employed are constant.

With the connections as in Fig. 82 the galvanometer current is

$$I_G = \frac{ER_S}{\left(R_B + Rh + \frac{R_G R_S}{R_G + R_S} + X\right)(R_G + R_S)}$$

If, by means of a switch, S be substituted for X and adjusted until the deflection is the same as before, then obviously

$$S = X.$$

Any error which might be due to the law of deflection of the galvanometer is eliminated. The shunt R_S serves to vary the

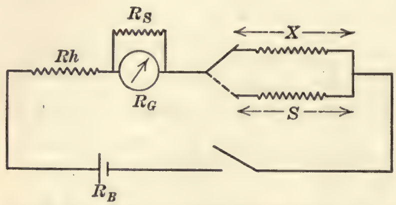


Fig. 82.—Substitution method for measuring resistance.

sensitivity of the galvanometer to suit different conditions. The resistances of the other parts of the circuit should be small compared with S and X; for this reason arrangements should be made so that the number of battery cells may be varied. The substitution method in a modified form is frequently used in dealing with very high resistances (see "Insulation Resistance").

Direct-deflection Method.—Two resistors which are to be compared may be connected in series and the potential differences between their terminals measured by voltmeters of the proper range. If the current be constant, a single instrument may be used; its deflection should be proportional to the current and it should be so arranged that the terminals can be quickly transferred from S to X, see Fig. 83.

R is a variable resistance for changing the range of the voltmeter; if the current taken by the voltmeter be negligible,

$$X = S\frac{D_X}{D_S}\left(\frac{R_V + R_X}{R_V + R_S}\right).$$

D_X and D_S are the readings, and R_X and R_S the resistances unplugged in R when the terminals are on X and on S respectively. R_V is the voltmeter resistance. If the current fluctuates, two voltmeters should be used, simultaneous readings being taken by two observers. This procedure, millivoltmeters being employed, is frequently used for testing rail bonds *in situ;* the resistance of a given length of rail including a bond being compared with that of the same length without a bond. The voltages measured are those due to the return current through the rail.

Potentiometer Method.—Instead of determining the potential difference between the terminals of S and of X by a voltmeter, the potentiometer (see page 271) may be used. Obviously the

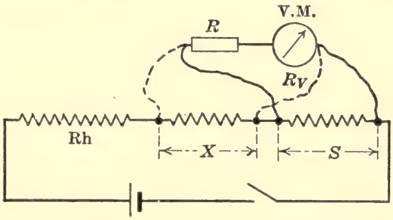


FIG. 83.—Direct deflection method for measuring resistance.

current in S, in X and that in the potentiometer must remain constant during the test. As the processes of balancing and checking the constancy of the potentiometer current require some time, this condition is very difficult of practical realization; while the method may be made to give accurate results, the measurement becomes a time-consuming operation. In very accurate measurements, the possibility of a heating error is considerable, for the current must be kept on continuously during the process of balancing.

In order that resistances may be determined with accuracy and despatch, it is necessary to have methods which are independent of fluctuations of the testing current. Such methods will now be discussed.

Differential-galvanometer Method.—A differential galvanometer has two distinct windings which are thoroughly insulated from each other, of equal magnetic strength, of equal resistance, and as nearly coincident as possible. To attain these conditions

the wires are wound throughout their length side by side and in layers. Any residual magnetic effect, if the instrument be of the Kelvin type, may be annulled by a small coil placed outside the case of the instrument and connected in series with the weaker coil in such a manner that its effect is additive; this adjusting coil is mounted so that its position may be altered by sliding it along a rod which is coaxial with the main coil.

The simplest method of using the instrument is shown in Fig. 84.

On the diagram R_{G_x} and R_{G_s} are the resistances of the two galvanometer coils; L_x and L_s the total lead resistances; R_x and R_s the resistances unplugged in the boxes. To avoid leakage and capacity effects, the positions of the boxes R_x and R_s should be such that the potential difference between the two galvanometer

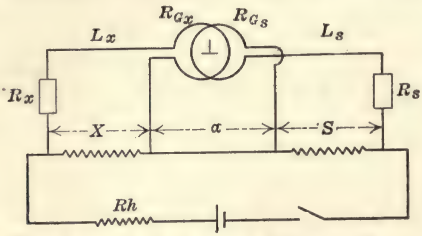


FIG. 84.—Simple method of using differential galvanometer.

coils is a minimum. The resistance of a should be low. High insulation of the leads, etc., is necessary.

First, the adjustment of the instrument must be tested. To do this the coils are connected in series and opposed magnetically; the maximum working current is then sent through them. No deflection should be observable. Perfect adjustment is obtained by adjusting the auxiliary coil.

With the connection shown in Fig. 84, after having unplugged a suitable resistance in R_x, the value of R_s is adjusted until the galvanometer stands at zero; then the currents in the two coils are equal, and

$$\frac{X}{S} = \frac{R_{G_x} + R_x + L_x + X}{R_{G_s} + R_s + L_s + S}$$

or

$$X = S\,\frac{R_{G_x} + R_x + L_x}{R_{G_s} + R_s + L_s}$$

The galvanometer and lead resistances must be known.

An alternative method is to unplug a small resistance, R_x, and obtain a balance by adjusting R_s, then to make R_x large and repeat the balance. If all other resistances remain constant, and the values unplugged be R_x, R'_x, and R_s, R'_s,

$$X = S\frac{R'_x - R_x}{R'_s - R_s},$$

The differential galvanometer was formerly in quite common use but was supplanted by the Wheatstone bridge. Of late years, however, the instrument has again come into use for measurements where the range to be covered is not large, for instance in resistance pyrometry and in the comparison of nominally equal resistances.

A practical difficulty is that the exact adjustment of a sensitive instrument is somewhat troublesome and when made is not permanent, being subject to changes in the levelling of the galvanometer. Therefore, methods have been suggested where the deflection is not brought exactly to zero.[1]

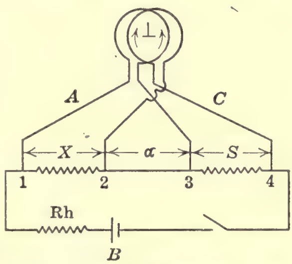


FIG. 85.—Diagram for Kohlrausch method of using a differential galvanometer.

Kohlrausch Method of Using a Differential Galvanometer.[2]—For work of the highest class, such as the precision comparison of resistance standards, the method employed must be free from errors due to variations in the resistances of the galvanometer circuits. These might be caused by changes of connections, involving the alteration of contact resistances, or they might be due to the inclusion of potential terminals of the resistances during the test but not when making the preliminary adjustment for differentiality.

Kohlrausch's method of overlapping shunts fulfils the desired conditions. It is designed for the comparison of nominally equal resistances. The scheme of connections is shown in Fig. 85. In carrying out the test some means of adjusting either S or X, as well as one of the galvanometer circuits, is required.

It is also necessary to be able to interchange B and α (equivalent to interchanging the galvanometer circuits A and C). This

may conveniently be done by a commutator with mercury contacts such as is shown in Fig. 86. The parts L are insulating strips of ebonite; the other parts of the rocker are of copper; two middle arcs are in electrical connection.

The parts of the commutator are so large that the resistance of the circuit, and therefore the battery current, is not appreciably altered by the interchange.

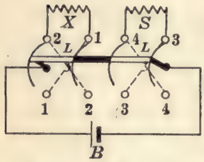


FIG. 86.—Commutator for interchanging the galvanometer coils.

Referring to Fig. 87, N is a shunt. A good resistance box may be used; in practice it is applied to the larger of the two resistances, S, X. By it, the resistance between 3 and 4 is reduced to equality with that between 1 and 2. The resistance of one of the galvanometer circuits is adjusted by means of g and n; trial determines which one should be varied.

As S and X are supposed to be nearly equal, the galvanometer is made as nearly differential as convenient but need not be exactly adjusted.

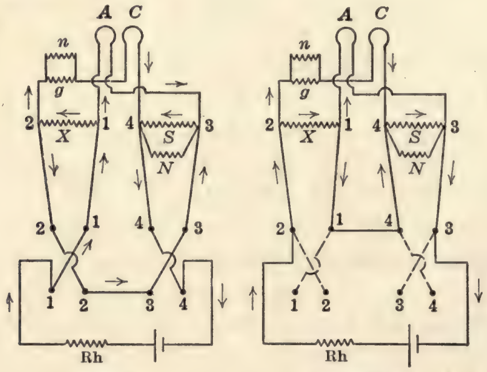


FIG. 87.—Showing connections for both positions of the commutator in Kohlrausch method.

The connections for the two positions of the commutator are shown in Fig. 87.

In order to find the conditions necessary for balance, the galvanometer deflections for the two positions of the rocker must be

determined. Let the connections be as shown in Fig. 88. The resistances of the various circuits are indicated on the diagrams; i_5 and i_6 are the galvanometer currents, I_B the battery current. Let S_1 be the resistance between 3 and 4; with the connections shown in the diagram, it is the parallel resistance of S and N (Fig 87).

$$i_6(r_6 + \alpha + X) - i_B(X + \alpha) + i_5\alpha = 0.$$
$$i_5(r_5 + \alpha + S_1) - i_B(\alpha + S_1) + i_6\alpha = 0.$$
$$i'_6(r_6 + S_1 + \alpha') - i'_B(S_1 + \alpha') + i'_5\alpha' = 0.$$
$$i'_5(r_5 + X + \alpha') - i'_B(X + \alpha') + i'_6\alpha' = 0.$$

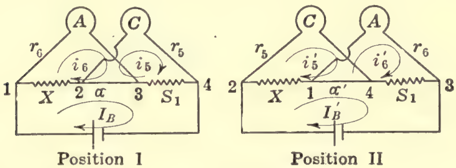


FIG. 88.—Mesh diagram for Kohlrausch method.

Solving for the desired currents and letting

$$M = \frac{i_B}{\alpha(r_5 + r_6 + S_1 + X) + (r_5 + S_1)(r_6 + X)}$$
$$M' = \frac{i'_B}{\alpha'(r_5 + r_6 + S_1 + X) + (r_5 + X)(r_6 + S_1)},$$
$$i_6 = M\{\alpha(r_5 + X) + X(r_5 + S_1)\}$$
$$i_5 = M\{\alpha(r_6 + S_1) + S_1(r_6 + X)\}.$$
$$i'_6 = M'\{\alpha'(r_5 + S_1) + S_1(r_5 + X)\}.$$
$$i'_5 = M'\{\alpha'(r_6 + X) + X(r_6 + S_1)\}.$$

Now let the deflection per ampere due to the coil carrying i_5 be A and that for the coil carrying i_6 be B. Then, as the two coils oppose each other and their effects are very nearly equal, the resultant deflection will be

For position I,

$$D_I = M[A\{\alpha(r_6+S_1)+S_1(r_6+X)\}-B\{\alpha(r_5+X)+X(r_5 + S_1)\}].$$

For position II,

$$D_{II}=M'[A\{\alpha'(r_6+X)+X(r_6+S_1)\}-B\{\alpha'(r_5+S_1)+S_1(r_5+X)\}].$$

Conditions for Balance.—Suppose that by adjusting the resistances the deflection is made *nil* for *both* positions of the rocker; that is,

$$D_I = D_{II} = 0.$$

Then with the first position of the rocker

$$\frac{A}{B} = \frac{\alpha(r_5 + X) + X(r_5 + S_1)}{\alpha(r_6 + S_1) + S_1(r_6 + x)}$$

and with the second position of the rocker

$$\frac{A}{B} = \frac{\alpha'(r_5 + S_1) + S_1(r_5 + X)}{\alpha'(r_6 + X) + X(r_6 + S_1)}.$$

Equating these two expressions for $\frac{A}{B}$ an equation results of the form

$$C(X - S_1) = 0.$$

C is a function of the various resistances; all the algebraic signs entering into it are $+$ so the condition $D_I = D_{II} = 0$ shows that

$$X = S_1$$

It will be noted that this result is obtained regardless of the values of A and B, that is, without making the galvanometer exactly differential.

Suppose D_I and D_{II} are equal but not zero, that is, that there are deflections of the same amount and toward the same end of the scale for both positions of the rocker. In the case where $i_B = i'_B$ and $\alpha = \alpha'$, that is, where there is no alteration of the circuit resistance, the relation between X and S is still

$$X = S_1 \text{ or } X = \frac{NS}{N + S}.$$

If the battery current and α alter slightly, due to the different positions of the rocker, and the adjustments are made so that $D_I = D_{II}$, the departure from the relation $S_1 = X$ is so slight that it may be neglected even in precision work.

The Kohlrausch method of employing the differential galvanometer is the only one adapted to work of the highest precision.

The Wheatstone Bridge.—This instrument which is so universally used in the determination of electrical resistance was invented by Mr. S. Hunter Christie, of the Royal Military Academy at Woolwich. He published an account of it in the *Philosophical Transactions*, under date of Feb. 28, 1833, calling

his invention "A Differential Arrangement." The variable ratio arms were added by Dr. Werner Siemens. In 1843 Sir Charles Wheatstone recalled attention to Christie's device, giving him full credit. At that time Wheatstone was one of the leading scientists of Great Britain, and his name became associated with the instrument and has so remained.

Having the conductors arranged as in the diagram, Fig. 89, the current through the galvanometer will be zero only when $\frac{M}{N} = \frac{X}{P}$, for to have zero current in the galvanometer, the potential difference between the galvanometer terminals must be zero, or, in other words, the fall of potential along M must be equal to that along X, and the fall along N equal to that along P.

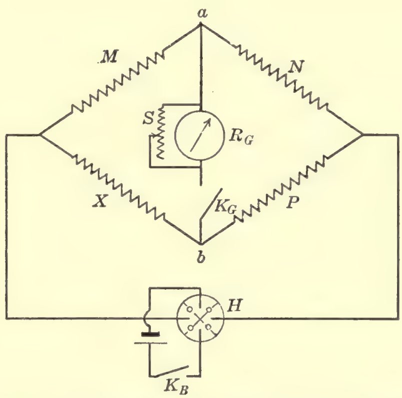


FIG. 89.—Diagram for Wheatstone bridge.

Let I_M, I_N, I_X, I_P be the unvarying currents in the respective branches; then when $I_G = 0$, $MI_M = XI_X$, also $NI_N = PI_P$,

$$\therefore \frac{MI_M}{NI_N} = \frac{XI_X}{PI_P}$$

but if no galvanometer current flows,

$$I_M = I_N \text{ and } I_X = I_P$$

so

$$\frac{M}{N} = \frac{X}{P} \text{ or } X = \frac{M}{N}P$$

Consequently, if three of the resistances are known, the fourth

may be determined. M and N are called the balance or ratio arms and P the rheostat arm of the bridge.

Auxiliary Apparatus.—Besides the bridge box, the other necessary pieces of apparatus are the battery (usually two or three cells), the keys K_B and K_G, the galvanometer, shunt, and commutator, see Fig. 89.

Keys.—Keys K_B and K_G are usually combined to form what is called a bridge key, which when depressed throws in first the battery and then the galvanometer (to eliminate the effects of inductance and capacity). To avoid the chance of leakage to the galvanometer and also thermo-electric effects, care must be taken when manipulating this key not to touch the metal work. The commutator H is used to reverse the battery current and so to eliminate the effects of thermo-electromotive forces.

The Galvanometer.—The galvanometer should be one which is not affected by variations of the local field and, if possible, should be critically damped; either a shielded Thomson or a D'Arsonval instrument may be used.

In selecting a D'Arsonval galvanometer for bridge work, the peculiarities of the instrument should be considered; for, suppose the resistance in the bridge arms between the galvanometer terminals is low and that the instrument is one, which, for critical damping, requires that it be in series with a high external resistance. On the passage of the current, as soon as the coil begins to move, an e.m.f. will be set up in the circuit and the motion will be damped. Consequently instead of a sharp, decided movement of the index the motion will be so deliberate as to greatly increase the difficulty of deciding when the bridge is in balance. Obviously it is impossible to select a galvanometer which will be critically damped for all combinations of the bridge arms, but with care a good working compromise may be secured. In general, with a given sensitivity, the shorter the period of the instrument, the more satisfactory will its action be.

The Shunt.—The shunt, S, is a bypass for the current and is placed between the galvanometer terminals. It is used during preliminary adjustments to protect the galvanometer against currents of abnormal strength. By means of the movable arm the value of the shunt resistance may be altered so that as the adjustment of the bridge nears perfection, a greater proportion

of the current can be sent through the galvanometer. During the final adjustment, when full sensitiveness is desired, the movable arm should be turned so far to one side that it breaks the shunt circuit and the entire current flows through the galvanometer. The various positions of the movable arm are usually so arranged that the fractional parts of the full current which can be sent through the galvanometer are 0.001, 0.01, 0.1 and 1. When using a Wheatstone bridge one should *always begin measurements with the galvanometer heavily shunted. Violent deflections of the instrument are thus avoided.*

The Null Method of Making a Measurement.—The coil of unknown resistance is inserted, as indicated in Fig. 90. All connections must be electrically perfect; all binding posts should be screwed up tightly, but without using undue force; all plugs should be firmly inserted and the galvanometer heavily shunted. A rough idea of the magnitude of X is obtained as follows: Make $M = N$; draw the 1-ohm plug in P; depress the key with care, being ready to release it immediately should the deflection of the galvanometer be violent. The deflection will be assumed to be toward the left. Note this deflection, plug up the 1-ohm coil, and draw the 5,000-ohm or other high resistance plug; proceed as before. The deflection may be toward the right; it is then known that P and consequently X is between 1 ohm and 5,000 ohms. If one deflection is greater than the other, it shows that the proper value of P is nearer the resistance which gives the smaller deflection. Next try 10 ohms in P. Suppose the deflection to be still toward the left; the proper value of P is between 10 ohms and 5,000 ohms. Proceed in this manner, always narrowing the limits between which the right value of P must be located. Having obtained an apparent balance, the shunt resistance is increased and a better approximation obtained. Suppose that the bridge finally balances with $P = 25 +$ ohms, 25 ohms being too small and 26 ohms too large; then X is between 25 and 26 ohms. It is obvious that the determination of X is good only to about 2 or 3 per cent. Suppose that X is desired to 0.1 per cent.; then as the smallest coil in P is 1 ohm, P must be between 2,500 and 2,600 ohms in order that the smallest step may represent most nearly the desired precision. Accordingly

make P, 2,500 ohms and *alter the balance arms M and N to correspond.*

Make $M = 10$ ohms and $N = 1{,}000$ ohms; gradually increase P from 2,500 ohms until exact balance is obtained, with shunt removed; then

$$X = \frac{10}{1{,}000} P.$$

In the above it has been supposed that with $M = N$ the deflections with $P = 1$ ohm and $P = 5{,}000$ ohms were one to the right, the other to the left. If they had both been to the right and the one with $P = 1$ ohm of the lesser magnitude, then the proper value of P would have been below 1 ohm and X less than 1 ohm. In such a case proceed at once to change the ratio of M and N so that 1 ohm in P balances 0.01 ohm in X. In other words, make $\frac{M}{N} = \frac{1}{100}$ and proceed as before with the adjustment of P. If P should be greater than 5,000 ohms, the proper procedure may be decided upon from the above discussion.

As a final precaution, all connections should be gone over to see if they are tight and all the plugs firmly in place; then the final balance should be taken. The battery current should be reversed and the test repeated; this is necessary in order to eliminate thermo-electric currents. The average result for P is used in calculating X.

The Deflection Method.—Referring again to the example just discussed, with $M = N$, P was between 25 and 26 ohms and X could be determined only to 2 or 3 per cent. Now suppose that with $P = 25$ ohms, the galvanometer deflects from its zero position thirteen divisions to the left, and with $P = 26$ ohms nine divisions to the right; then we may interpolate, for a change of 1 ohm in P causes the spot of light to vary twenty-two divisions, and the proper value of P for exact balance will be 25+ ohms or 25.59 ohms. As the readings of the deflections cannot generally be taken with great accuracy, 25.6 ohms would be the value of P to be accepted. It is obvious that if this procedure be followed X may be determined to ½ per cent. without changing the ratio from $M = N$. If the ratio be changed the precision may be still further increased. This method is used to gain precision when X is so small that P must be of small value. This

method is slower in its application than the null method, and the arms of the bridge are more likely to be overheated. The battery e.m.f. must remain constant.

Examples of Arrangements of Bridge Tops.—A very satisfactory form of bridge top is shown in Fig. 90. All the connections made in setting up the instrument are outside the box, the wires being attached to the appropriate binding posts.

From the plan of the top, it will be noted that there are three gaps at 1′, 2′, 3′ each marked Inf; removing the plug from any one of these gaps breaks the circuit. The 10,000-ohm coil may be used in either the balance or the rheostat arm. Reversal of the bridge arms is accomplished by placing X at 3′ instead of 1′, the galvanometer terminal being changed also. By removing the plugs at 1′ and 2′ and 3′, the coils are divided into two independent sections; occasionally this is very convenient where the box is used for general laboratory purposes.

Another design employing the series arrangement of coils is shown in Fig. 91. Here the connections are permanently made inside the box. The reversal of the ratio arms is effected by changing the plugs from positions 1′ to 2′.

In Fig. 92 is shown a bridge employing the dial arrangement for the rheostat arm. The unknown resistance is inserted at either 1 or 2 according to the ratio desired. The dial bridge in this form is not recommended because the condition of the top is not apparent at a glance and there is also difficulty in cleaning it.

A dial bridge with sliding in place of plug contacts in its rheostat arm is shown in Fig. 93.

In this bridge the ratio coils are arranged as suggested by A. Schöne[3] (see Fig. 94). There are two coils of each of the following denominations, 1, 10, 100, 1,000 ohms. All the coils have one terminal attached to the central copper bar which is inside the box, the other terminals being connected to the plug blocks. Two plugs are ordinarily used. When inserted as shown, $\frac{M}{N} = \frac{100}{10}$. The advantages of this arrangement are the ease with which the arms may be reversed, the reduction in the number of plug contacts, and the possibility of obtaining the the same ratio by using different coils of the same denomination, thus giving a check on the value of $\frac{M}{N}$.

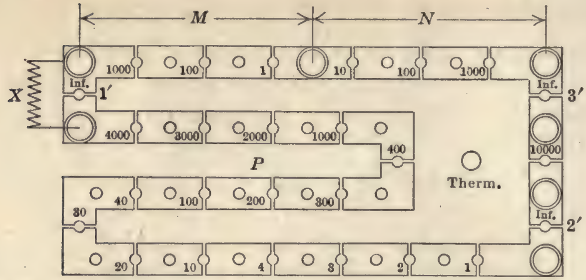


FIG. 90.—Wheatstone bridge top with plug connections.

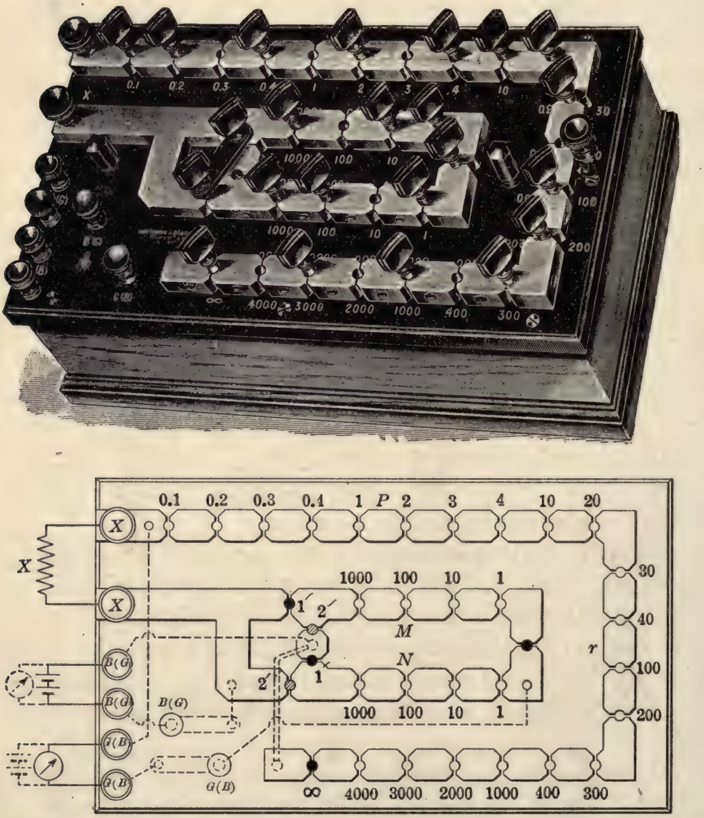
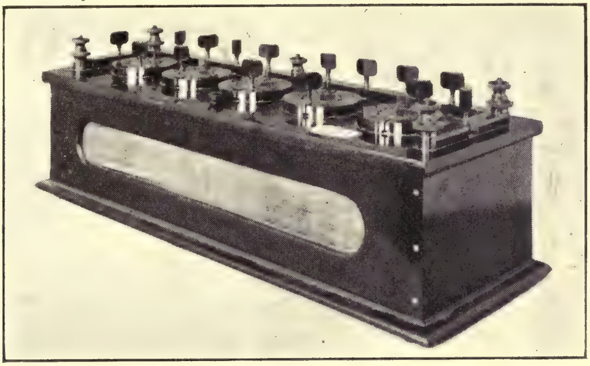


FIG. 91.—Wheatstone bridge with internal connections and reversible ratio arms.

For field work, portable bridges are used. One design is shown in Fig. 95. The carrying case contains the bridge proper, the galvanometer, which is of the D'Arsonval type, and a



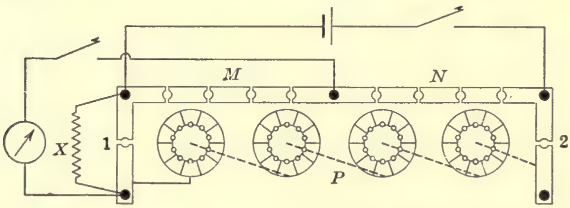


FIG. 92.—Wheatstone bridge with dial arrangement of rheostat coils.

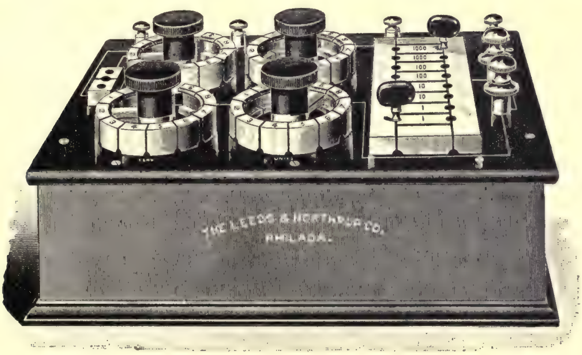


FIG. 93.—Wheatstone bridge with Schöne ratio coils and sliding-dial rheostat coils.

few cells of dry battery. In the example here shown the decade arrangement with sliding contacts is adopted.

Calibration of a Resistance Box.—In order that any changes in the coils may be detected, all resistance boxes and Wheatstone bridges should be calibrated occasionally. A convenient

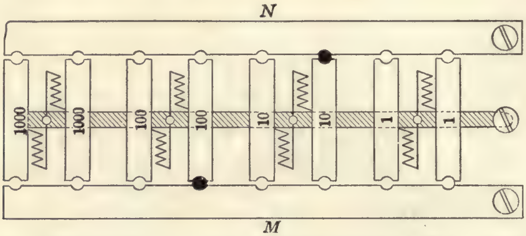


FIG. 94.—Schöne arrangement of ratio coils for Wheatstone bridge.

FIG. 95.—Portable Wheatstone bridge with sliding-dial coils.

method of doing this with sufficient accuracy for general laboratory work, and one for which the apparatus is readily assembled, is shown in Fig. 96. It is a substitution method involving the use of the bridge principle.

Definiteness is the only requirement in the balancing resistances. They must not change through heating or be erratic through ill-fitting plugs or defective sliding contacts.

The box to be calibrated is placed in series with the standard as shown. If the resistances of the equal extension coils, m, are properly adjusted to that of the slide wire, a given displacement of the slider may be made to correspond to an assigned percentage difference of X and S.

Suppose the slide wire has a length of 1 meter and a resistance of 10 ohms. It is desired that a difference of $\frac{1}{10}$ per cent. between X and S shall correspond to a displacement of the balance point of 10 cm. When $X = S$, the balance point is to be at the middle of the slide wire.

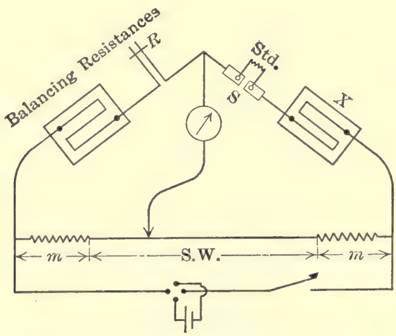


FIG. 96.—Connections for comparing resistance boxes.

A displacement of 10 cm. to the left takes 1 ohm from the left-hand side of the bridge and adds it to the right-hand side. Then

$$\frac{1}{1.001} = \frac{m + 4}{m + 6} \text{ and } m \doteq 1{,}996 \text{ ohms.}$$

Assume a balance to be obtained with the 1-ohm coil in X by unplugging the corresponding coil in the balancing resistance, the standard S having been cut out. If needful, R may be used to bring the balance point to the middle of the slide wire. The reading of the slider is taken and then the standard S is substituted for X. If $X = S$ there will be no change in the balance point; if X differs from S, balance is restored by moving the slider and another reading taken. The difference of the two readings and the known displacement of the slider corresponding to $\frac{1}{10}$

per cent. allows the percentage difference of the two coils to be calculated nearly enough for practical purposes. The 1-ohm coil is thus compared with the standard ohm, then the 2-ohm coil with the sum of the 1-ohm and the standard, then the second 2-ohm coil with the first and so on.

The 10-ohm coil may be compared with a 10-ohm standard and the values of the 20-ohm, 50-ohm, 100-ohm and other coils determined by comparison with those previously calibrated, or the whole series may be built up from the standard ohm.

After one box has been calibrated, others may readily be compared with it by this method.

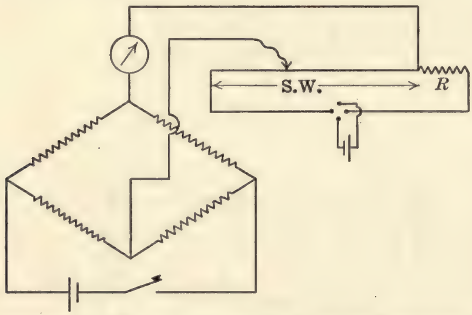


FIG. 97.—Arrangement for compensating large thermo-electromotive forces.

Compensation for Large Thermo-e.m.f.—Occasionally in special bridge arrangements the unavoidable inequalities of temperature in the apparatus may cause thermo-e.m.fs. of sufficient magnitude to drive the galvanometer spot off the scale. In many cases such e.m.fs. may be compensated, if they are reasonably constant and the galvanometer can be used on closed circuit.

Referring to Fig. 97, R is a high resistance of several thousand ohms, $S.W.$ a slide wire; by adjusting the slider, the galvanometer may be brought to zero; then the bridge is balanced as usual.

Slide-wire or Divided-meter Bridge.—In this simple form of Wheatstone bridge, shown in Fig. 98, two of the arms are replaced by the two sections of a uniform wire.

In the diagram the heavy lines represent copper strips of low resistance with gaps at a, b, c, d. On each side of each gap is a

binding post so that the gap may be closed by a strap of low resistance or by a resistance coil, as is desired. Between e and f is stretched a wire of high resistance. It is about a millimeter in diameter and, in this form of bridge, intended to be just 1 meter long. A slider, s, makes contact at any desired point along the wire and its position may be read off on a scale divided into millimeters. It is intended that e and the zero of the scale shall be coincident. The connections of battery and galvanometer are generally as shown, and this is usually the more sensitive arrangement. If the battery and galvanometer are interchanged, the resulting arrangement will be less disturbed by thermal e.m.fs. at the contact s.

To make a measurement, the simplest process would be to close the gaps at a and b by straps, place at d the unknown resistance

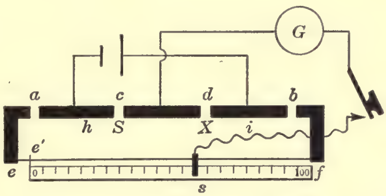


FIG. 98.—Diagram for slide-wire bridge.

X, and at c the known resistance S, the best value of which would be about the same as X. The slider s would then be pressed carefully down upon the wire at one point after another until one was found where the galvanometer remained undeflected, *i.e.*, where opening and closing at s did not cause motion of the galvanometer index. Let l represent the scale reading in millimeters, *i.e.*, l = distance $e's$. As $e'f = 1{,}000$ mm., $sf = 1{,}000 - l$. Then, by the bridge principle, $X = S\,\frac{\text{res. of } sf}{\text{res. of } l}\cdot$ But the wire is assumed to be of uniform resistance per unit length, so that the resistances of the two parts of the wire are proportional to their lengths. Hence, if the resistances of the leads from e' to c and from f to d are assumed to be zero, $X = S\,\frac{(1{,}000 - l)}{l}$.

Resistances at Ends of Bridge.—The leads at the ends of the bridge may not be of negligible resistance, and there may be

an inaccuracy in placing the zero of the scale opposite the end of the slide wire. The resistances between e' and the battery terminal h, and between f and the battery terminal i act as prolongations of the corresponding ends of the slide wire. Let n_1 denote the number of millimeters of the wire which would have the same resistance as $e'h$, and n_2 the corresponding number for fi. Then if n_1 and n_2 were known, their effect would be allowed for by writing

$$X = S\frac{1{,}000 - l + n_2}{l + n_1}$$

The values of n_1 and n_2 may be determined as follows:

After thoroughly cleaning the surfaces of contact insert the straps at a and b. Place at c a coil of A ohms and at d a coil of B ohms. A and B must be different. Call the reading of the slider when the balance has been obtained l_1. Then

$$\frac{A}{B} = \frac{l_1 + n_1}{1{,}000 - l_1 + n_2}.$$

Interchange A and B and obtain a new balance at l_2. Then

$$\frac{B}{A} = \frac{l_2 + n_1}{1{,}000 - l_2 + n_2}$$

$$\frac{A}{A + B} = \frac{l_1 + n_1}{1{,}000 + n_1 + n_2}$$

$$\frac{B}{A + B} = \frac{l_2 + n_1}{1{,}000 + n_1 + n_2}$$

$$\therefore n_1 = \frac{Bl_1 - Al_2}{A - B}.$$

Similarly

$$n_2 = \frac{B(1{,}000 - l_2) - A(1{,}000 - l_1)}{A - B}.$$

Extension Coils.—If, in measuring X, the balance point falls near one end of the bridge, the shorter section of the wire cannot be determined with accuracy. Again, if equal coils are being compared, greater accuracy in setting s may be desired. In either case, it would be advantageous to use a longer wire. As this would be inconvenient, the same result is attained by insert-

ing resistance coils at the gaps a and b. The effect of these, so far as the balance is concerned, is the same as if the wire had been extended on each side by the addition of such a length as would have the same resistance as the coils. Therefore, the equivalent lengths of these coils in millimeters of the slide wire must be determined. Let m_1 denote this quantity for the coil at a and m_2 that for the coil at b. Then when measuring X,

$$X = S\frac{1{,}000 - l + n_2 + m_2}{l + n_1 + m_1}.$$

The values of m_1 and m_2 may be determined as follows: place at c a resistance of E ohms and at d another known resistance of B ohms. Close both a and b by the straps and balance the bridge. Call the reading l_1. Then

$$\frac{E}{B} = \frac{l_1 + n_1}{1{,}000 - l_1 + n_2}.$$

Now insert at a the coil, the equivalent length of which is desired, and obtain a new balance. Call the reading l_2. Then

$$\frac{E}{B} = \frac{l_2 + n_1 + m_1}{1{,}000 - l_2 + n_2}$$

$$\therefore m_1 = (l_1 - l_2)\left(\frac{E}{B} + 1\right)$$

and similarly for m_2.

In using equal extension coils, it is necessary to have the known resistance S (at c) so nearly equal to X that the balance point will come upon the slide wire. If the extension coils are unequal, then the ratio of S to X must be such as to accomplish this; or if S is of a fixed value, then the ratio of m_1 to m_2 must be properly adjusted. If S, m_1 and m_2 are all fixed, the range of the apparatus is limited.

With a more elaborate construction, and when used with due precautions to eliminate thermo-currents, contact resistances, etc., the slide wire bridge becomes useful in work of the highest precision.

Carey Foster Method for Comparing Two Nearly Equal Coils.—This method is primarily designed for the comparison of nearly equal resistances; it therefore lends itself readily to the determination of temperature coefficients and to the verification of standard coils.

The coils to be compared are inserted at a and b (Fig. 98). Let their resistances be A and B; two approximately equal resistances S and S' are inserted at c and d; this insures that the balance point will fall near the middle of the slide wire. Let R be the resistance per unit length of the slide wire, L the total length of slide wire in divisions, l_1 and l_2 the readings of the slider at balance. Then

$$\frac{S}{S'} = \frac{A + n_1R + l_1R}{B + n_2R + (L - l_1)R}.$$

If A and B are interchanged,

$$\frac{S}{S'} = \frac{B + n_1R + l_2R}{A + n_2R + (L - l_2)R}.$$

So
$$\frac{S}{S + S'} = \frac{A + n_1R + l_1R}{A + B + n_1R + n_2R + LR}$$

and
$$\frac{S}{S + S'} = \frac{B + n_1R + l_2R}{A + B + n_1R + n_2R + LR}$$

$$\therefore A - B = R(l_2 - l_1) = RD$$

D is the difference of the two readings of the slider.

The difference of the resistances of the coils is seen to be equal to the resistance of the portion of the slide wire between the two balance points. It is to be noticed that this result is independent of S and S' and of n_1 and n_2, as well as of contact resistances, *if these factors remain constant during the test.* Some convenient device must be adopted for interchanging the coils without removing them from their cooling baths and without handling, and as it is the difference of two nearly equal quantities which is involved, extraneous resistances due to change of connections and contacts must be carefully avoided. Fig. 99 shows a form of bridge especially designed for carrying out the Carey Foster test.

The interchange of the coils is effected by raising the contact switch K, turning it through one-half a revolution and then lowering it.

To adapt the device to the comparison of high as well as low resistances, several pairs of coils (SS') must be provided in order that a sensitive bridge arrangement may be maintained. A number of slide wires of different resistance per unit length,

together with means for readily inserting them in the circuit, are also required. Three such wires are provided, any one of which may be used at will, and to obtain the effect of a wire of very low resistance, the slide wire proper may be shunted. This is effected by the link seen at the front of the switchboard.

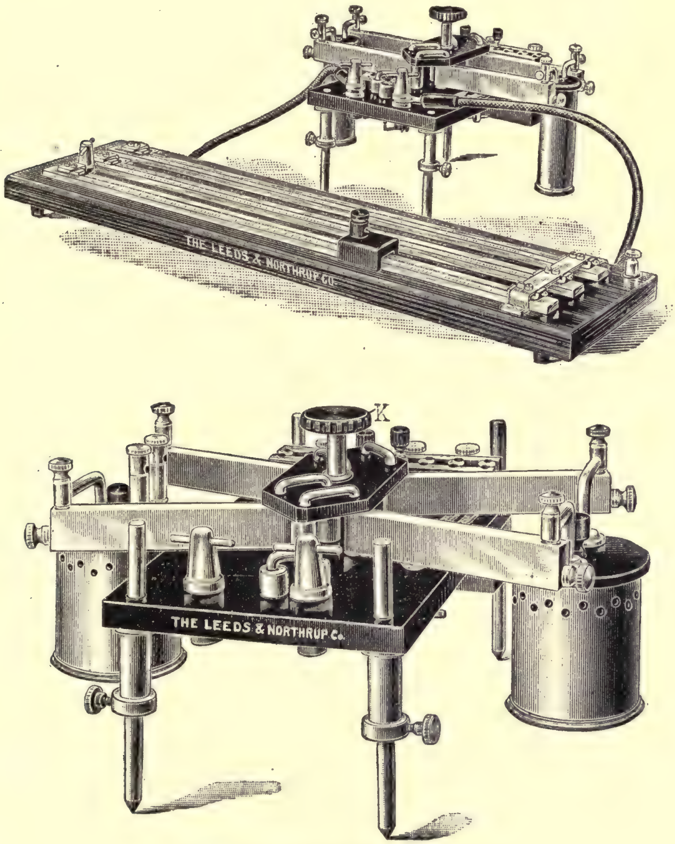


FIG. 99.—Carey Foster bridge.

Determination of the Resistance per Unit Length of the Slide Wire.—To determine R, the resistance per unit length of the slide wire, the process of measurement may be inverted. Let A and B be two coils of nearly equal resistance, so that the balance points will fall near the middle of the bridge; only one of the coils

need be known with exactness. A balance is effected in the usual way. Then by the law of the bridge,

$$A - B = RD_1.$$

The known coil is then shunted by a resistance C which is of such a value that the balance points fall near the ends of the slide wire. This shunted coil is then compared with the coil A which gives

$$A - \frac{BC}{B + C} = RD_2,$$

therefore

$$R = \frac{B^2}{(B + C)(D_2 - D_1)}.$$

When comparing coils of moderate resistance it may happen that their difference is so great that the slide wire is not of sufficient length; in such a case the larger resistance may be shunted, a comparison effected in the usual way and the shunt allowed for. The accuracy with which the shunt must be known depends on its ratio to the resistance of the coil around which it is placed.

Calibration of a Slide Wire.—In dealing with the slide-wire bridge, it has been assumed that the wire is of uniform resistance per unit length. In order that troublesome corrections may be avoided every effort should be made to have this assumption strictly true. If a case arises where the wire must be tested for uniformity, it may be divided into sections of equal resistance by one of several different methods. If the wire be uniform these sections will be of equal length; if they differ in length, corrections may be determined by which a reading on the wire may be reduced to the value it would have had if the wire had been of uniform resistance per unit length.

Carey Foster Method of Calibrating a Slide Wire.—Referring to the demonstration for the Carey Foster method of comparing two coils (page 176), it is seen that their difference is independent of S and S'; the relative values of S and S', however, determine the position of the balance points on the slide wire. Let A and B be two nearly equal resistances one of which is shunted by a resistance, C, so that $l_2 - l_1$ has the desired value. The two sections of a second slide wire replace the coils S and S' ordi-

narily used at the gaps c and d. The connections are then as shown in Fig. 100.

The contact q is placed at the zero of the slide wire and the contact O adjusted until balance is obtained. The coils are then interchanged and the slider, q, is adjusted. Then

$$A - \frac{BC}{B + C} = \text{resistance of section } l_2 - 0$$

The coils are returned to their original position, q remaining fixed, and the contact O adjusted until the balance is again attained. The coils are then interchanged and the balance obtained again by moving the contact q. The slide wire is thus divided into sections each having a resistance,

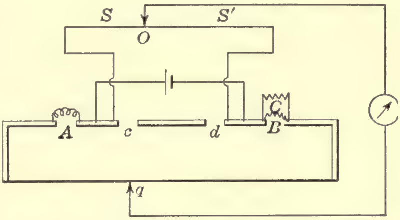


FIG. 100.—Diagram for Carey Foster method for calibrating a slide wire.

$$A - \frac{BC}{B + C}.$$

As this difference must remain constant, the coils employed should be of manganin.

Barus and Strouhal Method for Calibrating a Slide Wire.—This method is a simple application of the projection of potentials; the wire to be calibrated is placed in parallel with a series of movable coils having nearly equal resistances. For convenience they may be ranged along a board parallel to the slide wire with their terminals dipping into mercury cups.

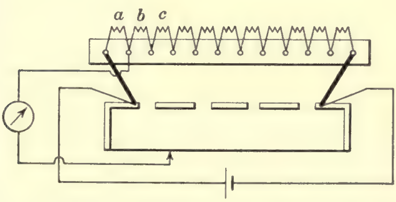


FIG. 101.—Diagram for Barus and Strouhal method for calibrating a slide wire.

One of the coils, a, is selected as a standard, the galvanometer attached to one terminal of it and balance obtained by use of the slider. The galvanometer wire is then transferred to the other terminal of the coil and a second balance obtained by use of the slider. Coil a is now interchanged with coil b and the operation repeated. Thus, by

successively interchanging the coils with a and finding the corresponding balance points, a series of length having equal resistances may be set off on the slide wire.

Thomson Bridge Method for Calibrating a Slide Wire.—Connections are made according to Fig. 102. CD is a resistance which determines that of the steps into which the slide wire is to be divided. It is connected in series with the slide wire by a resistance R which is considerably greater than that of the wire and which can be short-circuited. The coils M, N, m, n, should fulfill the condition $\frac{M}{N} = \frac{m}{n}$. If this condition is exactly fulfilled, the settings will be independent of the resistance between D and l_1; to make sure that this is so l_1 is set at any point on the slide wire, R is short-circuited and a balance obtained by adjusting l_2 The short-circuit around R is then removed. The balance should remain undisturbed and this should be true whatever the setting of l_1. A more severe test is to have a break at R, that is, to make $R = \propto$. If this test is satisfactory, the slider l_1 may be set at various points along the wire and balances obtained by moving l_2. The resistance of the sections will then be $r_{l_2 - l_1} = r_{CD}\left(\frac{M}{N}\right)$.

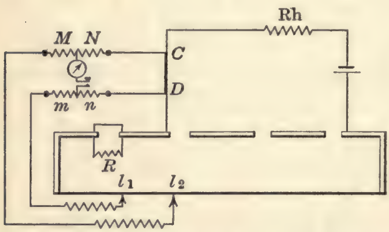


FIG. 102.—Diagram for Thomson-bridge method for calibrating a slide wire.

Calibration Corrections.—If it appears from the data obtained by any of these methods that the wire is not uniform, it is best to replace it by another. If circumstances preclude this, the calibration observations may be reduced as follows:

It is desired to find the corrected values of the readings to be inserted in the formulæ already given for the slide-wire bridge; that is, the readings on a uniform wire of the same total resistance as the wire actually used.

The lengths of the sections of equal resistance are plotted as ordinates. The positions of the lower ends of the sections are taken as abscissæ. A smooth curve is then drawn through

the points. This curve shows what the length of the section would be if its lower end were at any point on the slide wire.

Call the ordinate at $x = 0$, y_1. If the lower end of the section were at 0 the upper end would be at $x = y_1$. Look up the ordinate at that point, call it y_2, add it to y_1 and determine y_3 and so on. Let $y_1 + y_2 \ . \ . \ . + y_n = S$.

The resistances of the section S, denoted by R_s, and of the whole wire between 0 and 100, denoted by R_{100}, are then measured by any convenient method. Obviously, the section S is made up of n smaller sections each having a resistance $\frac{R_s}{n}$.

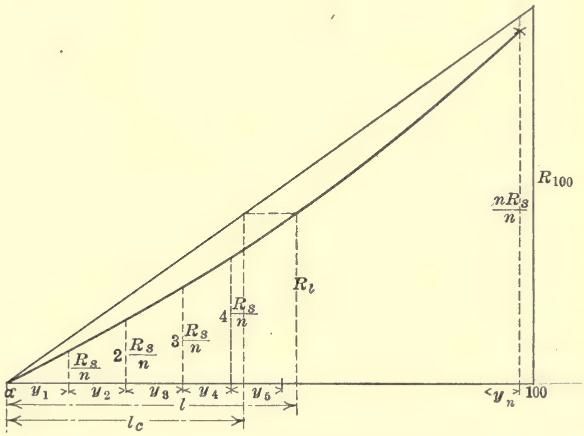


FIG. 103.—Pertaining to calibration of slide wire.

The resistance of the section y_1 is $\frac{R_s}{n}$, of section $y_1 + y_2$, $\frac{2R_s}{n}$, of section $y_1 + y_2 + y_3$, $\frac{3R_s}{n}$; of section $y_1 + y_2 + y_3 + \ . \ . + y_n$, $\frac{nR_s}{n}$. These resistances may be plotted as ordinates, using as abscissæ the values, y_1, $y_1 + y_2$, $y_1 + y_2 + y_3$, etc. (Fig. 103). At the 100 mark R_{100} is laid off; a straight line drawn through this point and the scale reading 0 is the line along which the points previously plotted would lie if the wire were uniform.

Take any scale reading, l; the resistance of the length l is R_l. The corresponding reading on the uniform wire is obtained

by projecting this resistance upon the straight line as indicated, giving l_c as the corrected value of the reading.

Instead of actually making the plot, the corrections are better determined as follows.

Assuming the wire to be a meter long, the resistance per cm. of the uniform wire would be $\frac{R_{100}}{100}$.

Let $(y_1)_c$ be the corrected value of y_1.

$(y_1 + y_2)_c$ be the corrected value of $(y_1 + y_2)$.

$(y_1+y_2+ \ldots +y_n)_c$ be the corrected value of $(y_1+y_2+ \ldots +y_n)$.

Then

$$(y_1)_c \frac{R_{100}}{100} = \frac{R_s}{n}$$

$$(y_1 + y_2)_c \frac{R_{100}}{100} = \frac{2R_s}{n}$$

$$(y_1 + y_2 + \ldots + y_n)_c \frac{R_{100}}{100} = \frac{nR_s}{n}$$

$$\therefore \ (y_1)_c = 1 \left(\frac{R_s 100}{n R_{100}}\right)$$

$$(y_1 + y_2)_c = 2 \left(\frac{R_s 100}{n R_{100}}\right)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$(y_1 + y_2 + \ldots + y_n)_c = n \left(\frac{R_s 100}{n R_{100}}\right).$$

The corrections are

$$C_1 = 1\left(\frac{R_s 100}{n R_{100}}\right) - y_1$$

$$C_2 = 2\left(\frac{R_s 100}{n R_{100}}\right) - (y_1 + y_2)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$C_n = n\left(\frac{R_s 100}{n R_{100}}\right) - (y_1 + y_2 + \ldots + y_n).$$

These corrections may be plotted using scale readings as abscissæ.

General Discussion of the Wheatstone Bridge.—In order to set up and use a Wheatstone bridge to the best advantage, certain

points brought out by a study of the theory of the instrument must receive attention.

Galvanometer Current.—The general expression for the current through the galvanometer in terms of the resistances of the various bridge arms and the electromotive force and resistance of the battery is readily deduced. However, a simpler and more useful formula is that connecting the total bridge current with the current through the galvanometer, for in many cases the bridge current is regulated by a rheostat rather than controlled solely by the e.m.f. of the battery and by the various resistances.

Suppose the connections to be those given in Fig. 104*A*, and assume, following Maxwell, that the meshes are traversed by currents $(x + y)$, x and I_B as shown in the figure, also that the

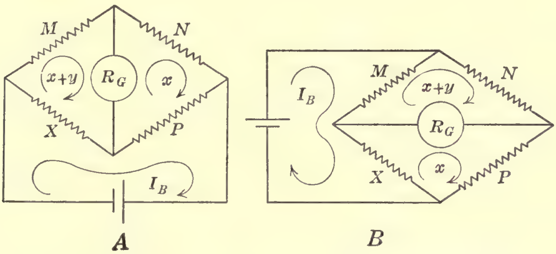


FIG. 104.—Mesh diagrams for Wheatstone bridge.

resistances of the various branches are M, N, X, P, R_G and B and that the electromotive force of the battery is E. By Kirchoff's corollaries

$$(x + y)\,(M + R_G + X) - xR_G - I_BX = 0$$

$$(x)\,(N + P + R_G) - (x + y)\,R_G - I_BP = 0.$$

Solving for y, which is the true galvanometer current, gives

$$y_1 = I_{G_1} = \frac{I_{B_1}(NX - MP)}{R_G\,(M + N + X + P) + (N + P)(M + X)} \quad (1)$$

The important case is when the bridge is nearly balanced. Then if B is the total resistance in the battery circuit outside of the bridge,

$$I_{B_1} = \frac{E}{B + \dfrac{(M + N)\,(X + P)}{M + N + X + P}} \text{ nearly enough.}$$

If the connections are as shown in Fig. 104B,

$$y_2 = I_{G_2} = \frac{I_{B_2}(MP - NX)}{R_G\,(M + N + X + P) + (M + N)(X + P)} \tag{2}$$

and

$$I_{B_2} = \frac{E}{B + \dfrac{(M + X)\,(N + P)}{M + N + X + P}} \text{ nearly enough.}$$

By use of these equations it is easy to determine whether or not any galvanometer will give results of the desired precision, for the maker will furnish a statement of the sensitivity of the instrument—that is, the deflection per unit current at a scale distance of 1 meter, the galvanometer being in proper adjustment.

The Best Resistance for a Thomson Galvanometer When Used with a Wheatstone Bridge.—In general terms, the galvanometer should have a high or a low resistance depending on whether high or low resistances are to be measured. The magnitudes of the bridge arms being fixed, the galvanometer having the best resistance is that one which will give the greatest deflection when the arm P is changed from the condition of perfect balance by a given amount. It has previously been shown that if the coils of a Thomson galvanometer are always wound on the same bobbin, the galvanometer constant is given by $G = K\sqrt{R_G}$, the effect of the insulation being neglected; consequently, if the time of vibration is kept constant, the deflection may be represented by

$$D = KI_G\sqrt{R_G} \tag{3}$$

K is seen to be the deflection per ampere for an instrument having a resistance of 1 ohm. Using (1)

$$D = KI_G\sqrt{R_G} = \frac{KI_{B_1}(NX - MP)\sqrt{R_G}}{R_G\,(M + N + X + P) + (N + P)(M + X)} \tag{3A}$$

This is to be made a maximum, R_G being the only variable. It will be found that

$$\frac{dD}{dR_G} = 0 \text{ if } R_G = \frac{(M + X)\;(N + P)}{M + N + X + P} \tag{4}$$

Inspection shows that this result corresponds to a maximum

value. Therefore, the galvanometer should have a resistance equal to the parallel resistance of the bridge arms between its terminals.

The effect of a departure from this best value of the galvanometer resistance may be seen from the following: Let the actual resistance of the galvanometer be n times that of the ideal instrument; that is, let

$$R_G = n \frac{(M + X)(N + P)}{M + N + X + P} \tag{5}$$

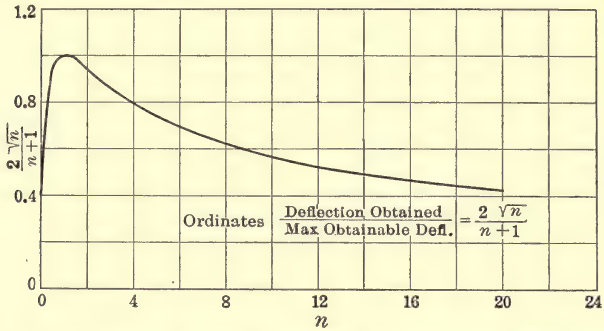


FIG. 105.—Showing effect of change of galvanometer resistance on the sensitivity of a Wheatstone bridge when a Thomson galvanometer is used.

Substitution of the value of R_G in (3A) gives as the corresponding value of the deflection,

$$D = \frac{KI_{B_1}(NX - MP) \qquad \sqrt{n}}{(n + 1)\sqrt{(M + X)(N + P)(M + N + X + P)}}.$$

The maximum deflection will be attained when $n = 1$. Consequently the ratio of the actual to the maximum possible deflection of the galvanometer is

$$\frac{D}{D_{max}} = \frac{2\sqrt{n}}{n + 1} \tag{6}$$

The values of this ratio are plotted in Fig. 105.

Best Position for the Galvanometer.—If the galvanometer be of fixed resistance and if a definite e.m.f. be impressed at the bridge terminals the magnitude of the galvanometer current may be considerably influenced by the relative positions of the galvanometer and battery.

For suppose the actual value of P differs from that necessary for a perfect balance by an amount δP, then referring to equations (1) and (2) and substituting the approximate values of I'_B,

$$I_{G_1} = \frac{-EM\delta P}{R_G(M+N)(X+P) + \dfrac{(M+N)(X+P)(N+P)(M+X)}{M+N+X+P}}$$

$$= \frac{-EM\delta P}{\text{Denom.}_1}$$

and

$$I_{G_2} = \frac{EM\delta P}{R_G(M+X)(N+P) + \dfrac{(M+N)(X+P)(N+P)(M+X)}{M+N+X+P}}$$

$$= \frac{EM\delta P}{\text{Denom.}_2}$$

The better arrangement will be the one corresponding to the equation having the smaller denominator.

$$\text{Denom.}_1 - \text{Denom.}_2 = R_G[(M+N)(X+P) - (M+X)(N+P)]$$
$$= R_G[(M-P)(X-N)] \qquad (7)$$

From this it is seen that if the opposite resistances are equal, the sensitiveness is the same for both arrangements. If the algebraic sign of the bracket is $+$, Denom.$_2$ is less than Denom.$_1$ and the second connection should be used; if it be $-$, the first is to be employed.

The better arrangement is the one where the galvanometer joins the junction of the two highest resistance bridge arms to the junction of the two lowest. An exact discussion of the more complete formula which includes the e.m.f. and the resistance of the battery gives

$$\text{Denom.}_1 - \text{Denom.}_2 = (R_G - B)(M-P)(X-N). \qquad (8)$$

Considering the battery and the galvanometer, the one having the higher resistance should join the junction of the two highest to the junction of the two lowest bridge arms. The importance of the proper relative position of the battery and galvanometer increases with the disparity of the bridge arms.

As an illustration, consider the bridge shown in Fig. 106 where there is a great difference in the bridge arms. Denom.$_1$ $-$ Denom.$_2$ is $+$, so the second connection should be used.

The second arrangement is about 23 times as sensitive as the first. However, as the total bridge current is about 44 times as great, the possibility of overheating a low-resistance arm of the bridge should be kept in mind. This is seen from the following, where E' has been assumed as 2 volts.

Watts dissipated in the bridge arms,

	M	N	X	P
With the first connection......	0.000004	0.004	0.00004	0.04
With the second connection....	0.9	0.0009	0.09	0.00009

In the second case there is a possibility that the arm M may be overheated if the current be kept on continuously.

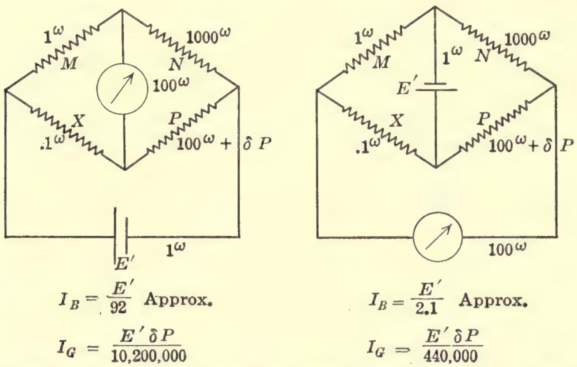


FIG. 106.—Showing effect of relative positions of battery and galvanometer on the sensitiveness of a Wheatstone bridge.

Sensitiveness Attainable with the Wheatstone Bridge.—The sensitiveness of a bridge arrangement may always be increased by increasing the bridge current. The limit to this increase is fixed by the carrying capacity of the bridge arm which will safely stand the least current.[4]

With coils of like construction but of different resistances the allowable energy losses are the same, that is,

$$(I')^2R = k \quad \text{or} \quad I'\sqrt{R} = \sqrt{k}, \text{ a constant.} \tag{9}$$

I' is the allowable current and k the allowable heating loss in watts; k depends upon the construction and manner of using the coils.

In planning new work it is frequently of importance to calculate

the deflection which would be obtained if some particular galvanometer were used.

Suppose that the maximum allowable bridge current, I'_B, is fixed by the heating in the arm X and that I'_X is the greatest current which can be employed in that arm.

If a Kelvin galvanometer of the best resistance is employed and the bridge is out of balance by a small amount, δP, then using equations (1), (3), (4) and the approximate relation

$$I_B = \left(\frac{M + X}{M}\right) I_X,$$

the deflection of the galvanometer is given by

$$D = KI_G \sqrt{R_G} = K\left(\frac{\delta P}{P}\right)(I'_X\sqrt{X})\frac{P}{2\sqrt{(P + X)(N + P)}}. \quad (10)$$

The sensitiveness which can be obtained is seen to be proportional to the square root of the allowable power loss in the arm which limits the bridge current.

Now suppose that a critically damped moving-coil galvanometer is to be used.

In this case let R'_G be the total resistance of the galvanometer branch of the circuit. R'_G will be made up of the resistance of the galvanometer itself plus any resistance which it is necessary to add in order to bring about critical damping. Let R_C be the *total* resistance of the galvanometer circuit which is required for critical damping, then as the bridge is nearly balanced,

$$R'_G + \frac{(M + X)(N + P)}{M + N + X + P} = R_C, \text{ a definite resistance,}$$

also

$$I_B = \left(\frac{M + X}{M}\right) I'_X$$

and

$$\frac{M}{N} = \frac{X}{P}.$$

Suppose that the resistance of the arm P is increased from the value necessary for a perfect balance by a small amount, δP, then the galvanometer current becomes

$$I_G = \frac{(M + X)I'_X\delta P}{R_C(M+N+X+P)} = \frac{XI'_X\delta P}{R_C(X+P)}$$

$$= \left(\frac{1}{R_C}\right)\left(\frac{\delta P}{P}\right)(I'_X\sqrt{X})\left(\frac{P\sqrt{X}}{X+P}\right),$$

and the deflection is

$$D = I_G S_I = S_V\left(\frac{\delta P}{P}\right)(I'_X\sqrt{X})\left(\frac{P\sqrt{X}}{X+P}\right) \qquad (11)$$

S_V is the volt sensitivity of the galvanometer when it is critically damped.

As in the previous case, the sensitiveness of the bridge is proportional to the square root of the allowable power loss in the arm which limits the bridge current.

If the resistance of the bridge (R_B) is so great that the galvanometer is under-damped, a resistance must be put in parallel with the bridge between the galvanometer terminals. Let the value of this resistance be R_S and let R_K be the resistance which it is necessary, in any case, *to put in series* with the galvanometer in order to attain critical damping. That is, let

$$R_C = R_G + R_K.$$

In this particular case R_K is the parallel resistance of the bridge and the resistance with which it is shunted, or

$$R_K = \frac{R_S R_B}{R_S + R_B}.$$

also

$$R_B = \frac{(M + X)(N + P)}{M + N + X + P}.$$

The galvanometer current when the bridge is slightly out of balance is

$$I_G = \frac{(M + X)I'_X R_S \delta P}{[R_G(R_S + R_B) + R_B R_S](M + N + X + P)}$$

$$= \left(\frac{\delta P}{P}\right)(I'_X\sqrt{X})\left(\frac{P\sqrt{X}}{X+P}\right)\left(\frac{R_K}{R_B}\right)\left(\frac{1}{R_G + R_K}\right)$$

$$\therefore D = S_I I_G = S_V\left(\frac{\delta P}{P}\right)(I'_U\sqrt{X})\left(\frac{P\sqrt{X}}{X+P}\right)\left(\frac{R_K}{R_B}\right) \quad . \qquad (12)$$

The sensitivity is seen to be diminished in the ratio $\frac{R_K}{R_B}$.

MEASUREMENT OF LOW RESISTANCE

Wheatstone Bridge Method.—When the value of X becomes small, 0.1 ohm or less, it is difficult to determine it accurately, owing to the uncertainty due to contact resistances introduced at the binding posts where X is clamped to the bridge. For example, such resistance for a No. 12 wire clamped by a binding post may be about 0.00015 ohm. Again, all standard low resistances and the shunts used in current measurements are provided with potential terminals, and the desired resistance is that between the points where these terminals are connected to the main resistance. The following method of comparing such resistances with a standard is very useful, since it requires no special apparatus.

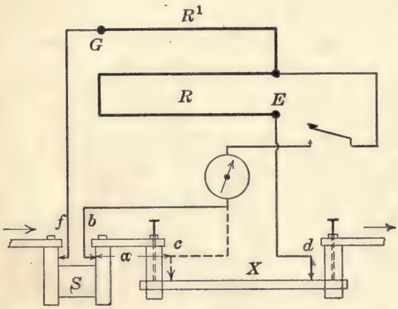


FIG. 107.—Wheatstone bridge method for comparing low resistances.

Let it be required to measure the resistance of a given length of the bar X. Connect it in series with the standard resistance, S, as indicated, and attach two potential terminals, c and d, at the proper points (Fig. 107). These terminals may be clamps making contact with the bar by a pointed screw or knife edge. In order to compare X and S, the intermediate resistance between b and c must be eliminated. To do this connect the galvanometer at b and balance; call the necessary value of R, R_b, then

$$\frac{R^1}{R_b} = \frac{S}{X + \alpha}.$$

Now change the galvanometer wire to c and balance again; call this value of R, R_c, then

$$\frac{R^1}{R_c} = \frac{S + \alpha}{X};$$

α can be eliminated from these two equations, giving as a result:

$$X = S\left(\frac{R_c}{R^1}\right)\left(\frac{R^1 + R_b}{R^1 + R_c}\right).$$

It has been assumed that the resistance a remains constant. Consequently all clamps and connections must be firmly set up and a itself should be of so large a cross-section that it will not heat with the largest current which is used. The following points should be attended to: the resistance of a must be made as small as possible; the total bridge current I_B should be as large as is consistent with absence of heating in the various bridge arms; R^1, and consequently R_c and R_b, should be small; their magnitude is limited by the fact that they are usually adjustable to single ohms, and a certain definite percentage precision is usually required in the results. They should be large enough so that there is no danger of heating, and so large that the resistances of the connection wires fG and dE are negligible. If it should prove that R_c and R_b are much smaller than R^1, it would be better to make the adjustment by changing R^1 rather than as suggested above. The standard resistance S should have ample current carrying capacity. It may be necessary to keep the temperature of X down by immersion in an oil bath.

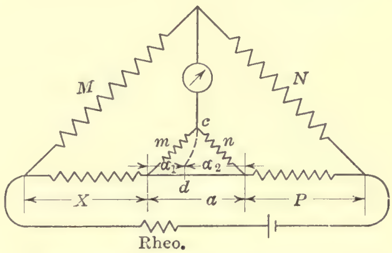


FIG. 108.—Circuit diagram for Thomson bridge.

Thomson Bridge or Kelvin Double Bridge.[6]—The elimination of the intermediate resistance a may also be accomplished by means of the Thomson bridge. The scheme embodied in this instrument is that most frequently employed for the precision comparison of low resistances; it is also commonly used for special bridges designed for the rapid measurement of the conductivity of samples of wire.

Inspection of the theoretical diagram will show that this arrangement differs from the Wheatstone bridge in the addition of two auxiliary resistances, m and n, which are placed in series and shunted around the resistance a, which is to be eliminated; one galvanometer terminal is connected to the junction of m and n. See Fig. 108.

The conditions necessary for a balance may be shown thus:

Assume that the galvanometer stands at zero. Between the terminals of α there must be a point at the same potential as c; let d be this point; suppose it to be joined to c by a connection of zero resistance, thus bringing the points c and d together; this is allowable from the manner of locating the point d. The arrangement has then become a Wheatstone bridge with arms

$$M, N, X + \frac{m\alpha_1}{m + \alpha_1} \text{ and } P + \frac{n\alpha_2}{n + \alpha_2}$$

$$\therefore \frac{M}{N} = \frac{X + \dfrac{m\alpha_1}{m + \alpha_1}}{P + \dfrac{n\alpha_2}{n + \alpha_2}}.$$

From the manner of locating d,

$$\frac{m}{\alpha_1} = \frac{n}{\alpha_2} \text{ or } \frac{m}{m + \alpha_1} = \frac{n}{n + \alpha_2}.$$

Consequently

$$\frac{M}{N} = \frac{X + \dfrac{n\alpha_1}{n + \alpha_2}}{P + \dfrac{n\alpha_2}{n + \alpha_2}}$$

and

$$X = \frac{MP}{N} - \left(\frac{n}{n + \alpha_2}\right)\left(\alpha_1 - \alpha_2\frac{M}{N}\right) = \frac{MP}{N} - \left(\frac{n\alpha_2}{n + \alpha_2}\right)\left(\frac{m}{n} - \frac{M}{N}\right).$$

Obviously, if the resistances are adjusted so that $\frac{m}{n} = \frac{M}{N}$ the second member becomes $\frac{MP}{N}$, and $X = \frac{MP}{N}$. The measurement is then independent of α.

For general laboratory purposes P may be a variable standard, and is frequently a slide wire or, better, a resistance divided into tenths, the last tenth being a slide wire of perhaps 0.001 ohm. This standard, shown diagrammatically in Fig. 109, should have ample carrying capacity, 50 amp. at least.

A convenient form of this bridge, employing such a low-resistance standard, and adapted to commercial work, is shown diagrammatically in Fig. 110. In this arrangement it is assumed

that the resistances of the connecting leads cc', dd', ee' and ff' are so small, compared with the resistance with which they are in series, that their effects are negligible.

With the Thomson bridge as it is actually used, X, P and α are always small, so that it is necessary to control the bridge current

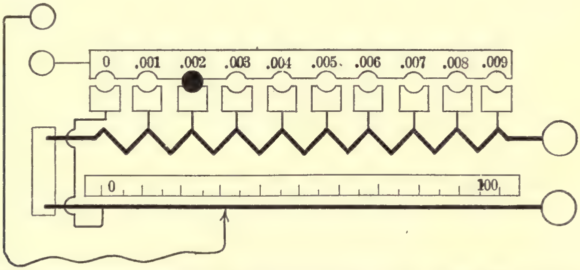


FIG. 109.—Variable standard resistance for use with Thomson bridge.

by a rheostat; the current should be as large as is compatible with accuracy. Its limiting value, and therefore the sensitivity of the bridge, is fixed by the carrying capacity of the resistances employed; consequently ample provision must be made for dissipating the heat generated in the various arms.

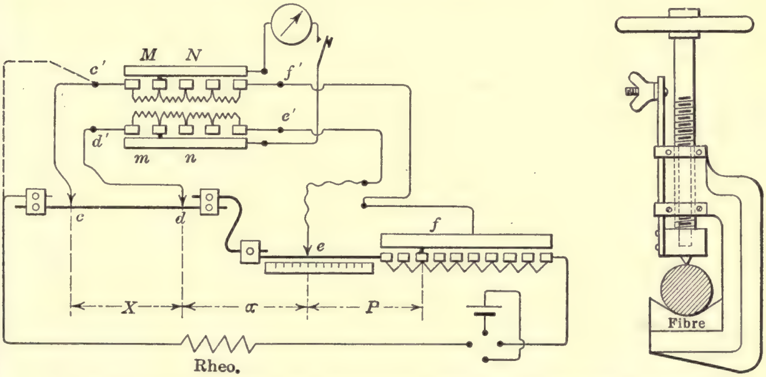


FIG. 110.—Diagram of laboratory form of Thomson bridge.

The resistance to be measured must be provided with potential terminals or their equivalent. In dealing with rods the contacts at c and d may be made by soldering small wires across the rod, the superfluous solder being carefully removed, or more conveniently, by point or knife-edge clamps.

With massive conductors it is absolutely necessary that the relative positions of the potential and current terminals be such that the stream lines between c and d are in their normal position; for instance, in measuring a low-resistance shunt, such as is used for large currents on switchboards, a serious error may be introduced, even if c and d are at the proper points, if the current is not led into the short, heavy terminals exactly as it is to be in the subsequent use of the instrument.

Expression for Galvanometer Current.—Let the arrangement of the conductors and the mesh currents be as shown in Fig. 111. The mesh equations are

$$(x + y)(M + R_G + m + X) - yR_G - zm - I_BX = 0$$
$$(y)(N + P + n + R_G) - (x + y)R_G - zn - I_BP = 0$$
$$(z)(m + n + \alpha) - (x + y)m - yn - I_B\alpha = 0.$$

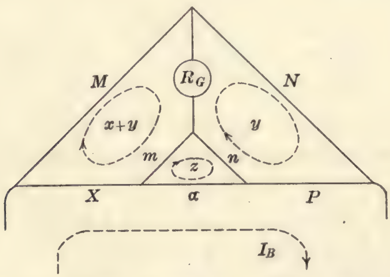


FIG. 111.—Mesh diagram for Thomson bridge.

Solving for x, which is the galvanometer current, gives:

$$x = I_G = I_B \frac{NX - MP + \dfrac{\alpha(mN - nM)}{m + n + \alpha}}{R_G\,(M + N + X + P) + (M + X)(N + P) + [\,]} \tag{13}$$

where [ ] =

$$\frac{n\alpha(M + X + R_G) + m\alpha(N + P + R_G) + mn(M + N + X + P + \alpha)}{m + n + \alpha} \tag{14}$$

If the galvanometer stands at zero, I_G is zero, and

$$X = \frac{MP}{N} - \frac{\alpha(mN - nM)}{N(m + n + \alpha)} = \frac{MP}{N} - \alpha n\frac{\left(\dfrac{m}{n} - \dfrac{M}{N}\right)}{m + n + \alpha} \tag{15}$$

Convenience dictates that the second term on the right-hand side of the equation be made zero; this is accomplished if m and n are so adjusted that $\frac{m}{n} = \frac{M}{N}$. In other words, M, N, m, n, should fulfil the conditions for the resistances in an ordinary Wheatstone bridge, in which case X is independent of the intermediate resistance α and of the auxiliary conductors n and m, as has just been shown in a somewhat less general fashion.

In the commercial instrument the ratio arms are usually mounted in the same box and are capable of variation, being made up of coils so chosen that the relation $\frac{m}{n} = \frac{M}{N}$ is conveniently attained. If this condition is not exactly fulfilled, the error due to neglecting the last term in (15) will diminish as α is decreased; therefore the resistance of the intermediate connection should be made as small as possible, especially when measuring small resistances. One obvious test for the accuracy of the relation $\frac{m}{n} = \frac{M}{N}$ may be made by temporarily increasing α, or better, by altogether removing the connection, thus breaking the circuit. If the galvanometer remains in balance with α both open and closed the adjustment is correct.

Best Resistance for a Thomson Galvanometer When Used with a Thomson Bridge.—The best resistance for the galvanometer may be found as follows: The resistance α is always made as small as possible; assume that it is negligible in comparison with both m and n, then [ ] in equation (14) reduces to

$$[\ ] = \left(\frac{mn}{m+n}\right)(M + N + X + P)$$

and

$$I_G = I_B \frac{(MP - NX)}{\left(R_G + \frac{mn}{m+n}\right)(M + N + X + P) + (M + X)(N + P)}.$$

With Thomson galvanometers having coils of equal dimensions the relation between the current, the resistance and the deflection is

$$D = KI_G\sqrt{R_G}.$$

If the resistance of the arm P differs from that necessary for

a perfect balance by a small amount, δP, the galvanometer deflection is

$$D = KI_B \frac{M\delta P\sqrt{R_G}}{\left(R_G + \frac{mn}{m+n}\right)(M+N+X+P) + (M+X)(N+P)}$$

D is to be made a maximum by adjusting R_G. On differentiating and equating the result to zero it will be found that the maximum value of D will be obtained when

$$R_G = \frac{mn}{m+n} + \frac{(M+X)(N+P)}{M+N+X+P} \tag{16}$$

That is, the galvanometer resistance should be equal to that of the remainder of the circuit in which it is placed.

Sensitiveness Attainable in Measurements With the Thomson Bridge.—The sensitiveness of the bridge increases proportionally to I_B, the limit being reached when one of the arms begins to heat unduly. As the bridge is usually employed, the limit will be set by either X or P.

When the bridge is nearly balanced and the resistance α is small compared with m and n,

$$I_B = I_X\left[\frac{M+X}{M} + \frac{\alpha}{M+N}\right] = \left[\frac{M+X}{M}\right]I_X \text{ approx.}$$

If a Thomson galvanometer of the best resistance is employed, the value for R_G reduces to

$$R_G = \frac{P(M+X+m)}{X+P}.$$

and

$$I_G = \frac{I_B M\delta P}{2(N+P)(M+X+m)}.$$

Substituting these values in the expression for D and reducing gives

$$D = K\left(\frac{\delta P}{P}\right)(I'_X\sqrt{X})\frac{P\sqrt{X}}{2\sqrt{(P+X)[X(N+P)+mP]}} \tag{17}$$

If $m = 0$ this reduces to equation (10) which applies to the Wheatstone bridge.

If the same conductors M, N, X, P are arranged as a Wheatstone bridge and then as a Thomson bridge it will be found from

(10) and (17) that with a Thomson galvanometer the Wheatstone bridge arrangement will be the more sensitive in the ratio,

$$\frac{D_W}{D_T} = \sqrt{1 + \frac{n}{N+P}}.$$

If a critically damped moving-coil instrument is to be used and the resistance of the bridge is so low that the instrument is over-damped, it will be necessary to increase the resistance of the galvanometer circuit. It will be assumed that the resistance α is low compared with m and n. The arrangement then becomes the equivalent of a Wheatstone bridge where the resistance of the galvanometer branch is

$$R'_G = R_G + \frac{mn}{m+n} + R$$

R being the resistance added in order to obtain critical damping.

Let the resistance of the circuit necessary for critical damping be R_C, then

$$R_C = R_B + R'_G$$

where

$$R_B = \frac{(M+X)(N+P)}{M+N+X+P}.$$

When the bridge is slightly out of balance

$$I_G = \frac{I'_X(M+X)\delta P}{(R_C - R_B)(M+N+X+P) + (M+X)(N+P)}$$

$$I_G = \frac{1}{R_C}\left(\frac{\delta P}{P}\right)\left(I'_X\sqrt{X}\right)\left(\frac{P\sqrt{X}}{X+P}\right)$$

$$D = S_I I_G = S_V\left(\frac{\delta P}{P}\right)\left(I'_X\sqrt{X}\right)\left(\frac{P\sqrt{X}}{X+P}\right) \tag{18}$$

If the instrument is under-damped, it will be necessary to place a shunt around the bridge between the galvanometer terminals. In this case, if R_G is the resistance of the galvanometer and R_S that of the shunt,

$$I_G = \frac{I'_X(M+X)R_S\delta P}{\left[\left(\frac{mn}{m+n}\right)(R_G+R_S) + R_G R_S\right](M+N+X+P) + (M+X)(N+P)(R_G+R_S)}.$$

The bridge resistance is

$$R_B = \frac{mn}{m+n} + \frac{(M+X)(N+P)}{M+N+X+P}$$

$$I_G = \frac{I'_X(M+X)R_S\,\delta P}{[R_G\,(R_S+R_B)+R_S\,R_B]\,(M+N+X+P)}.$$

The resistance which, in any case, it is necessary to add to that of the galvanometer to produce critical damping is R_K, that is,

$$R_C = R_G + R_K.$$

The value of R_K is

$$R_K = \frac{R_S R_B}{R_S + R_B}$$

$$\therefore\ I_G = \frac{I'_X\, X R_K \delta P}{(R_G+R_K)\,(X+P)R_B}$$

and

$$D = S_I I_G = S_V\left(\frac{\delta P}{P}\right)\left(I'_X\sqrt{X}\right)\left(\frac{P\sqrt{X}}{X+P}\right)\left(\frac{R_K}{R_B}\right) \qquad (19)$$

Compare formula (12) on page 189.

Precision Measurements with the Thomson Bridge.[7]—In the proofs already given M, N, m, n, are the total resistances of the various bridge arms, that is, they are the sums of the resistances of the coils in the arms and of the necessary leads. The lead resistance can never be zero; consequently, though the coils themselves are adjusted so that the relation $\frac{M}{N} = \frac{m}{n}$ is fulfilled, yet when they are connected into circuit by the necessary leads this relation will be slightly disturbed and the elimination of α from the results will not be complete. In the careful comparison of resistance standards this matter is of importance, for of necessity the resistances of the potential terminals are included in the bridge arms.

If the resistances in the arms are separated into two parts, that in the coils being denoted by the subscript $_C$ and that in the leads by the subscript $_L$, equation (15) becomes

$$X = P\left(\frac{M_C+M_L}{N_C+N_L}\right) - \frac{\alpha\left(\frac{m_C+m_L}{n_C+n_L} - \frac{M_C+M_L}{N_C+N_L}\right)(n_C+n_L)}{m_C+m_L+n_C+n_L+\alpha} \qquad (20)$$

If the elimination of α from the result is complete, the balance of the bridge will not be upset when α is greatly increased or even made infinity by breaking the connection between X and P. Therefore the test for the proper adjustment of the auxiliary ratio is that the bridge remains in balance with α closed and with α open. In precision measurements it is essential that α be made as low as possible, and less than X.

Reeves Method for Adjusting the Ratio Arms to Eliminate α.—Based on the foregoing, the process of adjustment to eliminate α is, with α in place, to adjust the main ratio $\frac{M}{N}$ until the bridge is balanced, then to remove α and rebalance by changing the ratio $\frac{m}{n}$. This second adjustment will throw out the first, so α must be replaced and $\frac{M}{N}$ readjusted and so on until by successive approximations such an adjustment is attained that the balance is maintained with α either closed or open. This process of successive balances eliminates all questions as to the exact values of m and n and their leads.

When the elimination of α is complete,

$$X = P\left(\frac{M_C + M_L}{N_C + N_L}\right).$$

The lead resistances to M and N must be determined and allowed for.

Wenner Method for Eliminating the Effects of Lead Resistances and α.—For this method of working the Thomson bridge it is necessary that the slides on the main and auxiliary ratio arms be mechanically connected so that the relation $\frac{M_C}{N_C} = \frac{m_C}{n_C}$ is always maintained. The coils are adjusted with this in view.

Inspection of formula (20) shows that if in addition the resistances of the leads to the ratio coils are adjusted so that

$$M_C N_L = N_C M_L$$

and

$$m_C n_L = n_C m_L$$

then

$$X = P\left(\frac{M_C}{N_C}\right).$$

Therefore M_L and m_L are made adjustable by including in each a mercury slide resistance (a and b in Fig. 112). This consists of an ebonite tube about 12 cm. long with a 3-mm. bore. The terminals are at the upper and lower ends of the tube and an amalgamated copper plunger serves to displace and short-circuit more or less of the mercury. This form of adjustable resistance is remarkably definite in its action.

To carry out the adjustment it is necessary to add two switches, S_1 and S_2, as shown in Fig. 112, by which the arms $M_C + N_C$ and $m_C + n_C$ may be short-circuited.

The final balance is attained by four steps:

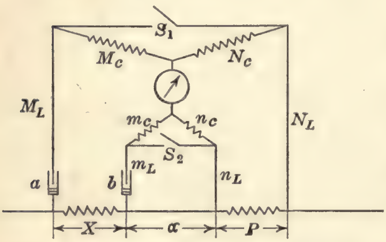


FIG. 112.—Wenner arrangement for eliminating the effect of lead resistances in the Thomson bridge.

1. With both S_1 and S_2 open, the bridge is balanced as usual by adjusting the ratio arms.

2. The switch S_1 is closed; the balance will be upset; it is restored by adjusting M_L by means of the rheostat a. This makes

$$M_L N_C = N_L M_C, \text{ very closely}.$$

3. The switch S_1 is opened and S_2 is closed and the balance restored by adjusting the value of m_L by means of the rheostat b. This makes

$$m_C n_L = n_C m_L, \text{ very closely}.$$

4. With both S_1 and S_2 open, the bridge is finally balanced by adjusting the double ratio slides. If this last adjustment requires a considerable change in the setting of the ratio slides, the adjustments are repeated.

Measurement of Resistances in Permanently Closed Circuits.—For a method of measuring a resistance which is included in a circuit which cannot be opened, see page 96.

MEASUREMENT OF HIGH AND OF INSULATION RESISTANCE

The measurement of insulation resistance, using direct-current potentials of a few hundred volts, is of great practical importance because of its utility as a means of separating the

good from the defective insulated wires during the process of manufacture. Also, specifications as to insulation resistance as measured by direct currents are inserted in contracts.

It is to be understood that the results of this test are not "resistances" in the same sense as those obtained for metallic conductors by use of the Wheatstone bridge. As the test is ordinarily carried out, the results give no means of calculating the current which will finally flow through the insulation of the wire under the prolonged application of a direct-current electromotive force (see page 207, Absorption Effects). It is to be understood that the current which will flow through the di-

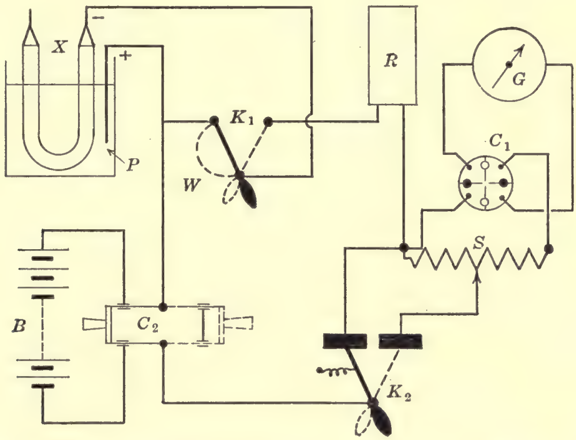


FIG. 113.—Connection for measuring insulation resistance.

electric when the applied electromotive force is periodic, especially at the high frequencies employed in telephony, involves a very different "resistance" from that determined by this method, at a given voltage it is distinctly lower, due to the energy dissipated in the dielectric.

Direct-deflection Method.—Insulation resistances have very high values and may be several hundred or several thousand megohms, a megohm being 1,000,000 ohms. The method usually employed in these measurements is really one of substitution; the necessary apparatus is shown in Fig. 113.

The galvanometer G should be of the D'Arsonval type and very sensitive; an instrument having a sensitivity of about

1×10^9 and of approximately 1,000 ohms resistance will be satisfactory. This galvanometer must have a good law of deflection; that is, the deflection must be proportional to the current, and it must have a definite zero reading. It should be so supported that it is free from mechanical vibration and thoroughly insulated electrically. C_1 is an ordinary four-part commutator for reversing the galvanometer current, as it is necessary to keep the deflections in the same direction on account of possible irregularities of the zero reading due to the coil being

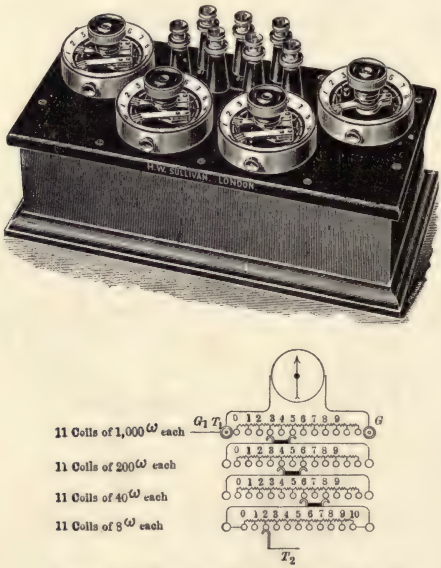


FIG. 114.—Universal shunt box.

slightly magnetic. The shunt S should be of the Ayrton universal type, for by selecting one of the proper resistance the galvanometer may be critically damped, thus enabling the readings to be taken in the shortest possible time. With this type of shunt, when used in the manner indicated, the damping of the galvanometer is independent of the multiplying power, consequently the instrument will not be overdamped even though it be heavily shunted. Also, though there will usually be some thermal electromotive forces due to inequalities of temperature in the galvanometer circuit, G, C_1, and S, no dif-

ficulty will arise, as this circuit is of constant resistance; the only effect will be that the deflections will be read from a zero which may differ slightly from the mechanical zero of the instrument.

R is a fixed known resistance of 100,000 ohms or $\frac{1}{10}$ megohm.

An exceedingly convenient form of Ayrton shunt is shown in Fig. 114. The constant resistance in the box between the galvanometer terminals is 10,000 ohms. By means of the four handles, the movable terminal may be carried from 0 to 10,000 by steps each one of which changes the multiplying power by $\frac{1}{10{,}000}$ part of the value it has when all the slides are at the extreme right hand, which value is taken as unity; for with the Ayrton-Mather arrangement we are concerned only with relative multiplying powers. The multiplying power to be used is obtained by dividing 10,000 by the sum of the readings of the four slides. The three posts at the rear are the shunt terminals; the four in the second line connect with the resistance R, of 100,000 ohms, which is divided into three sections of 10,000, 30,000 and 60,000 ohms respectively.

The key K_2 (Fig. 113) is kept in the position shown, by a spring; when in this position no current can flow through the galvanometer and the instrument is thoroughly protected. If the key be held in the dotted position the galvanometer is in service. The construction should be such that the circuit is not broken when the key is thrown from one position to the other.

The key K_1 serves to throw the cable in circuit and, when in the position shown, short-circuits it, thus ensuring thorough discharge. The battery B is connected to one side of a double-pole, double-throw switch C_2, the other side being short-circuited to enable discharge deflections to be taken. One of the middle connections of C_2 is carried to the tank plate P, the other to the middle post of K_2. The battery should be fairly well insulated to prevent its running down. It must be capable of giving a constant e.m.f. of, possibly, 500 volts.

It is usual to connect the negative pole of the battery to the core of the cable, the idea being that with this connection the effect of electrolysis is to open up any fault which may exist in the insulation. Many specifications require tests with both

the − and + poles connected to the core and require that the two results check.

To measure X the "constant" of the apparatus must first be determined; this is, the number of megohms at X which will correspond to a deflection of 1 mm. on the galvanometer scale. To do this, short circuit X at K_1 by a piece of wire W; the resistance of the circuit will then be $R + P_R$, where P_R is the resistance of the shunted galvanometer, the leads and the battery. S should be set at its smallest value and the deflection of the galvanometer noted. Now, if necessary, alter S to obtain a good reading; call this D_R and let m_R be the corresponding multiplying power of the shunt. Then $E = I_R(R + P_R)$. When X is in place $E = I_X(X + R + P_X)$. As the deflection of the galvanometer is proportional to the current,

$$I_R = Km_RD_R,$$

$$I_X = Km_XD_X,$$

therefore $X = \dfrac{m_RD_R}{m_XD_X}(R + P_R) - (R + P_X)$.

In general, $R + P_X$ is negligible compared with X, and P_R negligible compared with R; so,

$$X = \frac{Rm_RD_R}{m_XD_X}.$$

The quantity Rm_RD_R is the "constant" of the apparatus. As R is expressed in megohms, this is the number of megohms for unit deflection of the galvanometer when the relative multiplying power of the shunt is unity.

The resistance R is left in circuit continuously, in order that there may be no possibility of a current being sent through the galvanometer of sufficient strength to burn it out. The magnitude of R, $\frac{1}{10}$ megohm, is so small that no material error is introduced by this procedure, as X is some hundreds or thousands of megohms.

Precautions.—In order that a measurement of insulation resistance may possess any value, one must be sure that the only current which passes through the galvanometer is that which flows through the insulation of the cable. Therefore, the galvanometer *must* be connected to the core of the cable and

not to the tank plate. As the tank is grounded, any leakage current due to imperfect insulation of battery and leads will be measured by the galvanometer, if it be connected to the tank plate, and the results of the test vitiated.

Any current which leaks over the surface of the insulation from the projecting wire of the cable to the tank will cause error in the results. This leakage current *must* be reduced to zero. Therefore any protective covering, such as braid or armor, should be removed from the ends of the wire for a distance of at least 18 in., thus laying bare the insulating coating. This latter should not be handled and must be kept scrupulously clean. As an additional safeguard, the insulation should be cut back from the end of the wire, as indicated in Fig. 113. This must be done with a sharp, *clean* knife, so as to leave a perfectly clean surface of considerable length (2 or 3 in.). In order to be absolutely certain that all surface leakage has been eliminated, recourse should be had to Price's guard wire. This device is shown in Fig. 115.

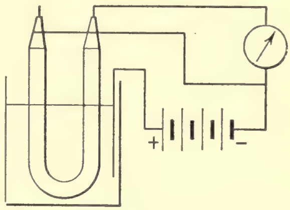


FIG. 115.—Diagram for Price guard wire.

A few turns of bare and flexible wire are closely wrapped around the insulation a few inches from its end so that they make perfect contact with it; the wire is then carried to the battery side of the galvanometer. The potential difference between the core and the guard wire is practically *nil*, so that any leakage will be from the guard wire to the tank; consequently the leakage current will not be measured by the galvanometer. The guard wire may be used for a variety of purposes; for instance, to protect the galvanometer lead if it be long and cannot be made an air line, or to protect a part or all of the testing apparatus.

The shunt S, the commutator C_1, the galvanometer G, the resistance R, and the key K_1 *must* be thoroughly insulated. They should all be mounted on posts of polished hard rubber at least 4 in. high, and may be protected by a guard wire. The lead from K_1 to the cable should, if possible, be an air line.

Even after all precautions have been taken, it will not do to assume that leakage is not present. A test must be made to

determine this point. To do this, wire up exactly as for a test, close K_1, then disconnect the lead from the cable and leave it hanging free. Be sure not to introduce a new source of error by the arrangement for supporting the free end. Now throw on the battery. If the e.m.f. be high the galvanometer will probably give a slight deflection and then settle back to its original reading. If it does this the deflection is due to the electrostatic action incident to charging the apparatus to a high potential. Note, with the key K_2 as shown, if there is a permanent deflection of the galvanometer and its direction. Now put K_2 in the dotted position and adjust S for full sensitiveness ($m = 1$), and again note the deflection if there be any. Leakage may occur on the lead between its free end and the shunt; if so, the deflection will be positive in direction and will not appear until the key K_2 is placed in the dotted position. S should be adjusted so that $m = 1$. If the leakage occurs on the leads from the shunt box to the galvanometer or at the galvanometer terminals, the deflection may be either positive or negative, as follows:

Ground on left-hand lead—

Shunt adjusted for $m = \infty$, $D = +$ and small.
Shunt adjusted for $m = 1$, $D = +$ and large.

Ground on right-hand lead—

Shunt adjusted for $m = \infty$, $D = -$ and large.
Shunt adjusted for $m = 1$, $D = -$ and small.

The test should be repeated with C_1 reversed.

When measuring X the key K_2 must not be thrown to the dotted position until the cable is charged electrostatically. Therefore, after closing K_1, allow at least 20 sec. to elapse before throwing K_2 to the right; this will prevent injury to the galvanometer.

Immersion.—After the specimen has been immersed, sufficient time must be allowed before the test is made for the cable to become thoroughly saturated, for moisture to work into any defects that may exist in the insulation, and for the insulation to attain the temperature of the tank. In practice no tests should be made until after 12 hours.

Absorption Effects.—When the battery is first applied to the cable there will be a sudden rush of current due to the charging of the cable electrostatically. Therefore it is absolutely necessary to have the key K_2 in the position shown when the circuit is made, in order to prevent possible injury to the galvanometer. After the static charging, a current flows into the cable, rapidly diminishing to a nearly constant value. This current furnishes the "absorbed" charge and includes the current which actually flows through the insulation. The first portion diminishes toward zero, while the latter tends to become constant. If

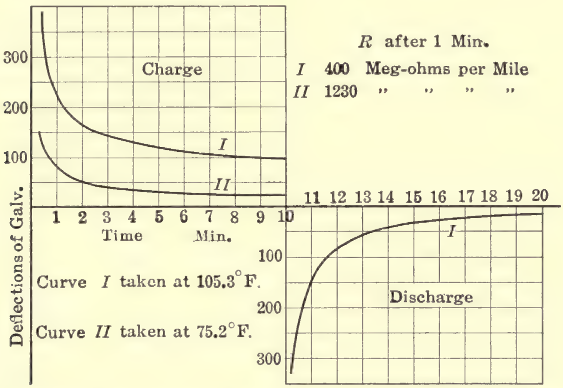


FIG. 116.—Showing the effect of time of electrification on the galvanometer deflection when measuring insulation resistance.

the switch C_2 were now thrown to the discharge position (dotted), C_1 reversed (to keep the deflections in the same direction), and the deflection observed, it would be found that at first there is a sudden rush due to the condenser discharge of the cable. This is followed by a current which gradually diminishes toward zero, this latter being due to the gradual working out of the absorbed charge. The various phenomena are illustrated by the curves shown in Fig. 116.

From the curves it is seen that the apparent resistance of the insulating covering is a function of the time of electrification and that it is necessary to state this time when quoting values of the insulation resistance; otherwise they possess no meaning. It is customary to calculate the resistance at the end of 1 min.

electrification; from this result and the known length of the sample the insulation resistance per mile or per 1000 ft. is determined.

Effect of Temperature.—The substances classed as insulators have very large *negative* temperature coefficients; that is, an increase of temperature lowers their resistance. This is shown in Fig. 117 which gives the result of tests on a sample of rubber-covered wire. In this work it is customary to express the temperature in degrees Fahrenheit.

For purposes of comparison it is necessary to reduce the results of insulation resistance measurements to some standard temperature, 15°.5C. This is usually done by dividing the resistances at the temperatures of observation by experimentally determined factors, the values of which will be different for different insulating compounds. Consequently, the various reduction factors quoted in electrical handbooks should not be applied indiscriminately.

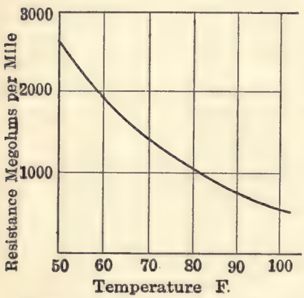


FIG. 117.—Illustrating effect of temperature on insulation resistance.

The great difficulty with tests for insulation resistance as a guide to the condition of underground feeders after installation, is the uncertainty as to the temperature, due to the feeders having been in use, or to the heating by currents in neighboring ducts.

Insulation Testing by Voltmeter.—Insulation resistances of the magnitude of 1 or 2 megohms may be measured by aid of a direct-current voltmeter of known resistance. Two readings are taken, the first when the instrument is directly across the line, the second when the line voltage is applied to the instrument and the unknown resistance in series. The testing voltage must remain constant.

Call the reading when the voltmeter is across the line, D_1, and when it is in series with X, the unknown resistance D_2. Then if the resistance of the voltmeter be R_V, and the constant of the instrument considered as a current galvanometer be K,

$$\frac{E}{R_V} = KD_1$$

$$\frac{E}{R_V + X} = KD_2 = \frac{ED_2}{R_V D_1}$$

$$\therefore X = R_V\left(\frac{D_1 - D_2}{D_2}\right).$$

If the power may be shut off, this method lends itself to the determination of the insulation resistance between the conductors of a two-wire distribution circuit and the ground; that is, the water and gas pipes. Such tests are necessary when investigating the wiring of buildings. The circuit may be opened by removing the main fuses, and the necessary e.m.f. obtained by using either the supply voltage or a portable battery of dry cells.

Loss of Charge Method.—The loss of charge method of measuring insulation resistance is based on the following considerations.

If a perfect condenser is charged and then discharged through a resistance, it can be shown that $V_t = V_0\epsilon^{-\frac{t}{CR}}$ where V_0 is the initial P.D. of the plates, V_t the P.D. at any subsequent time t, C and R the capacity of the condenser and resistance of the circuit. Solving this for R,

$$R = \frac{t}{C \log_e \frac{V_0}{V_t}} = \frac{t0.4343}{C \log_{10} \frac{V_0}{V_t}}.$$

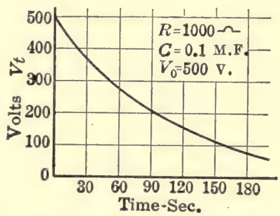


FIG. 118.—Illustrating fall of potential in loss of charge method for measuring insulation resistance.

The relation of its units is such that if C is in microfarads, and t in seconds, R will be given in megohms. If R be large, several hundred or thousand megohms, the time of discharge will be sufficiently prolonged, so that it is possible to follow the variation of V_t with an electrometer or electrostatic voltmeter. From the above it is seen that it is possible to measure a large resistance by discharging through it a condenser of known capacity and noting the P.D. at the beginning and end of a time t.

The curve illustrates the phenomena for the case where R = 1,000 megohms, C = 0.1 microfarad, V_0 = 500 volts.

If the resistance be exceedingly high, the P.D. of the condenser may fall so slowly that in any reasonable time, V_t may not

differ greatly from V_0, and consequently the ratio $\frac{V_0}{V_t}$ will be seriously affected by errors of observation. By observing V_0 and the *fall* of potential, much more accurate results may be obtained. Let the fall from V_0 in time t be denoted by v, then

$$V_0 = V_t + v,$$

and

$$R = \frac{t0.4343}{C \log_{10} \frac{V_0}{V_0 - v}}.$$

Only the ratio of voltages enters the formula, and it is possible to use a ballistic galvanometer instead of an electrometer or electrostatic voltmeter in the work. Connections are made so that the condenser is *charged* from the battery through the ballistic galvanometer. Let the elongation be $D_0 = KQ_0 = KCV_0$. After the cable has leaked for t seconds, it is again connected to the battery through the galvanometer and the elongation due to the quantity which is necessary to replace that which has leaked out is observed. Call this elongation D_t, then

$$D_t = KQ_t = KCv$$

consequently

$$R = \frac{t0.4343}{C \log_{10} \frac{D_0}{D_0 - D_t}}.$$

In this formula an error in D_0 does not seriously affect the results, as it occurs both in the numerator and denominator, and D_t is a comparatively small quantity directly observed, so inaccuracies in it will not greatly affect the results. The ballistic galvanometer should be so sensitive that D_t will be large, and consequently can be read with accuracy. This will probably mean that the instrument is used with the Aryton shunt adjusted for $m = 1$. When D_0 is observed, it will be necessary to shunt the galvanometer (the Ayrton shunt is used as it keeps the damping constant) in order to keep the deflection on the scale. In this case D_0 and D_t are the deflections corrected for the multiplying power of the shunts; that is, *both* are reduced to the value they would have if $m = 1$.

To apply this method to the determination of insulation resistance, the connections shown in Fig. 119 are required.

Care should be taken that the connections are such that the current necessary to charge the wiring does not pass through the galvanometer; the connection from K_1 to the core should be short. Air lines should be used when possible.

It is first necessary to determine the capacity of the cable C, for as a matter of convenience the cable is charged and allowed to leak through its own insulation resistance. The measurement is made by the direct deflection method; K_1 is placed in the mid-position; by throwing K_3 to the right the condenser C_1 is charged

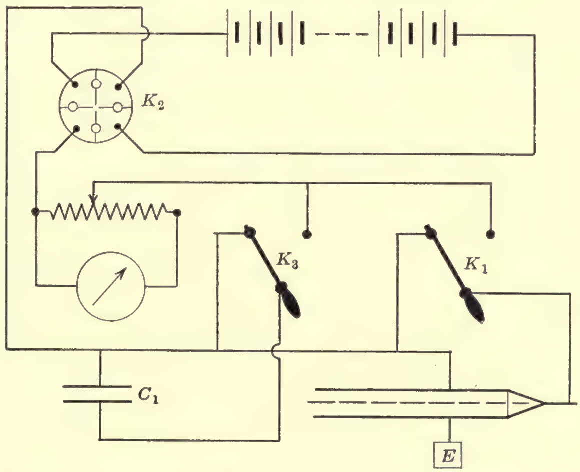


FIG. 119.—Connections for measuring insulation resistance by loss of charge method.

through the galvanometer, and the elongation noted. When the key is against the left-hand stop the condenser is discharged.

The key K_1 is now thrown to the position shown, thus making sure that the cable is discharged. The elongation, D_0, on charging the cable is now taken by throwing K_1 to the right; several observations should be made; between them the cable should be short-circuited long enough to ensure its thorough discharge. The elongation D_0 is used both in computing the capacity C and the resistance R. To determine D_t charge the cable, and at a noted minute and second, place K_1 in the mid-position, thus

insulating the cable and allowing it to discharge by leakage through its own insulation resistance. At a noted minute and second again charge the cable, noting the elongation D_t, which is due to the passage of the quantity Q_t through the galvanometer to replace that which has leaked out.

In thus applying the loss of charge method to cables, the assumption has been made that the current flow through the insulating covering obeys Ohm's law. On account of the non-fulfilment of the assumed conditions, the results are subject to errors; but in many cases of industrial testing the results attained by a definite method of procedure are sufficient.

It will be noticed that for a given value of $\frac{V_0}{V_t}$ the time of discharge is proportional to C. In general, the capacity of the cable is great enough to sufficiently prolong the discharge; however, when short lengths are tested it may be necessary to employ an auxiliary condenser. If this be done, a special determination of its effective resistance must be made. The condenser being in parallel with the cable, it is customary to compute the resistance of the latter from their combined resistances by the law of divided circuits. From what has been stated it will be seen that this procedure cannot be expected to give results of great accuracy.

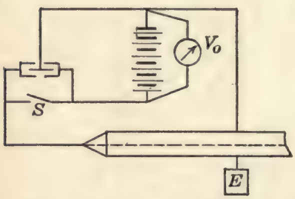


FIG. 120.—Connections for loss of charge method, using quadrant electrometer.

Loss of Charge Method, Using Quadrant Electrometer.—The fall of potential may also be obtained by the use of a quadrant electrometer, the needle of which is charged to a high and constant potential. The connections are shown in Fig. 120.

By closing the switch S the cable is charged and both sets of quadrants brought to the *same* potential. The electrometer will therefore remain undeflected. When S is opened, the right-hand quadrants are kept at a potential V_0 by the battery, while the potential of the left-hand quadrants gradually falls, as the cable discharges through its own resistance. A deflection appears which is sensibly proportional to v.

Evershed "Megger."—It is highly desirable to have some convenient portable instrument for measuring insulation resistance, which shall be rugged in construction, direct reading, capable of giving results with rapidity and so simple in manipulation that it may be employed by persons not accustomed to the use of delicate instruments. In addition, the device must be capable of furnishing a testing voltage so high that it will search out defects of high resistance.

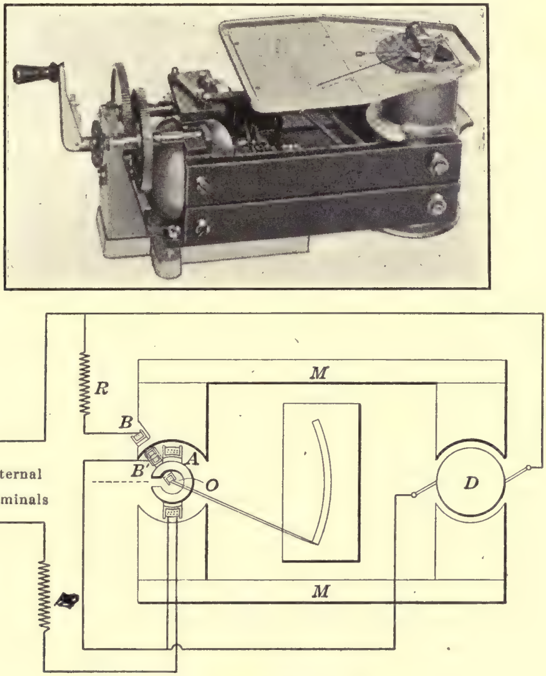


FIG. 121.—Evershed Megger.

The Evershed Megger, Fig. 121, was designed with these requirements in view.

Referring to the scheme of connections, M and M are permanent magnets which furnish the fields both for the instrument proper and for the small hand-driven magneto D, which gives the testing voltage. The movable element consists of the coils BB' and A which are rigidly connected; flexible leads are taken to the coils but there are no controlling springs.

The coils BB' are in series and together with a suitable resistance are connected across the terminals of the magneto D. The coil A is so connected that it is traversed by any current which flows through the specimen when the latter is joined between the external terminals. If the external circuit is open, the movable element, under the action of the current through R, will take up such a position that the plane of the coils BB' coincides with the dotted line. This position corresponds to an infinite external resistance and is marked infinity on the scale. If there be a current in the external circuit, due to the imperfect insulation resistance of the specimen, a current will flow through the coil A and in such a direction that it turns the movable system toward the right, carrying the coils BB' with it until the latter, which move in a non-uniform field, are in a field so strong that the turning moments due to A and BB' are balanced. The coil A may thus be considered to furnish the directive moment which acts on the system.

The indications are independent of the voltage of the magneto, for if that changes both the deflective and directive moments are altered in the same ratio.

In one design of the instrument, the magneto is driven through a clutch arrangement controlled by a centrifugal governor so that the voltage cannot rise above a definite maximum; then, when the crank is turned fast enough, a constant testing voltage is obtained. This is of importance when apparatus having a considerable electrostatic capacity is tested.

MEASUREMENT OF INSULATION RESISTANCES OF COMMERCIAL CIRCUITS WHEN POWER IS ON

It is sometimes necessary to measure the insulation resistance to "ground" (the water and gas pipes) of a distribution system, for instance, that of an office building, where the conditions are such that the power is on and the supply must not be interrupted. This case is illustrated by Fig. 122.

Voltmeter Method.—The resistances to ground will be represented by x_1 and x_2.

If the insulation resistances are not above 1 or 2 megohms, recourse may be had to the voltmeter method (page 208). Three voltage measurements suffice to determine both x_1 and x_2.

The supply voltage should be constant and the readings made as expeditiously as possible. First measure E, the supply voltage, then V_1 and V_2, the voltages to ground from leads 1 and 2. When the voltmeter is connected from lead 1 to ground, the voltage E sends a current, I_1, through the resistance x_2 plus the paralell resistance of x_1 and the voltmeter. Then if R_V is the voltmeter resistance,

$$I_1 = \frac{E}{x_2 + \dfrac{x_1 R_V}{x_1 + R_V}}.$$

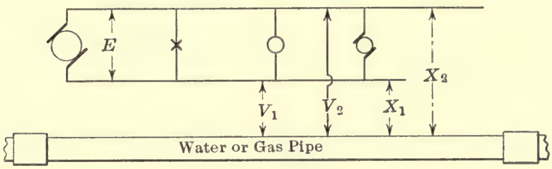


FIG. 122.

Similarly, when the voltmeter is between lead 2 and the ground,

$$I_2 = \frac{E}{x_1 + \dfrac{x_2 R_V}{x_2 + R_V}}$$

also

$$E - I_1 x_2 = V_1$$

$$E - I_2 x_1 = V_2.$$

Hence:–

$$x_1 = R_V \left(\frac{E - V_1 - V_2}{V_2}\right)$$

and

$$x_2 = R_V \left(\frac{E - V_1 - V_2}{V_1}\right).$$

As the voltages enter as a ratio, any galvanometer which gives a deflection proportional to the current through it may be used in place of the voltmeter, a suitable series resistance being employed. In this case, the scale readings may be used in place of the voltages. Inspection of the formulæ shows that the method is not applicable when one side of the circuit is grounded. The method may be used to measure the insulation

resistances of two insulators which are used in series; for instance, a trolley wire insulator and its accompanying strain insulator.

Northrup Method.—If the insulation resistances are too high for the voltmeter method, a galvanometer may be used according to a scheme due to Northrup.[11]

The connections are shown in Fig. 123.

A fairly high resistance, *ab*, which is provided with a sliding tap, *s*, is placed across the circuit. The galvanometer *G* is shunted, preferably by an Ayrton shunt as shown, so that its sensitivity may be varied to suit the conditions.

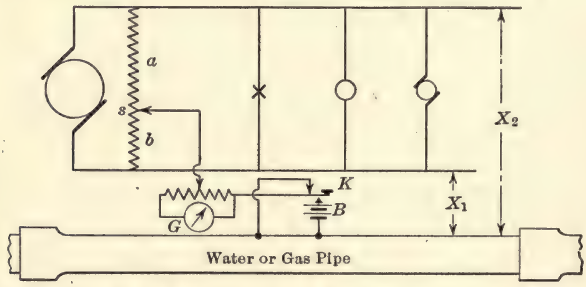


FIG. 123.—Northrup connections for measuring resistance to ground when power is on.

When the key *K* is against the back stop, connection is established between the ground and *s*. By moving *s*, a position may be found where the galvanometer will stand at zero, the potential of *s* being that of the ground. It will be seen that this is a Wheatstone bridge arrangement. Consequently

$$\frac{X_1}{X_2} = \frac{b}{a}$$

and

$$\frac{X_1}{X_1 + X_2} = \frac{b}{a + b},$$

$$\frac{X_1 + X_2}{X_2} = \frac{a + b}{a}.$$

If the key be depressed, any deflection will be due to the current from the battery *B* which flows to *s* and there divides, part going

through the resistances $a + x_2$ and part through $b + x_1$, and back to the battery via the ground connection.

The total resistance encountered will be

$$R = R_G + \frac{1}{\dfrac{1}{a + x_2} + \dfrac{1}{b + x_1}}.$$

R_G, a and b are negligible with respect to x_1 and x_2, so

$$R = \frac{x_1 x_2}{x_1 + x_2} \text{ very approximately.}$$

$$\therefore x_1 = R\left(\frac{a + b}{a}\right)$$

$$x_2 = R\left(\frac{a + b}{b}\right).$$

If the voltage of the battery B is E, then

$$I_G = \frac{E}{R} = KD$$

where K is the current necessary for unit deflection of the galvanometer and D is the deflection.

Then

$$R = \frac{E}{KD}$$

$$\therefore x_1 = \frac{E}{KD}\left(\frac{a + b}{a}\right)$$

$$x_2 = \frac{E}{KD}\left(\frac{a + b}{b}\right).$$

Changes in circuit conditions, such as throwing on or off appliances not perfectly insulated, will change x_1 and x_2 and shift the neutral point. This may introduce difficulties in the execution of the test.

Other Methods of Measuring Resistance to Ground.—The current which flows to ground from one of the line wires may be measured by the method given on page 94 for measuring the current in a permanently closed circuit. The connections are shown in Fig. 124. By varying r_2 the P.D. between

the points a and b may be made zero; when this adjustment is complete the deflection of the galvanometer G is zero and the current which then flows through x_2 is given by the galvanometer G_2. It follows that

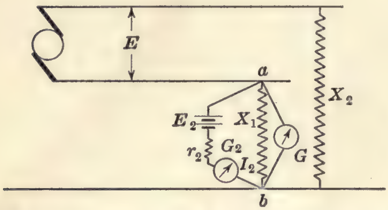


FIG. 124.—Connections for measuring resistance to ground.

$$x_2 = \frac{E}{I_2} = \frac{Er_2}{E_2}.$$

Mance's method, originally designed for measuring the internal resistance of batteries, may be so modified that it can also be used for measuring insulation resistances while under the working voltage.

THE TEMPERATURE COEFFICIENT OF ELECTRICAL RESISTANCE

The effect of change of temperature on the electrical resistance of various materials is shown in Fig. 125. It is seen that, with few exceptions, the resistance increases with increase of temperature. For pure metals an approximate figure is 0.4 per cent. per degree C. It may be noted that metals which, at certain temperatures, undergo changes of structure, for instance, iron at about 780°C., show alterations of curvature in the temperature-resistance curve at those points.

From this it is obvious that if a statement of an electrical resistance is to possess definiteness, the temperature at which the measurement was made must be given. Also, when a resistance is measured, it is necessary to know the temperature and temperature coefficient of the coils with which it is compared, as well as their value at some particular temperature, in order that their true resistance at the time of use may be known.

Experiments show that, in general, when the temperature is altered, the change of the electrical resistance of a conductor to which terminals are rigidly attached, and which therefore possesses a constant mass, is not quite linear; that is, the plot connecting resistance and temperature is not exactly a straight line. For the ordinary ranges of atmospheric temperatures the curvature is very slight, but may be considerable if the temperature be carried to high values.

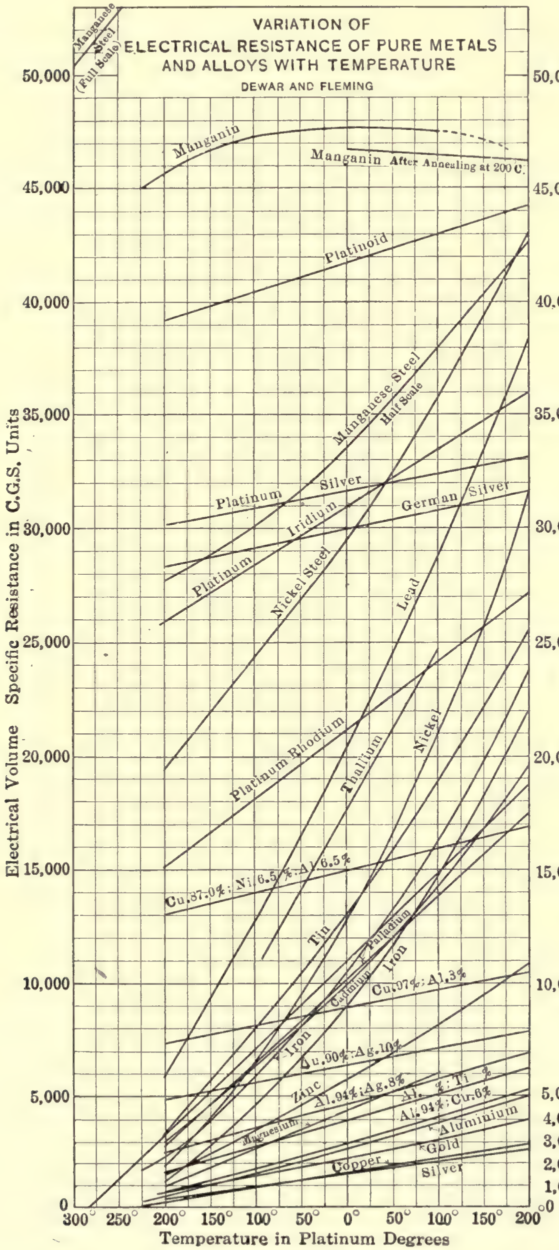


FIG. 125.—Showing effect of temperature on electrical resistance.

In this and similar cases, an empirical relation of the form here given is frequently employed to represent the results of physical measurements:

$$R_t = R_0(1 + at + bt^2 + ct^3 + \ . \ . \ . \ ). \qquad (21)$$

R_t is the resistance at $t°$, and R_0 that at 0°. The departure from a straight line is determined by the constants b and c. For the special case of copper, the variation of which is linear, between

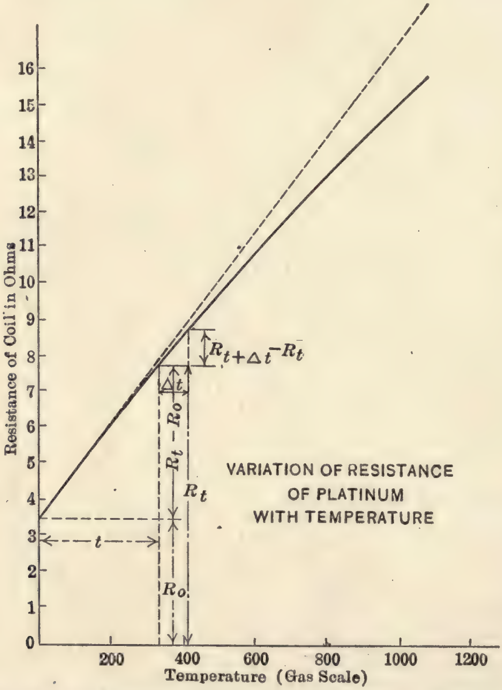


FIG. 126.—Temperature-resistance curve of platinum.

10° and 100°C., the constants b and c are zero. The constants a, b, c are best determined by applying the method of least squares to a series of measurements of the resistance made at different temperatures.

The temperature-resistance curve for a coil of very pure platinum is shown in Fig. 126; it is represented by the following equation.

$$R_t = R_0(1 + 0.00392t - 0.000000588t^2);$$

therefore

$$a = +\ 0.00392$$
$$b = -\ 0.000000588.$$

On account of its use in resistance thermometers, the variation of the resistance of platinum with temperature is important.

Mean Temperature Coefficient.—Denoting particular temperatures by subscripts, it is usual to write formula 21 thus

$$R_{t_1} = R_0(1 + \beta_{t_1} t_1) \tag{21a}$$

where

$$\beta_{t_1} = a + bt_1 + ct_1^2.$$

It will be seen that β_{t_1} is the mean, or average fractional rate of increase of resistance per degree between 0° and t_1, referred to the resistance at 0°, for

$$\beta_{t_1} = \frac{R_{t_1} - R_0}{R_0 t_1}.$$

β_{t_1} is called the *mean temperature coefficient of resistance increase.*

To compute the value of a resistance at any temperature, $t°$, from that at some given temperature, $t°_1$,

$$R_{t_1} = R_0(1 + \beta_{t_1} t_1);$$
$$R_t = R_0(1 + \beta_t t);$$

therefore

$$R_t = R_{t_1} \frac{1 + \beta_t t}{1 + \beta_{t_1} t_1} \tag{22}$$

Temperature Coefficient of Resistance.—As in general the graph connecting R_t and t is curved, the true rate of increase of resistance will have a particular value at each temperature, consequently a very small temperature interval must be used in computing it.

The *temperature coefficient of resistance increase* at any temperature is the fractional rate of increase of resistance for a very small temperature increment referred to the resistance at that temperature. It will be denoted by α; then

$$\alpha_{t_1} = \frac{1}{R_{t_1}} \left(\frac{dR_t}{dt}\right)_{t_1} = \frac{a + 2bt_1 + 3ct_1^2 + \ . \ . \ .}{1 + at_1 + bt_1^2 + ct_1^3 + \ . \ . \ .} \tag{23}$$

$$\alpha_{t_1} = \frac{R_{t_1 + \Delta t} - R_{t_1}}{R_{t_1} \Delta t} \text{ approximately.}$$

For a small finite temperature interval, $\Delta t = t - t_1$,

$$R_t = R_{t_1}(1 + \alpha_{t_1}[t - t_1]). \tag{24}$$

On account of the approximations involved, equation (24) applies in general only to short ranges of temperature.

Strictly speaking, in order to compute α or β one must know the values of the constants a, b, and c for the particular sample of material under discussion; such data are rarely at hand.

Special Case: Temperature Correction for Copper.—Careful experiments made at the Bureau of Standards[12] upon commercial copper of high conductivity, such as is used for electrical purposes (varying from 96 per cent. to 101 per cent. conductivity), show that between 10° and 100°C. the *variation of resistance with temperature is linear*, and also that the temperature coefficient at 20° is directly proportional to the per cent. conductivity.

That is,

$$\alpha_{20} = \frac{R_t - R_{20}}{R_{20}[t - 20]} = 0.00393 \times (\text{per cent. conductivity}) \tag{25}$$

In this formula the per cent. conductivity is expressed decimally.

The *temperature coefficient of resistance* at any other temperature may be calculated from that at 20° as follows, the variation of resistance being linear:

$$\alpha_t = \frac{1}{R_t}\left(\frac{dR_t}{dt}\right)_t = \frac{a}{1 + at} = \frac{1}{\frac{1}{a} + t};$$

similarly

$$\alpha_{t_1} = \frac{1}{\frac{1}{a} + t_1}$$

and

$$\frac{1}{a} = \frac{1}{\alpha_{t_1}} - t_1;$$

$$\therefore \alpha_t = \frac{1}{\frac{1}{\alpha_{t_1}} - t_1 + t}.$$

Let the original temperature of reference, t_1, be taken as 20°; then

$$\alpha_t = \frac{1}{\frac{1}{0.00393 \times (\text{per cent. cond.})} + [t - 20]} \tag{26}$$

As the resistance variation of copper is linear, measurements to determine α_{t_1} may be made at any two temperatures which may both differ from t_1, the temperature of reference; for let the measurements be made at temperatures t_2 and t_3, then

$$R_{t_1} = R_{t_3} - \frac{(R_{t_3} - R_{t_2})}{t_3 - t_2}[t_3 - t_1]$$

and

$$\alpha_{t_1} = \frac{R_{t_3} - R_{t_2}}{R_{t_1}[t_3 - t_2]} = \frac{R_{t_3} - R_{t_2}}{R_{t_2}[t_3 - t_1] - R_{t_3}[t_2 - t_1]}.$$

By use of (25) and (26) the following table has been calculated. The standard 100 per cent. conductivity copper is taken as having a resistivity of 0.15328 ohm (meter, gram) at 20°C. (see page 230).

Table.—Temperature Coefficients of Standard Annealed Copper at Various Temperatures of Reference

Ohms (meter gm.) at 20° C.	Per cent. conductivity	α_0	α_{15}	α_{20}	α_{25}	α_{30}	α_{50}	Inferred absolute zero. T.
0.16134	95.0	0.00403	0.00380	0.00373	0.00367	0.00360	0.00336	−247.8
0.15966	96.0	0.00408	0.00385	0.00377	0.00370	0.00364	0.00339	−245.1
0.15802	97.0	0.00413	0.00389	0.00381	0.00374	0.00367	0.00342	−242.3
0.15753	97.3	0.00414	0.00390	0.00382	0.00375	0.00368	0.00343	−241.5
0.15640	98.0	0.00417	0.00393	0.00385	0.00378	0.00371	0.00345	−239.6
0.15482	99.0	0.00422	0.00397	0.00389	0.00382	0.00374	0.00348	−237.0
0.15328	100.0	0.00427	0.00401	**0.00393**	0.00385	0.00378	0.00352	−234.5
0.15176	101.0	0.00431	0.00405	0.00397	0.00389	0.00382	0.00355	−231.9

Experiments have shown that distortions of the wire, such as occur in winding and ordinary handling, do not alter the temperature coefficient.

For windings in general, *annealed* copper, conductivity 100 per cent., may be assumed. Hard-drawn copper may be assumed to have a conductivity of 97.3 per cent. These values are approximations to be used only when data concerning the copper in question cannot be obtained.

Measurement of Rise of Temperature.—In the testing of electrical machinery it is customary to find the average rise of temperature of copper windings by measurements of their

resistance. This is facilitated by the linear relation between temperature and resistance; for, *assume* that this relation holds for all temperature intervals, and prolong the line connecting temperature and resistance downward. It will cut the axis of temperatures to the left of the origin at a temperature $-T°$. Now let R_{t_1} and R_{t_2} be the resistances of the windings at temperatures t_1 and t_2, then

$$\frac{R_{t_2} - R_{t_1}}{t_2 - t_1} = \frac{R_{t_1}}{T + t_1}$$

or

$$t_2 - t_1 = \frac{(R_{t_2} - R_{t_1})}{R_{t_1}}[T + t_1]. \qquad (27)$$

Again from the above,

$$\frac{R_{t_2}}{R_{t_1}} = 1 + \frac{t_2 - t_1}{T + t_1}.$$

The quantity T is usually called the "inferred absolute zero." It may be calculated from the data given in the table. For example,

$$R_t = R_{20}(1 + \alpha_{20}[t - 20])$$

if the resistance becomes zero and 100 per cent. conductivity be taken.

$$t - 20 = -\frac{1}{\alpha_{20}} = -\frac{1}{0.00393} = -254.5;$$

$$\therefore T = -234.5.$$

These values are entered in the last column of the table on page 223. When used in the above formulæ the minus sign is omitted.

The Resistance Pyrometer.[13]—Following a suggestion made in 1871 by W. Siemens, the variation of the electrical resistance of platinum with temperature is utilized in pyrometry. The first experiments in this direction were not successful, and in 1874 the British Associaton report on the instruments submitted was not favorable, it being found that after exposure to a high temperature the platinum coils did not return to their original resistances and that the changes were progressive. In 1886 Callendar proved that these changes were due to the absorption by the platinum of silica from the coil support and of

furnace gases which at high temperatures penetrated the iron tubes then employed to protect the resistance coils. In view of these facts, the coils are now wound on very light frames of mica, which touch each turn of the wire at only four points; the areas of the surfaces of contact are thus reduced to a minimum. The coils are now protected by porcelain tubes glazed externally.

In order to make the relation of resistance to temperature comparatively simple and one which may be determined by measurements at three known temperatures, pure platinum must be employed.

If the resistance of the coil be measured at 0° and at 100°C., the average change per degree will be

$$\frac{R_{100} - R_0}{100}.$$

and if R be the resistance at some other temperature,

$$100\,\frac{R - R_0}{R_{100} - R_0}$$

will be the corresponding temperature, on the assumption that the change of resistance is linear. A temperature so defined is called the "platinum temperature," so

$$t_p = 100\,\frac{R - R_0}{R_{100} - R_0}.$$

Callendar showed that for high temperature measurements it was not correct to assume a linear variation of the resistance, but that a correction must be applied in order to give the proper temperature on the scale of the gas thermometer. He found that if t be the temperature on that scale, the difference of t and t_p could be expressed by the following empirical relation:

$$t - t_p = \delta\left\{\left(\frac{t}{100}\right)^2 - \left(\frac{t}{100}\right)\right\},$$

where if *pure* platinum be used δ is a constant, its value being about 1.505. This form of expression holds with great exactness for pure platinum, but is not general; for instance, it fails in the case of palladium. For impure platinum δ must be expressed thus: $\delta = a + bt$.

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To determine δ, pure platinum being used, the resistance must be measured at three known temperatures, which for high temperature measurements are usually taken as 0°, 100°, and 444°.70, the latter being the boiling point of sulphur under carefully specified conditions, this having been determined with great care by many experimenters. With the δ correction applied, the platinum thermometer reproduces temperatures on the gas (nitrogen) thermometer scale to within the limits of accuracy of the latter between −80° and +1,100°C.

The above empirical formula does not admit of extrapolation downward below −100°, so for measurements of extremely low temperatures the formula must either be modified or the coil

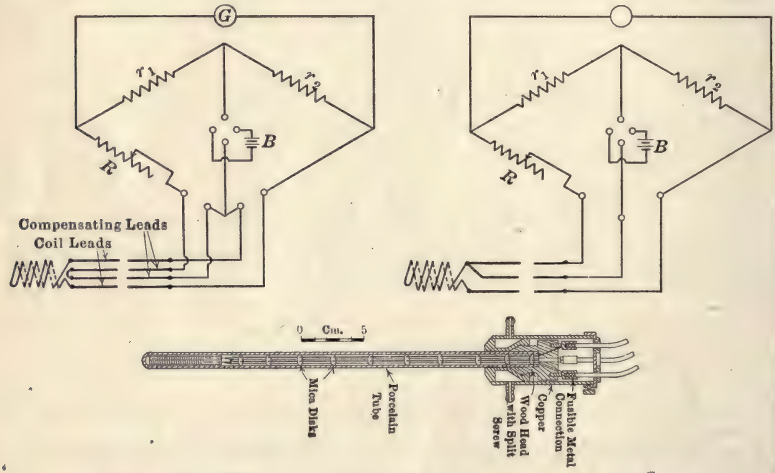

FIG. 127.—Bridge connections for resistance pyrometers.

resistance taken at such points as the temperature of melting ice, solid CO_2, and the boiling point of oxygen, in order that the extrapolation may not be so excessive. The platinum thermometer, when used for precise work at high temperatures, must be frequently calibrated; the purer the platinum, the less the likelihood of a change in its constants. Also the changes in the coils are minimized if the wire be large and be supported free from strains. Annealing at a temperature above that at which the instrument is to be used contributes to constancy.

The coil and the bridge connections for measuring the resistance are shown in Fig. 127.

To compensate for the changes in the resistance of the coil leads, due to temperature, compensating leads as nearly like the coil leads as possible and in a similar environment are inserted in the adjustable bridge arm, or three leads are used as shown at the right hand of Fig. 127.

RESISTIVITY, CONDUCTIVITY

It is well known that the electrical resistances of conductors having the same dimensions, but made of different materials, will differ. Each metal has its characteristic resistance, which, when determined for some unit specimen of the material and at a stated temperature, gives the resistivity or specific resistance. Various methods of expressing this property are used; the three most commonly employed are shown below.

1. The resistivity is frequently expressed as the resistance, in ohms, or microhms, of a wire 1 cm. long and having a cross-section of 1 sq. cm. This has commonly been called the centimeter cube resistivity. This name frequently gives rise to false notions as to the relations of the quantities involved. This quantity is better designated as the ohm (cm.) or microhm (cm.) resistivity, which terms are descriptive of the units involved. If it be represented by S_A, then a wire L cm. long and A sq. cm. in section will have a resistance

$$R = \frac{S_A L}{A}. \tag{28}$$

2. Another unit in common use is the resistance in ohms of a wire 1 ft. long and 1 mil or 0.001 in. in diameter. This is commonly called the foot-mil resistivity. It should, however, be designated as the ohms (mil, foot) resistivity. If it be represented by S_D, then for circular wires, if the length L be expressed in feet and the diameter D in mils,

$$R = \frac{S_D L}{D^2}. \tag{29}$$

3. The third method of expressing resistivity is in terms of the resistance in ohms of a wire of uniform cross-section, 1 m. long and 1 gm. in mass. This has commonly been called the meter-gram resistivity. The designation ohm (meter, gram) resistivity

should, however, be used. If this quantity be represented by S_M, then when the length L is expressed in meters and the mass M in grams,

$$R = \frac{S_M L^2}{M}. \tag{30}$$

S_A and S_D are frequently referred to as the "length-section" or "volume" resistivities, and S_M as the "length-mass resistivity" or the "mass resistivity."

The relation of S_A to S_M involves the density, d; for, expressing S_M in ohms,

$$S_M = \frac{RM}{L^2_{\text{meters}}} = \frac{10{,}000 RAL_{\text{cm.}}d}{L^2_{\text{cm.}}} = 10{,}000 S_A d.$$

A careful study by the Bureau of Standards[12] of all available data gives for the density of commercial copper having a conductivity of over 94 per cent., 8.89 at 20°C. The limits of variation, neglecting extreme values, were found to be 8.87 and 8.91. The density of annealed and hard-drawn copper is sensibly the same.

To find S_A, S_D, or S_M, the resistance of the sample, R, must be determined at some standard temperature, and the necessary mechanical measurements made in the units above stated. To determine with accuracy the section of a wire of supposed uniform cross-section from measurements of its diameter is a very difficult matter and laborious of execution, for a great many observations must be taken by means of the micrometer caliper, and even then the result is likely to be seriously in error, since the square of the diameter enters the formula. An error of 1 per cent. in the diameter means 2 per cent. in the area. For this reason diameter measurements should be used only when the wire is large. In case the cross-section of the sample is uniform but irregular in outline, such measurements are of course impossible. If the wire be small, its average area or average diameter is best determined by weighing a known length of it in air and then in water, thus finding the mass of the displaced water and consequently the volume of the wire. This, however, requires that all the precautions taken in an exact specific gravity determination be observed.

The determination of S_M, or the length-mass resistivity, presents

the least experimental difficulty, as the mechanical measurements are simply the determination of a length and a weight, both of which can be very accurately made; consequently the ohm (meter, gram) resistivity is very commonly employed, and is recommended in preference to the "length-section" or "volume" resistivities denoted above by S_A and S_D.

The standard temperature at which results are expressed should not differ greatly from the ordinary average atmospheric temperature; therefore 20°C. is to be taken.

The electrical qualities of copper are frequently stated in terms of the conductivity, which is the reciprocal of the resistivity, and in business transactions guarantees are given as to per cent conductivity. In order that such guarantees may possess definiteness, some standard must be adopted. Obviously, the conductivity of pure copper would be the most proper standard, but this is unknown. So for the greatest convenience some reasonable figure must be agreed upon. What this figure may be is not of consequence, but that it should be universally recognized is of the greatest moment.

Many different standard values of the resistivity of annealed copper have been in use and sanctioned by various electrotechnical societies. Generally these values have been based on the work of Matthiessen on supposedly pure copper (1862), but the results on *annealed* copper involve an assumption as to the ratio of the resistivity of the hard-drawn to the annealed wire. There are also uncertainties as to the temperature coefficients, the many digits usually given in the constants being without significance; consequently the values derived by various persons from Matthiessen's work do not agree, and the so-called Matthiessen standard has had no universal significance.

To obviate this difficulty the Bureau of Standards, at the request of the American Institute of Electrical Engineers, has investigated the subject, and as the result of measurements upon 89 samples of commercial copper, procured from 14 different refiners, an average result of 0.15292 ohm (meter, gram) at 20°C. was obtained. This value is seen to be in close agreement with the figure 0.15302 ohm (meter, gram) at 20°, which had previously been adopted by the Bureau. This latter figure was suggested for international adoption, but the German

engineers had already in use a standard of conductivity, 58 $\frac{1}{\text{ohm}}$ (meter, mm.²) at 20°, which is slightly different from the above. The corresponding figure for the resistivity was the one finally recommended for adoption in America and Germany, with a likelihood of its immediate adoption in other countries. It is:

International Annealed Copper Standard

Mass resistivity.....	0.15328 ohm (meter, gram) at 20°C.
	875.20 ohms (mil, pound) at 20°C.
Volume resistivity...	1.7241 microhm (cm.) at 20°C.
	0.017241 ohm (meter, mm.²) at 20°C.
	0.67879 michrom (inch) at 20°C.
	10.371 ohms (mil, foot) at 20°C.
Density (grams per cubic cm.).........	8.89 at 20°C.

Resistivity-temperature Constant.—The changes in the resistivity of copper due to alterations of temperature are complicated by the expansion of the material. This effect is very small and is readily allowed for.

Assuming that the resistance is measured between terminals rigidly attached to the specimen, in general for the ohm (meter, gram) resistivity,

$$S_M = \frac{MR}{L^2}.$$

Introducing the temperatures and denoting the coefficient of linear expansion by γ,

$$[S_M]_t = \frac{MR_{20}(1 + a_{20}[t - 20])}{L^2_{20}(1 + \gamma[t - 20])^2}.$$

For copper, γ is a very small quantity, 0.000017, and so

$$[S_M]_t = [S_M]_{20}\left\{1 + (a_{20} - 2\gamma)[t - 20]\right\} \text{ approximately.}$$

For standard copper, 100 per cent. conductivity,

$$\begin{aligned}[S_M]_t &= 0.15328\left\{1 + (0.00393 - 0.000034)[t - 20]\right\};\\ &= 0.15328 + 0.000597[t - 20].\end{aligned}$$

The change per degree in the resistivity is seen to be 0.000597 *ohm;* this figure is independent of the temperature of reference, and in consequence of (25) applies to coppers of all conductivities. It is called the "resistivity-temperature constant."

If the effects of expansion had been neglected, the result would have been 0.000602.

Using the volume resistivity, in general,

$$S_A = \frac{RA}{L}.$$

At $t°$,

$$[S_A]_t = \frac{R_{20}A_{20}(1 + a_{20}[t - 20])(1 + 2\gamma[t - 20])}{L_{20}(1 + \gamma[t - 20])};$$

$$= [S_A]_{20}\left\{1 + (a_{20} + \gamma)[t - 20]\right\} \text{ approximately.}$$

Using microhms,

$$[S_A]_t = 1.7241\left\{1 + (0.00393 + 0.000017)[t - 20]\right\};$$
$$= 1.7241 + 0.00681[t - 20].$$

In this case the "resistivity-temperature constant" is 0.00681. Again, using the ohm (mil, foot) resistivity,

$$[S_D]_t = 10.371 + 0.0409[t - 20].$$

Here the "resistivity-temperature constant" is 0.0409.

Relation Between Resistivity and the Temperature Coefficient of Resistance.—As shown above, the change in the ohm (meter, gram) resistivity per degree C. is 0.000597; consequently, the temperature coefficient of the ohm (meter, gram) resistivity $= \dfrac{0.000597}{[S_M]_{t_1}}$.

The resistance of a wire at $t°$, if t_1 is the temperature of reference, is given by

$$R_t = \frac{[S_M]_{t_1}L^2_{t_1}}{M}\left\{1 + \frac{0.000597}{[S_M]_{t_1}}(t - t_1)\right\}\left\{1 + 2\gamma(t - t_1)\right\}$$

$$= R_{t_1}\left\{1 + \left(\frac{0.000597}{[S_M]_{t_1}} + 0.000034\right)(t - t_1)\right\} \text{ approx.}$$

For the copper met with in practice this is approximately

$$R_t = R_{t_1}\left\{1 + \frac{0.000602}{[S_M]_{t_1}}(t - t_1)\right\},$$

$\therefore$ the temperature coefficient of resistance at the reference temperature $t°_1$ is $\dfrac{0.000602}{[S_M]_{t_1}}$.

Similarly for S_A,

$$a_{t_1} = \frac{0.00678}{[S_A]_{t_1}}.$$

For S_D it is

$$a_{t_1} = \frac{0.0407}{[S_D]_{t_1}}.$$

Per Cent. Conductivity.—The per cent. conductivity is obtained by dividing the resistivity of the annealed copper standard at 20° by that of the sample at 20°.

It is to be noticed that on account of the relation of the temperature coefficient to the conductivity, the per cent. conductivity of a sample when referred to the standard copper will depend somewhat on the temperature at which the conductivity is computed. For instance, if copper of resistivity 0.15328 ohm (meter, gram) at 20° be taken as a standard, the resistivity at 0° will be $0.15328 - 0.000597 \times 20 = 0.14134$; a copper which has 95 per cent. conductivity at 20° will have a resistivity of 0.16134, and at zero a resistivity of $0.16134 - 0.000597 \times 20 = 0.14940$. Therefore the per cent. conductivity at 0° is

$$100\frac{0.14134}{0.14940} = 94.6 \text{ per cent.}$$

To avoid possible confusion, per cent. conductivities are to be computed at 20°C.

Resistivity of Aluminum.—The Aluminum Co. of America furnishes the following data concerning the resistivity of their commercial product of hard-drawn aluminum.

Mass resistivity.....	0.0764	ohm (meter, gram) at 20°C.
	436.0	ohm (mils, pound) at 20°C.
Volume resistivity...	2.828	microhm (centimeter) at 20°C.
	1.113	microhm (inch.) at 20°C.

Density, grams per cubic cm. 2.70.
Mass per cent. conductivity 200.7%.
Volume per cent. conductivity 61.0%.

Conductivity Bridges.—In wire works and in the testing laboratories of large consumers of wire, it is necessary to have special apparatus for the determination of conductivity, the requirements being:

1. Convenience of manipulation; allowing speed to be attained.
2. No calculation required; that is, the instrument must be direct reading in terms of the accepted standard material.
3. Freedom from all temperature corrections.
4. Accuracy to $\frac{1}{10}$ or $\frac{1}{20}$ per cent.

One form of such an apparatus is the Hoopes conductivity bridge. This device is an adaptation of the Kelvin double bridge, the scheme being as follows:

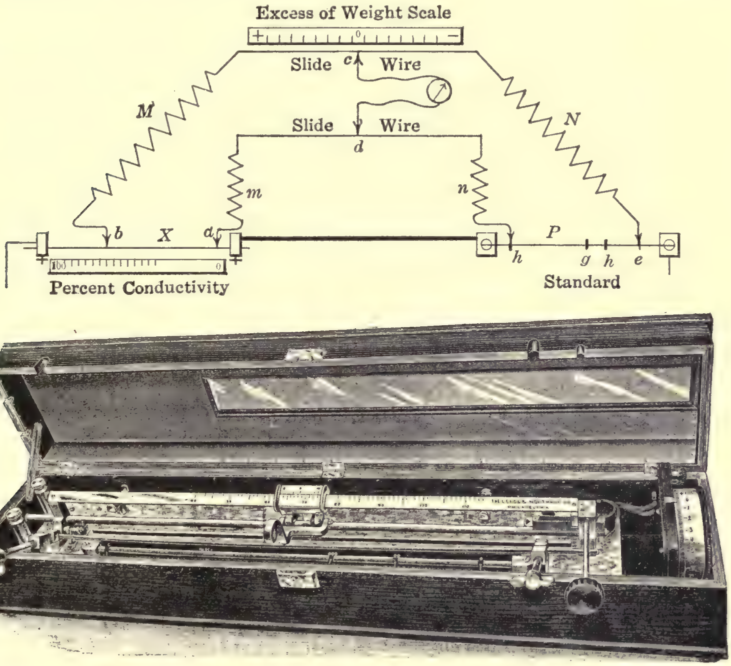

FIG. 128.—Hoopes conductivity bridge.

Hoopes Conductivity Bridge.—The standard P and the unknown X are of the same metal; consequently *if care be taken that they are at the same temperature*, all corrections due to temperature are avoided. The arms M, N, m, n are in the same case and made of material of low temperature coefficient, so that their relative values will not change. The sliders c, d, are rigidly connected so that when they are moved the ratio $\frac{M}{N}$ is altered while the relation $\frac{M}{N} = \frac{m}{n}$ is maintained.

The sample shown at X is placed alongside a scale divided into 100 equal parts; the graduations therefore represent percentages of the total length of the scale.

Consider X to be of uniform cross-section and 100 per cent. conductivity, that a and b are set at 0 and 100, respectively, and that the resistance of P equals that of X; for balance c and d must be set so that $M = N$. Now suppose that the sample at X is changed for one of the same diameter, but of 50 per cent. conductivity; the length required to balance P, contacts c and d remaining fixed, will be only 50 per cent. as great as in the first case, and b must be moved along the percentage scale to the 50 per cent. mark.

However, the samples X vary in diameter, while the resistance of P is fixed; consequently the ratio $\frac{M}{N}$ must be variable, so that it may be made to correct for the cross-section of the sample. To obtain the relative diameters of wires it is more accurate and convenient to weigh samples of the same length than to caliper them; accordingly, all samples are cut to a length of 38 in. in a special machine and weighed. The contact c is then set at the graduation corresponding to the excess or defect in the weight, referred to a sample of correct size, thus making $\frac{M}{N}P$ equal to the resistance of a sample of 100 per cent. conductivity—length 0–100 on the percentage scale—and of the same diameter as X. The bridge is then balanced by moving b, and the conductivity is read from the scale.

Several standards are provided; they are removable, and by the use of the taps e, f, g, each has a range of three consecutive numbers on the B. & S. gage. For rapid work the stock of samples must be kept at the temperature of the testing apparatus.

The coarse adjustment of the slider b is effected by the handle projecting toward the front of the bridge; the final balancing is made by turning the milled head at the front.

The percentage scale is seen at the back of the instrument, the excess of weight scale at the right hand.

The instrument is covered by a metal-lined wooden case, which serves to keep the temperature constant; the reading is made

through a glazed window in the top. Provision is made in the case for storing the samples to be tested.

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CHAPTER V

THE MEASUREMENT OF POTENTIAL DIFFERENCE AND ELECTROMOTIVE FORCE

The most obvious method of determining the potential difference between two points in a circuit is to connect them through a suitable galvanometer which is in series with a high resistance. The potential difference, P.D., is the product of the galvanometer current and the total resistance of the galvanometer circuit. The galvanometer and the resistance may be combined in a

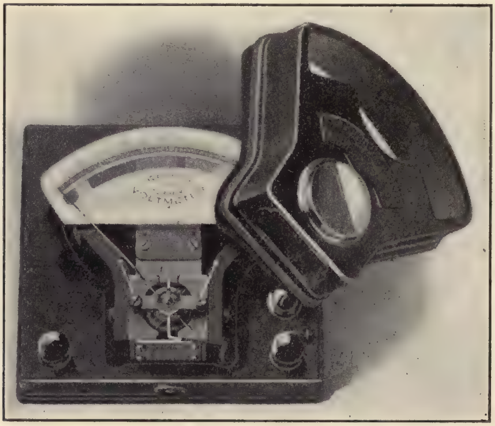


Fig. 129.—Weston moving-coil voltmeter for direct-current circuit.

single instrument and such "potential galvanometers" were in common use before the introduction of voltmeters or instruments where the products are taken once for all and marked on the scales. The various types of instruments which have been described as ammeters may be used as voltmeters, provided the total resistance of the instrument be made sufficiently high by the use of proper windings and series resistances. There are differences of detail; for example, the resistance of the controlling

springs need not be kept as low as in ammeters. For direct-current work, the moving-coil type of instrument has become the standard; for alternating currents, the electrodynamometer type is usual, though there are some soft iron and induction instruments.

In direct-current voltmeters the resistance is about 100 ohms per volt of full scale reading; the resistance of alternating-current voltmeters of the electrodynamometer type is much lower, about 20 ohms per volt. It is customary to make the final adjustment of the instruments by altering the series resistance, but it is more convenient, when multipliers are to be used, to have a definite resistance per volt and to effect the adjustment in some other manner.

In direct-current portable instruments having ranges up to about 750 volts, it is usual to place the series resistance within the base. Self-contained alternating-current portable voltmeters having a range of 300 volts may be obtained.

When using a voltmeter, it should be kept in mind that it only shows the P.D. between its own terminals and that this is not necessarily the same as the P.D. which previously existed between the points on the circuit to which the terminals are applied. For example, suppose there is a large resistance, 32,000 ohms, across which the drop is 200 volts and that it is desired to measure the P.D. between one terminal and a tap at the middle of the resistance. Obviously, the P.D. in question is 100 volts; however, if a voltmeter of 16,000 ohms resistance is applied between one terminal and the tap, it will read 66.6 volts. The application of the voltmeter has changed the quantity which it is desired to measure by 33 per cent. The disturbance of the circuit conditions diminishes as the resistance of the voltmeter is increased and would be *nil* with an instrument which operated on open circuit, that is, an electrostatic voltmeter. In engineering work this difficulty is not often met but one should not lose sight of the possibility.

Effect of Temperature.—It is evident that a high resistance, which must not be subject to changes due to the heating action of the current or to alterations of room temperature, is an essential part of any electromagnetic voltmeter. As the instrumental errors should be practically independent of tem-

perature, the major portion of the resistance must be of a material having a negligible temperature coefficient. In addition, the effect of temperature on the controlling springs, and in direct-current instruments the effect on the magnets, must be small. The springs grow weaker by about 0.04 per cent. per degree C. as the temperature is raised. Usually the magnets grow weaker with an elevation of temperature, an average value for the temperature coefficient being -0.025 per cent. per degree C. The net effect of temperature on a 150-volt direct-current portable voltmeter of good design is about $+ 0.012$ per cent. per degree.

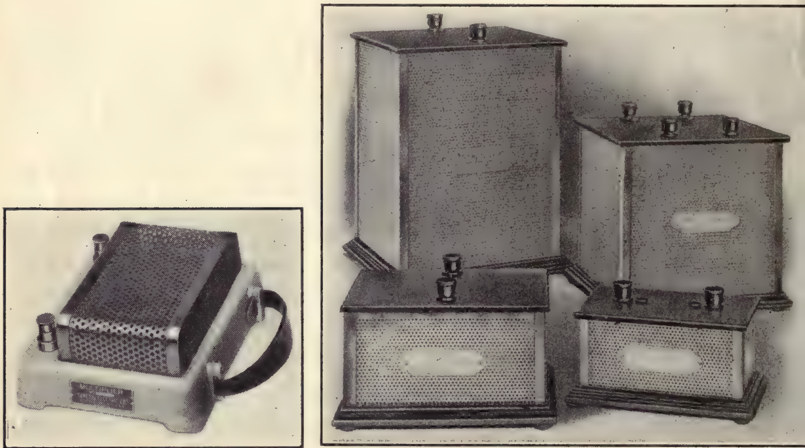


FIG. 130.—Multipliers for extending the range of voltmeters and wattmeters.

The effect of temperature variation becomes *more important when low-range instruments are used* for in them the movable coil, which is always wound with copper wire, forms a relatively larger proportion of the total resistance.

Multipliers.—Frequently it is necessary to measure potential differences higher than those for which the voltmeter was originally intended. In this case a properly constructed resistance is joined in series with the instrument so that the voltage necessary to force a given current through the voltmeter circuit

is increased; then if R_M and R_V be the resistances of the multiplier and of the voltmeter respectively,

$$\text{P.D.} = \text{reading times } \left(\frac{R_V + R_M}{R_V}\right).$$

When the range is very greatly extended, to several thousand volts, the multiplier is subdivided and mounted in a number of boxes, thus reducing the voltage drop between neighboring wires and rendering it easier to insulate them. Also, capacity effects which might be serious in alternating-current work are much reduced, for high-range multipliers may be subject to errors due to distributed capacity and capacity to ground.

The condensation of moisture on the resistors of very high-range multipliers, when they are used in air, frequently gives rise to burnouts. These may be obviated by immersing the multiplier in transformer oil. Multipliers often contain soft rubber insulation, this must be removed before the immersion.

Electrodynamometer Voltmeters.—These instruments are primarily designed for the measurement of alternating potential differences. The indicating portion is a comparatively delicate electrodynamometer with a pivoted movable coil.

The current through an alternating current voltmeter is given by

$$I = \frac{V}{\sqrt{R^2 + \omega^2 L^2}}$$

where R and L are respectively the total resistance and inductance of the voltmeter and ω is $2\pi f$. At the ordinary commercial frequencies, the indications must be practically independent of the frequency but as L can never be zero, the resistance must be made so high that the reactance can be neglected in comparison with it. The instrument may then be used for both direct and alternating potential differences. This is a convenience, for it may then be calibrated with direct currents, using reversals.

As the electrodynamometer is a comparatively insensitive instrument, considerable current is required and it is not possible, while retaining the characteristics of portability and solidity of construction necessary in order that the instrument may have a long life and maintain its accuracy under the trying conditions of everyday work, to give this form of voltmeter as high a re-

sistance as is common in direct-current instruments. This is a disadvantage, for it increases the liability of altering the circuit conditions by the application of the instrument. The resistance of a 150-volt instrument of this type is from 2,500 to 3,000 ohms. Low ranges are obtained by reducing the series resistance, and as the inductance remains the same, the likelihood of a frequency error is much increased.

In investigation work it is sometimes necessary to measure voltages on circuits of abnormally high frequency, 500 to 1,000 cycles per second. In this case, if a dynamometer voltmeter is used, the inductance term will *not* be negligible. Its value will depend on the position of the movable coil. Eddy currents in the metal frames as well as capacity effects between the coils and in the series resistance also modify the action of the instrument.

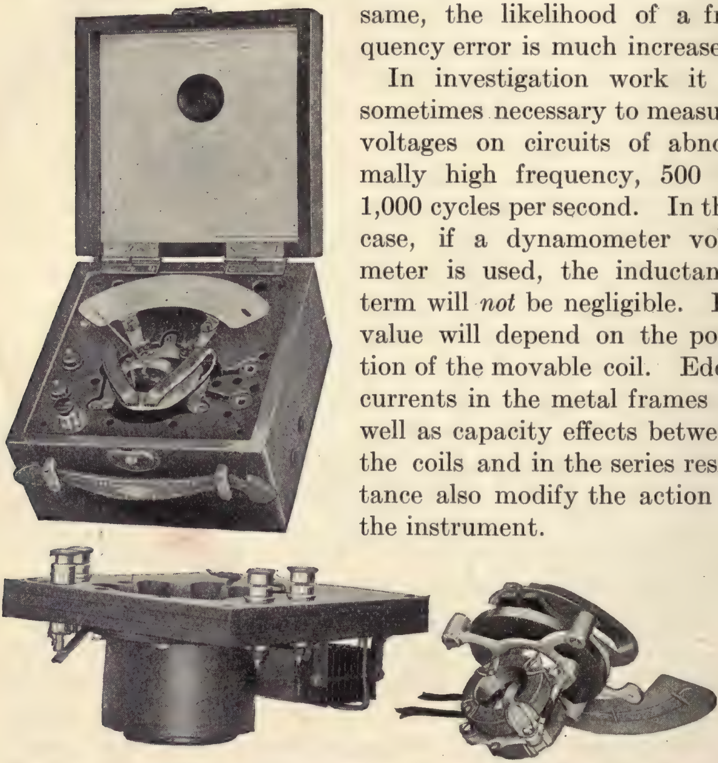


FIG. 131.—Shielded dynamometer voltmeter, General Electric Co.

A certain alternating-current voltmeter of the electrodynamometer type had a resistance, when measured with direct current, of 1,557 ohms. This agreed with the resistance measured with an alternating current of 16 cycles per second. The effective inductance at 16 cycles per second (and 110 volts deflection) was 0.061 henry. At 3,000 cycles per second the effective resistance was 1,675 ohms while the effective inductance

was 0.053 henry. While there was no appreciable error at 16 cycles per second, the error in the indication at 3,000 cycles per second was 25+ per cent.

Fig. 131 shows the general construction of a dynamometer voltmeter. The working parts of the instrument are surrounded by a laminated magnetic shield. Electromagnetic damping is obtained by having the movable system carry a fan-shaped sector of aluminum, which swings between the poles of two small permanent magnets. The shielded voltmeter made by the Weston Instrument Co. is very similar in general design to the wattmeter shown in Fig. 177; it has a very efficient air damper, consisting of two light, symmetrically disposed vanes which are enclosed in carefully finished chambers in the base of the framework which supports the coils. The vanes are of exceedingly thin metal stiffened by ribs stamped into them and by the edges which are turned over. The useless leakage to the outside air is reduced to a minimum and the desired degree of damping attained by a suitably designed clearance space between the vanes and the walls of the chamber (see page 71). The moment of inertia of the moving parts of this arrangement is very small.

Hot-wire Voltmeters.—The first instrument particularly adapted to the measurement of alternating potential differences was the Cardew voltmeter, invented by Major Cardew, R. E.

In this instrument the current was passed through a long thin wire of platinum-silver, and by a suitable mechanism the expansion of this wire, due to its rise of temperature, caused the index to move over the scale. Later designers have been able to improve on Cardew's arrangement for translating the expansion of the wire into the movement of the index, so that a much shorter wire may be employed, thus rendering the instrument less cumbersome.

The ingenious multiplying device used by Hartmann and Braun is shown in principle in Fig. 132.

The active wire *ADB* is of platinum-iridium, *DEC* is a very fine phosphor-bronze wire, and *EFG* is a silk fiber which passes once around the drum *F* and is drawn taut by the spring *S*. On the passage of the current, *ADB* is heated and expands, the slack is taken up by the spring *S* and the index is thus moved

over the scale. The vane W moving in the air gap of the permanent magnet M serves as a damping device. The moving system is carried by insulating studs at A, B and C. These are

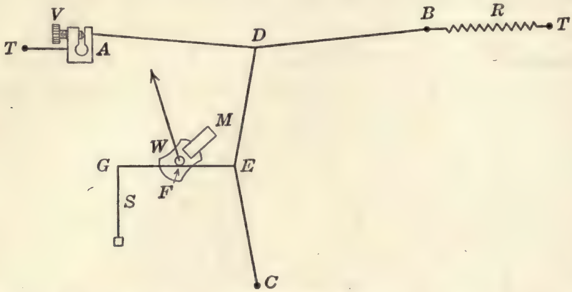


FIG. 132.—Diagram for Hartmann & Braun hot-wire voltmeter.

supported on a back plate constructed of two metals in such a proportion that the net coefficient of expansion is the same as that of the wire, so the effect of changes of room temperature is minimized. The position of the end A of the working wire may be altered by turning the screw V and the zero position of the pointer thus adjusted. The coefficient of expansion of the platinum-iridium wire is less than that of the platinum-silver wire formerly used; it can be run, however, at a much higher temperature, the result being a considerable reduction of the zero shift, which is troublesome in hot-wire instruments.

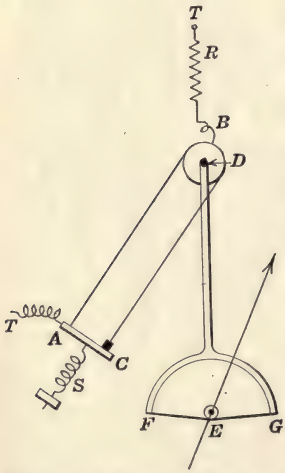


FIG. 133.—Diagram for Roller hot-wire voltmeter.

The working parts of the Roller hot-wire voltmeter are sketched in Fig. 133. The pivot D carries a pulley B and the forked arm DFG. A silk fiber is stretched between F and G. This passes around and is made fast to the pivoted drum E, to which the pointer is attached. It is evident that any rotation of the drum D will cause the pointer to move over the scale. The wire ABC passes around and is made fast to the pulley B. The current flows through the wire AB since the end

C is insulated. The wire is in tension due to the spring S. Normally the tension of the sections AB and CB is the same; on the passage of the current, AB expands and the equality of tension is restored by the rotation of B, which at the same time moves the pointer.

As the wires AB and CB are of the same material and mass, the effect of variation of room temperature is compensated, even though it be rapid.

The whole movable system is mounted on a plate which can be rotated about the point E; the zero adjustment is thus provided.

In American practice, hot-wire instruments are not used on switchboards. They are very useful in the laboratory, for being adapted to the measurement of both alternating and direct currents, they can be used as transfer instruments in the calibration of alternating-current ammeters and voltmeters. The alternating-current instrument may be compared, under normal conditions of frequency and wave form, with the hot-wire instrument and the latter calibrated immediately by use of the direct-current potentiometer.

When instruments of this type are first placed in circuit, enough time should be allowed for them to come to their stable condition before the readings are taken.

The advantages of hot-wire instruments are: they have no self-heating errors, are not influenced by local fields and are uninfluenced by changes of frequency and wave form;* the last is in consequence of their low inductance. Their disadvantages are that they are sluggish in action, the zero is unstable, they are easily burned out by overloads, and the resistance of the voltmeter is low.

ELECTROSTATIC INSTRUMENTS

In instruments of this class, advantage is taken of the electrostatic attraction existing between bodies charged to different potentials. The magnitude of the force depends on the geometry

* In strictness this remark applies only to instruments where the whole current is taken through the hot wire. There are certain forms of instruments which are equivalent to shunted ammeters in their construction. They will show frequency errors at the very high periodicities used in radiotelegraphy (see page 63).

of the system of conductors, the relative potentials of its parts and the dielectric coefficient of the medium separating the attracting bodies.

The Attracted-disc Electrometer.—The first suggestion of the attracted-disc electrometer was due to Sir William Snow-Harris, who used an instrument of this sort. Its essential members were a fixed circular plate electrode supported by an insulating standard, and a movable plate electrode hung directly over the fixed plate from the arm of a gravity balance. By putting weights in the scale pan, a balance could be secured and a measure of the electrostatic attraction and consequently of the P.D. between the electrodes obtained. A defect of any such simple arrangement, which renders it useless as an absolute instrument, is that owing to the influence of the edges of the plates the distribution of the charges over the surfaces will not be uniform. This renders inexact the application of a simple formula for the attraction between the two plates, based on the assumption of a uniform density of distribution of the charge.

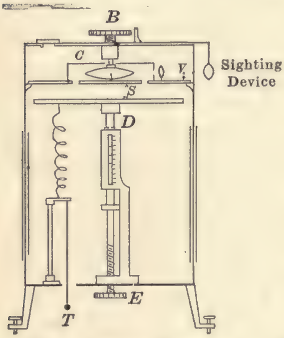


FIG. 134.—Elements of the Kelvin absolute electrometer.

The distribution of the charges over the central portions of two parallel plates, whose dimensions are large compared with their distance apart, will be practically uniform. Therefore, if the force exerted on the central portion of one of the plates is measured, the use of a formula which assumes a uniform distribution will be legitimate.

Lord Kelvin secured a practically uniform distribution by the use of the guard ring. This is a broad ring closely surrounding, but not touching, the movable member and in electrical connection with it. The stationary, or attracting plate, has the same diameter as the guard ring.

Absolute electrostatic instruments are not of industrial importance; however, the application of the guard-ring principle will be illustrated by the Kelvin absolute electrometer, the elements of which are shown in Fig. 134.

In this instrument the attraction on the movable member is

weighed by a calibrated spring balance. The guard ring and attracted disc are supported from the sides of a large glass jar which serves as a case for the instrument; the disc is made of aluminum and is carried by three small springs, having the shape of coach springs. These are supported from an insulating rod, which by the use of a micrometer screw, B, can be raised or lowered through known amounts. In order that the attracted disc may always be returned to its proper zero position, with its lower surface coplanar with that of the guard ring, a sighting arrangement is provided. A fine hair is stretched between two small pillars at the center of the disc; this hair is at the focus of a lens which forms an image between the points of two small screws carried by the guard ring. The image of the hair and the points of the screws are viewed through a lens. When the disc is in its zero position the hair appears to bisect the distance between the points.

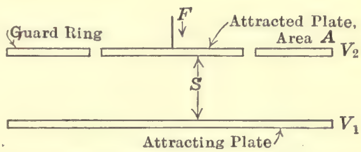


FIG. 135.—Pertaining to attracted-disc electrometer.

The attracted disc is shielded from extraneous action by the removable box C. The attracting plate is carried on an insulating glass rod D and can be moved vertically through known amounts by means of the micrometer screw E. T is the well-insulated terminal connecting with the attracting plate. Connection between the guard ring and the attracted disc is made by a flexible wire.

The relation between the difference of potential of the plates and the force of attraction may be established thus:

Referring to Fig. 135, the attracted plate has an area of A sq. cm. and is distant S cm. from the attracting plate. V_2 and V_1 are the potentials of the two plates. The arrangement forms an electrical condenser and if S is small compared with the size of the plates, the capacity will be

$$C = \frac{A}{4\pi S}.$$

The energy necessary to raise one plate to the potential V_1 and the other to the potential V_2 will be

$$E = \tfrac{1}{2}Q(V_1 - V_2) = \tfrac{1}{2}C(V_1 - V_2)^2 = \frac{A(V_1 - V_2)^2}{8\pi S}.$$

Suppose the upper plate is given a small displacement, V_1 and V_2 being kept constant by connection to a source of potential difference. There will be a change in the energy of the condenser which will be numerically equal to the mechanical work necessary to displace the plate in the direction S.

$$dE = FdS$$

$$F = \frac{dE}{dS} = \frac{A(V_1 - V_2)^2}{8\pi S^2}$$

$$\therefore V_1 - V_2 = S\sqrt{\frac{8\pi F}{A}} \text{ in electrostatic units.} \qquad (2)$$

If $(V_1 - V_2)$ is in volts, P.D. $= 300S\sqrt{\frac{8\pi F}{A}} = 1{,}504S\sqrt{\frac{F}{A}}$ (3)

It has been assumed that all of the electrostatic lines of force are straight and normal to the plane of the disc. A few lines will stray into the very narrow gap between the guard ring and the attracted plate. They may be assumed to divide equally between the ring and the plate, so to make an approximate allowance, the effective area of the plate may be taken as the area of the plate plus one-half the area of the air gap.

It is well to emphasize the fact that unless the voltages are high, the forces to be dealt with in electrostatic instruments are small. If

$$A = 100 \text{ sq. cm.}$$
$$S = 1 \text{ cm.}$$
$$\text{P.D.} = 150 \text{ volts}$$

then

$$F = 1 \text{ dyne, approximately.}$$

That is, the force is about the same as the attraction of gravity on a mass of 1 mg. To increase this force, the plates must be brought very near together, or the use of the instrument restricted to measuring high potentials.

Before using the absolute electrometer, the spring balance must be calibrated; to do this the terminals of the instrument are short-circuited and the disc brought to its zero position by means of the micrometer head. Known weights are then placed upon the disc and the number of turns which it is necessary to give B in order to return the disc to the zero position is noted. The number

of dynes corresponding to one division of the micrometer head can then be calculated.

The natural procedure in taking a measurement would be first to short-circuit the instrument and bring the attracted disc to the zero position by means of the micrometer head B, the reading of which is noted. The potential to be measured would then be inserted between the terminals of the instrument. This would cause the attracted disc to move downward. The disc would then be returned to its zero position by turning the head B, the final position of which is read. The difference of the micrometer readings gives the stretch of the spring. The force F is determined from this stretch and the calibration of the spring.

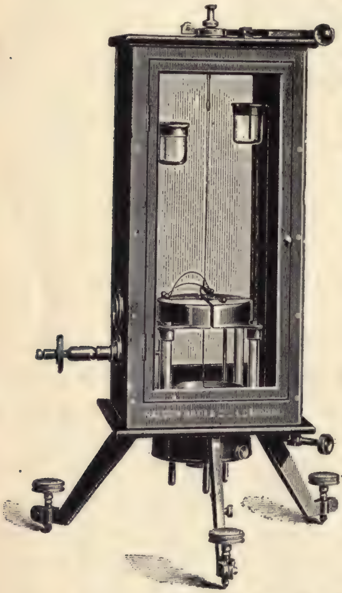
It will be found that if the P.D. is small, the lower plate must be brought very near to the attracted disc. For example, if the potential difference is 500 volts, F, 100 dynes, and A, 100 sq. cm., the distance between the plates will be only 3 mm. It is practically impossible to make and adjust the plate and disc so that such a small value of S can be measured with certainty. Slight irregularities of the surfaces and lack of parallelism of the plates would vitiate the results.

This difficulty has been overcome by Lord Kelvin's method of using an auxiliary high potential. Suppose that the guard ring and disc are charged to a potential of 10,000 volts, the attracting plate being connected to earth. When the disc has been brought to its sighted position, corresponding to $F = 100$, $S = 6.666$ cm. If the plate be now connected to earth through a 500-volt battery, thus making the potential applied to the instrument 9500 volts, to return the disc to its standard position, B remaining fixed S must be made 6.366 cm. That is, the lower plate has to be moved through a distance equal to that which would exist between the plate and disc if the P.D. were directly measured. The advantage attained is that in both measurements the attracting plates are so far apart that there is practically no uncertainty as to the value of S. When used as just suggested, the instrument is said to be employed *heterostatically;* when only the P.D. to be measured is employed the electrometer is said to be used *idiostatically.*

The expression for the P.D. when the instrument is used heterostatically is P.D. $= 1{,}504(S - S')\sqrt{\frac{F}{A}}\cdot$ The distance

through which the attracting plate must be moved in order to again bring the cross-hair to its standard position after the application of the P.D. to be measured is $S - S'$.

The glass jar which forms the case of the instrument is coated with tin foil both inside and outside, and serves as a Leyden jar to hold the auxiliary charge, which is obtained from an electrophorus and is kept constant by the use of a small influence machine called the replenisher. It will be noticed that it is not necessary to know the value of the auxiliary potential; it must, however, remain constant throughout the experiment. To test this a small attracted disc electrometer called a gage is provided. As the electrometer is constructed, both the replenisher and the gage are included within the case.



The attracted-disc principle is used in secondary electrostatic voltmeters intended for high-tension work (see page 258). In such instruments the guard ring is omitted.

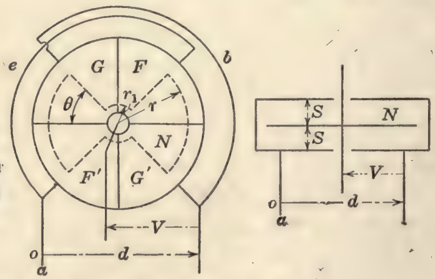


Fig. 136.—Simple quadrant electrometer.

The Quadrant Electrometer.—The quadrant electrometer, also the invention of Lord Kelvin, is more sensitive and of much greater practical importance than the absolute instrument just described. Of late years it has been used in investigations concerning the energy losses in dielectrics intended for high-voltage insulations.

The instrument, reduced to its elements, is shown in Fig. 136,

while a perfected form, designed at the National Physical Laboratory, and intended for work of high precision, is shown in Fig. 189, page 324.

The arrangement of conductors is shown in Fig. 136. G,G', F,F' are the quadrants, usually made by cutting into four parts a shallow metal box with its cover; for low potentials, the diameter of the box is about 3 in., the depth about ½ in. The quadrants are supported on insulating standards and are cross-connected electrically by the wires b and e.

The needle N, made of thin aluminum and of the form indicated, is suspended within the box equally distant from the top and the bottom.

The directive moment is usually obtained by a torsion wire or by a bifilar suspension.

To deduce the formula for this electrometer, it will be assumed that as the needle deflects, the rates of variation of the capacities of the condensers formed by the needle and the quadrants are uniform and independent of the angular displacement of the needle, that the radial cuts dividing the quadrants are of negligible width and that only the five conductors, F, F', G, G', N need be considered.

The distribution of potentials will be assumed as indicated in Fig. 136. Starting from the point a the fall of potential to the second set of quadrants is d units. The further fall from the second set of quadrants to the needle is V units.

The needle N and the quadrants G, G' form a condenser which is charged by a potential difference, V, while the needle and F, F' form a condenser charged by a potential $(V + d)$. The total area of the needle under G and G' is A.

The energy of the condenser, considering both sides of the needle, is

$$E = \tfrac{1}{2}CV^2 = \frac{AV^2}{4\pi S} \tag{4}$$

$$A = \left(\frac{\pi r_1^{\,2}}{4} + \tfrac{1}{2}r^2\theta - \tfrac{1}{2}r_1^{\,2}\theta\right)2 = \frac{\pi r_1^{\,2}}{2} + (r^2 - r_1^{\,2})\theta$$

so

$$E = \left[\frac{\dfrac{\pi r_1^{\,2}}{2} + (r^2 - r_1^{\,2})\theta}{4\pi S}\right]V^2 \tag{5}$$

If the needle be given a slight angular displacement, V being

kept constant, a small amount of work will be done which is numerically equal to the change of electrical energy. If M is the moment causing the displacement of the needle, then as

$$M = \frac{dE}{d\theta},$$

$$M_G = \left(\frac{r^2 - r_1^2}{4\pi S}\right) V^2$$

Similarly $$M_F = \left(\frac{r^2 - r_1^2}{4\pi S}\right) (V + d)^2.$$

The net turning moment acting on the needle will be

$$M = \left(\frac{r^2 - r_1^2}{4\pi S}\right) \left((V + d)^2 - V^2\right) = \left(\frac{r^2 - r_1^2}{4\pi S}\right) (2Vd + d^2).$$

This moment is balanced by the torsion of the suspension, so if D is the deflection and τ the torsion constant

$$\tau D = \frac{r^2 - r_1^2}{4\pi S} [2Vd + d^2] \text{ in absolute electrostatic units.} \tag{6}$$

If volts are used,

$$\tau D = \frac{1}{1{,}130{,}000} \cdot \frac{r^2 - r_1^2}{S} [2Vd + d^2] \tag{7}$$

The quadrant electrometer is always used as a secondary instrument, but the formula (7) is useful as an aid in preliminary design.

The instrument is read by one of the mirror and scale methods, so if D be taken in scale units,

$$D = K [2Vd + d^2] \tag{8}$$

An equivalent expression is frequently used,

$$D = K\left[(V_1 - V_2)\left(V - \frac{V_1 + V_2}{2}\right)\right] \tag{9}$$

Here, V_1, V_2 and V are the *potentials* of the two sets of quadrants and of the needle respectively.

If the potential differences are alternating, the instantaneous value of the turning moment is proportional to $2Vd + d^2$ and

$$D = K \frac{1}{T}\int_0^T (2Vd + d^2)dt.$$

The above demonstration is in general sufficient; in it all contact differences of potential have been neglected and it has been assumed that the rates of change of the capacities of the condensers formed by the quadrants G and F and the needle, as the latter turns, are the same and equal to that when the needle is in its zero position. If this is not so there will be an additional moment on the needle, proportional to the difference of the rates of change of the capacities of the two condensers, F and G, and to V^2; as this moment is dependent on the voltage applied to the needle it will vary with the uses to which the instrument is put.[1]

The Mechanical and Electrical Zeros.—When the needle and the two sets of quadrants are at the same potential, there will be no electrical turning moment acting on the needle and it will take up a position due to the suspension alone; this is the mechanical zero of the instrument.

When the electrometer is perfectly symmetrical, if the two sets of quadrants are kept at the same potential and voltage is applied between them and the needle, it will be seen from (6) that there should be no deflection from the mechanical zero. However, the slightest departure from symmetry will cause a deflection and the needle will move to the electrical zero. The difference of the two zeros should be small and in a carefully made instrument they can be made to coincide by adjusting the symmetry of the arrangement. In the instrument shown in Fig. 189, this may be done by tilting the upper quadrants by the use of the vertical screw. The deflection should be read from the electrical zero.

General Considerations.—To obtain a good law of deflection over a long range, it is necessary that the needle be bounded by arcs of circles and straight lines as indicated in Fig. 136. Raising or lowering the needle (or any tilting of the needle) will cause a change in the constant; for this reason, it is sometimes preferable to have a considerable distance between the upper and lower quadrants. The consequent decrease in the deflecting moment must be compensated by using a more delicate suspension. Even when the greatest care is exercised, the constant of the instrument will not be the same for all deflections.

If the instrument is to be used for alternating-current meas-

urements, the calibration should be made by using alternating potential differences. The effects of contact differences of potential are then eliminated. Conditions which distort the

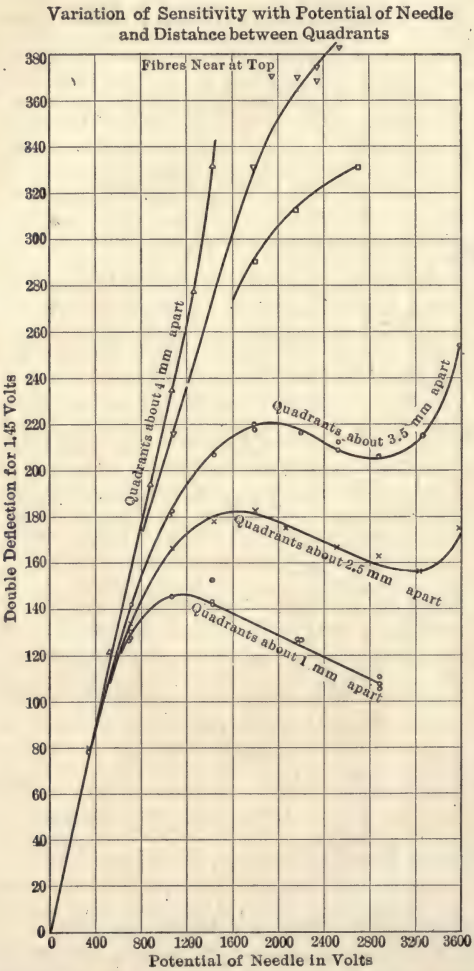


FIG. 137.—Illustrating effect of distortion of the electrostatic field at the needle of a quadrant electrometer.

electrostatic field in which the needle swings cause departures from the theoretical law. For instance, in one of the older designs of instrument, a guard tube was used which surrounded

the stiff wire supporting the needle, the intent being to protect the movable system from extraneous electrostatic attraction. The whole tube was supported from above, and carried down through the circular opening at the center of the quadrants, being cut away at the sides to allow free motion of the needle. This construction resulted in the peculiar law of deflection shown in Fig. 137.

Electrostatic Voltmeters.—On the supposition that the law of the quadrant electrometer is that previously deduced, if the needle and one set of quadrants are connected together, $V = 0$ and

$$D = Kd^2.$$

Or, if the P.D. is rapidly alternating,

$$D = K\frac{1}{T}\int_0^T (d^2)dt.$$

As one pair of quadrants produces no effect, it may be omitted in an instrument designed primarily for voltage measurements.

As the deflections are proportional to the mean square value of the P.D., electrostatic voltmeters are particularly applicable to the measurement of alternating potential differences.

These instruments absorb no power and at ordinary frequencies produce no disturbance of the potential difference to which they are applied. Their action is not complicated by inductance effects so there are no frequency or wave-form errors. They have no self-heating errors and are uninfluenced by local magnetic fields. On the other hand, at low voltages the forces to be dealt with are very small and consequently the instruments are much more delicate than those based on the electrodynamometer principle. When they are used on direct-current circuits, the effect of contact differences of potential must be eliminated by reversals.

Ayrton, Mather and others have developed the electrostatic voltmeter so that it has become an instrument of great value in laboratory work. Fig. 138 shows one of Ayrton and Mather's instruments for low voltages, up to 16 volts.

This instrument is intended to be read by one of the mirror and scale methods. The suspended system consists of a light

aluminum needle, made in the form of a portion of a cylinder. The "quadrants" are portions of two cylinders, concentric with the needle. The needle is drawn into the space between them by

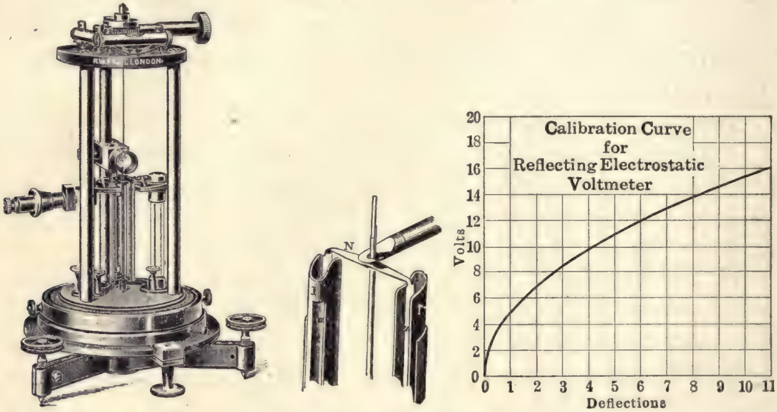


FIG. 138.—Low-range electrostatic voltmeter and the calibration curve. Damping magnet not used in this instrument.

the electrostatic attraction. The controlling force is given by a flat strip suspension and the zero may be set by means of a tangent screw.

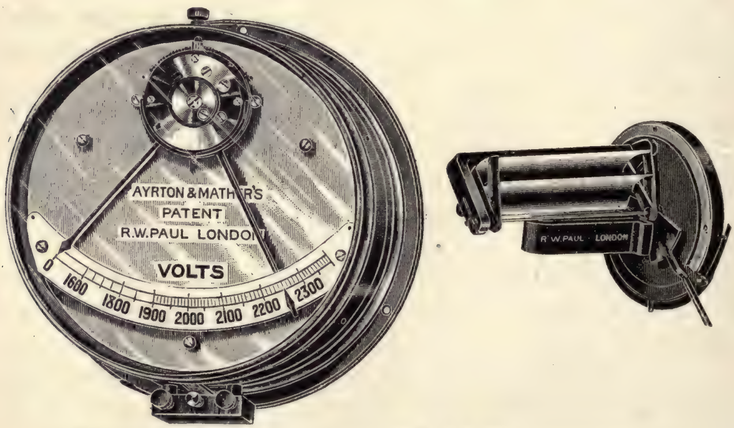


FIG. 139.—Electrostatic voltmeter for switchboard work.

In order to damp the instrument, the needle, which from its construction forms a closed loop, is frequently arranged to turn between the poles of a permanent magnet.

To prevent extraneous electrostatic action, the needle, the magnet and the case are connected together electrically. With this construction, the outside case is at the potential of one side of the line. In later instruments, the shielding is accomplished by an inner case which is insulated from the outside or protective case. The construction of the instrument is such that accidental contact between the quadrants and the movable system is impossible.

Instruments of this general design are listed, which give a full-scale deflection with 7 volts.

In cases where the voltage is high, above 800 volts, the forces become great enough so that the cylindrical needle may be pivoted on jewelled bearings with its axis horizontal (see Fig. 139).

The controlling moment is obtained by using a small weight attached to a short arm which projects from the axis. Electrical connection with the needle is made by a very fine wire, wound in a flat spiral.

Some control over the law of deflection may be obtained by shaping the quadrants.

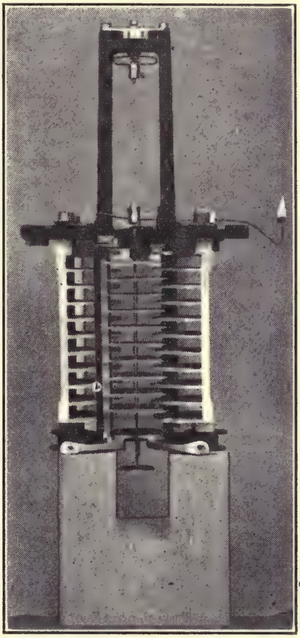


FIG. 140.—Working parts of Kelvin multicellular electrostatic voltmeter.

To prevent arcing between the needle and the quadrants in event of a great increase of voltage, a spark gap is provided between the terminals and within the case. It is intended that it act when the voltage has risen to twice the full-scale value. Fuses which are enclosed in the removable terminals are thus blown and the instrument automatically taken out of circuit.

A reference pointer and means for clamping the movable system during transportation are provided. The damping is obtained by having attached to the needle an aluminum sector which moves between the poles of a permanent magnet. All of the

working parts of the instrument are insulated from the outside or protecting case. This removes the possibility of incurring shocks when the instrument is touched.

To increase the forces acting on the movable systems of electrostatic voltmeters, up to about 1,000 volts, Lord Kelvin devised the multicellular instrument, an example of which is shown in Fig. 140.

A torsion wire suspension is used, and the increase of the deflecting force, which is in proportion to the number of cells, is sufficient so that a pointer and scale may be used for reading the deflections. The instrument, however, is not portable in the ordinary sense. In the voltmeter shown in Fig. 140 the damping is effected by a disc which turns in a viscous oil contained in a little glass vessel at the bottom of the instrument.

Two vertical plates are connected to the movable system and screen it from the action of the set of quadrants near which they are placed.

Use of a False Zero Reading.—It is sometimes desirable to measure a small direct-current voltage without drawing any current. The use of the electrostatic voltmeter suggests itself but the normal curve of the reflecting form of this instrument is very nearly a parabola, as is shown by Fig. 138, so that a low potential difference gives a very small deflection which cannot be read with accuracy. The difficulty may sometimes be overcome by superposing the P.D. to be measured on a fixed and higher voltage. For instance, if the instrument gives a scale reading of 50 cm. with 50 volts, a potential difference of 2 volts applied directly to the instrument will give a deflection of 0.08 cm. However, if it be superposed on a P.D. of 50 volts the *increase* in deflection will be 4.08 cm. As the upper part of the calibration curve is nearly straight, the deflections from the false zero are practically proportional to the voltage.

Condenser Multipliers for Extending the Range of Electrostatic Voltmeters.—The range of an electrostatic voltmeter may be extended by means of condenser multipliers as indicated in Figs. 141 and 142.

A condenser of the proper capacity may be joined in series with the voltmeter. Then $V = V_2\dfrac{(C_M + C_V)}{C_M}$ where V_2 is the reading

of the instrument and C_M and C_V are the capacities of the condenser and the voltmeter respectively. As the capacity of the instrument, C_V, depends on the deflection, the factor $\frac{(C_V + C_M)}{C_M}$ will not be a constant and the voltmeter must be calibrated with the condenser in place.

An alternative method is to join a number of condensers in series and to place the electrostatic voltmeter around one of them, as shown in Fig. 142.

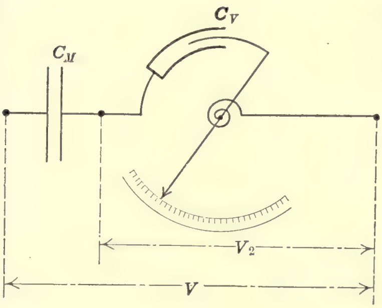


FIG. 141.

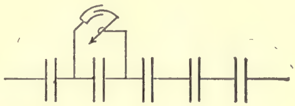


FIG. 142.

FIGS. 141 AND 142.—Condenser multipliers for electrostatic voltmeter.

Here again the whole arrangement should be calibrated as a unit.

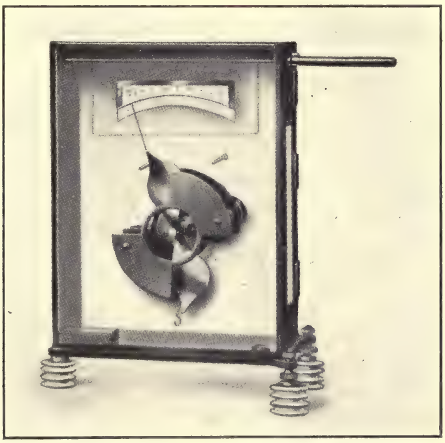


FIG. 143.—Electrostatic voltmeter with variable range.

A simple form of electrostatic voltmeter with a gravity control, a very obvious development from the quadrant electrometer, is shown in Fig. 143.

The range of the instrument is up to 10,000 volts, without the use of a condenser multiplier, and up to 30,000 volts if the multiplier is employed. The deflection per volt may be varied by means of weights which are hung on the hook at the lower end of the needle. The oscillation of the needle can be checked by bringing a silk thread into contact with the pointer.

Fig. 144 shows a form of electrostatic voltmeter made by Siemens and Halske for voltages up to 150,000.

One electrode, *A*, is under the base of the glass jar *C*; this jar is filled with oil and the movable electrode *B* is suspended in it. The restoring force is a spiral spring. The pull on the electrode *B* is transmitted to the pointer by a mechanism which is so

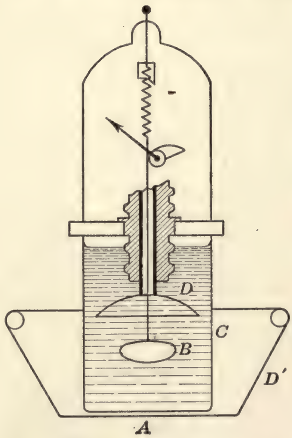


FIG. 144.—Siemens and Halske high-range electrostatic voltmeter.

arranged that the upper 70 per cent. of the scale is practically uniformly divided. The use of the oil reduces the risk of an arc forming between the electrodes *A* and *B* and permits them to be brought nearer together, thus increasing the force and permitting the instrument to be made smaller. The damping is by the fluid friction of *B*. The shields *D* and *D′* are to protect the instrument from the influence of surrounding objects.

The Westinghouse Co. manufactures the high-range voltmeter[2] shown diagrammatically in Fig. 145.

The voltage is applied at I and I'; between ab and bc are two condensers which can be short-circuited at will, thus altering the range of the instrument. They are placed within the highly insulated condenser terminal, T, and are brought into action by means of a silk cord. The fixed attracting elements are at B and B'. The movable element consists of two hollow metal cylinders M and M' which are united by a suitable web. This system hangs freely from a single pivot resting in a jewel. The usual controlling spring and zero adjustment are provided.

There is no electrical connection to the movable element; on the application of the voltage, charges are induced on it.

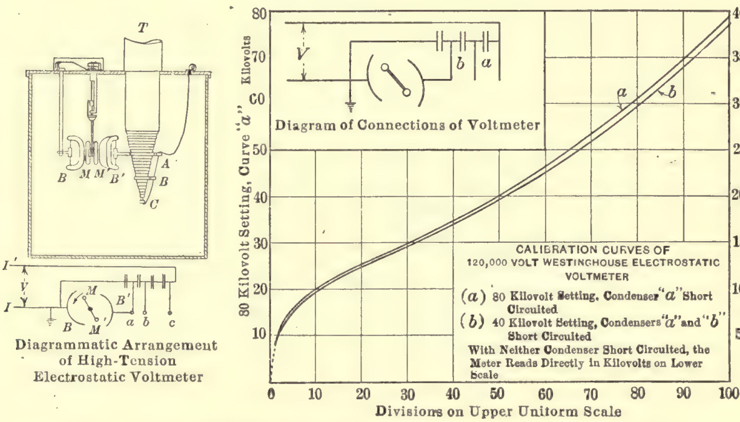


FIG. 145.—Westinghouse high-range electrostatic voltmeter.

As the curved plates B and B' and the movable parts are not concentric, the latter will move so as to increase the electrostatic capacity of the arrangement; that is, in the direction of the arrow. The plates B and B' are so bent that the scale is approximately uniform over a considerable portion of its length. All the working parts are immersed in oil, and as the movable element is hollow, the weight is practically removed from the jewel and pivot.

These voltmeters are made for potentials as high as 200,000 volts. Instruments having a range of 25,000 volts, both condensers being short-circuited, will read up to 100,000 volts with both condensers in service.

THE SPARK GAP METHOD OF MEASURING HIGH PEAK VOLTAGES

It is difficult to design indicating instruments for directly determining extra high voltages because of corona effects, disruptive discharges, and extraneous electrostatic attractions. Also in testing the dielectric strengths of insulations it is desirable to know the maximum rather than the effective voltage to which any sample is subjected. Consequently a method of measurement depending on the dielectric strength of air has been developed and has been employed for many years. The neces-

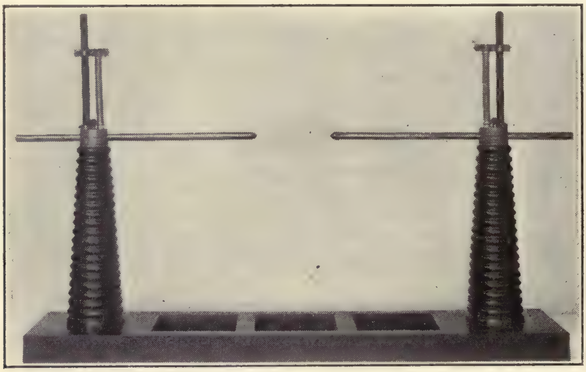


FIG. 146.—Needle-point spark gap.

sary apparatus is termed a spark gap and its use as a means of determining high voltages is sanctioned by the American Institute of Electrical Engineers[3]. This method is frequently used in acceptance tests of new apparatus where the dielectric strength of the insulation is guaranteed.

The construction of a needle-point spark gap is shown in Fig. 146. According to the standardization rules of the American Institute of Electrical Engineers, "the spark points should consist of new sewing needles supported at the ends of linear conductors which are each at least twice the length of the gap. There should be no extraneous body near the gap within a radius of twice its length."

The arrangement is such that the points of the needles may be set at any desired distance apart. The function of the carbon

resistances placed vertically above the supporting pillars is to limit the current when the gap breaks down. The establishment of surges in the circuit due to a sudden change in its constants is thus avoided. Water-tube resistances are more reliable for this purpose than carbon rods, which may have low resistances at high voltages.

The current after the gap has broken down should not be greater than 1 amp. The needles are set in accordance with the table given below.

To use the gap, it is set to correspond with the appropriate voltage and placed in parallel with the apparatus under test. The applied voltage is then gradually raised until the gap breaks down. The reading of the voltmeter on the low-tension side of the testing transformer and the adjustment of the voltage regulating apparatus at the instant of breakdown are noted. A new set of needles is then inserted and the gap set for a voltage about 20 per cent. too high. The former reading of the voltmeter or the adjustment of the regulating apparatus is then reproduced and the voltage applied for the required time.

The gap breaks down at the peak of the wave and therefore gives a measure of the maximum voltage to which the insulation has been subjected. It will do this irrespective of the wave form, but the following table is for sinusoidal waves and effective voltages. If the gap is to be set for peak voltages, the values in the table must be multiplied by $\sqrt{2}$.

TABLE I.—NEEDLE-POINT SPARK-OVER VOLTAGES WITH NO. 00 SEWING NEEDLES

(At 25°C. and 760 mm. barometer—relative humidity 80 per cent.)

R.m.s. kilovolts	Millimeters	R.m.s. kilovolts	Millimeters
10	11.9	40	62
15	18.4	45	75
20	25.4	50	90
25	33.0	60	118
30	41.0	70	149
35	51.0	80	180

The American Institute of Electrical Engineers sanctions the use of the needle-point spark gap for voltages between 10 and 50 kv.

Mechanically, the needle-point spark gap is about the simplest electrical measuring device but this simplicity of construction is no guarantee of simplicity of action and the needle-point gap must be used by skilled experimenters if reliable results are required. Its indications are open to many sources of error.

Like all spark gaps, it is influenced by the distortion of the electrostatic field in its neighboorhood, due to surrounding objects.

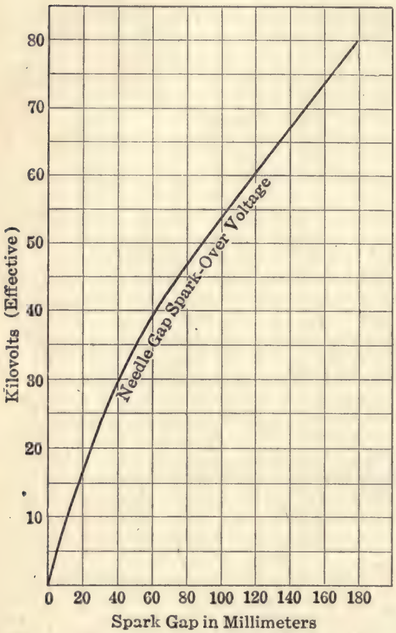


FIG. 147.—Plot of A. I. E. E. table for needle-point spark gap.

For high voltages, therefore, the arrangement must necessarily occupy a large space. In an apparatus for 200 kv., if designed as indicated above, the distance between the needle points will be about 52 cm.; the supports for the needles will each be about 104 cm. long, so the length of the apparatus will be at least 9 ft., and as no object should be nearer the gap than 3½ ft., the space occupied would be about 9 by 7 by 7 ft.

In addition to this large space factor, which is disadvantageous, there are irregularities in the action of this form of gap arising from the varying sharpness of the needles. A new set of needles must be inserted after each breakdown of the gap.

When the voltage is gradually raised, the points are seen to be surrounded by a bluish glow or corona, more or less spherical in form. This happens long before the gap breaks down and means that the air about the points has become conducting. Serious errors may be introduced by this preliminary breaking down of the air, for irregularities due to heating are thus introduced. Again, it is found in all cases where the corona forms

before the passage of the spark that the dielectric strength of an air gap is very dependent on the humidity. This is illustrated by Fig. 148.

The density of the air affects the results obtained with any form of spark gap.

The irregularities of the needle-point gap have proved so troublesome that a substitute for this form of gap has been sought. For voltages above 50 kv. the use of two spherical electrodes of equal diameters is now recommended by the American Institute of Electrical Engineers. The advantage is that if the distance between the electrodes is less than three times the radius of the spheres, the corona does not form before the gap breaks down. The erratic effects due to broken down air around the electrodes are thus avoided. Humidity has no influence on the results.

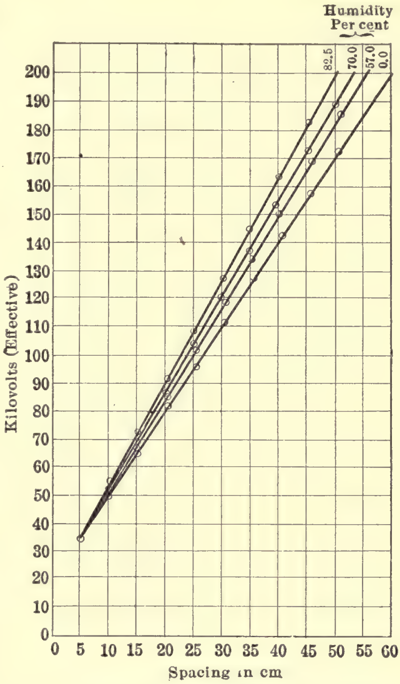


FIG. 148.—Showing effect of humidity on the action of a needle-point spark gap.

An additional advantage is that the length of the gap is much reduced; with spheres 25 cm. in diameter, the sparking distance for 200 kv. is about 13 cm. against about 52 cm. for the needle-point gap.

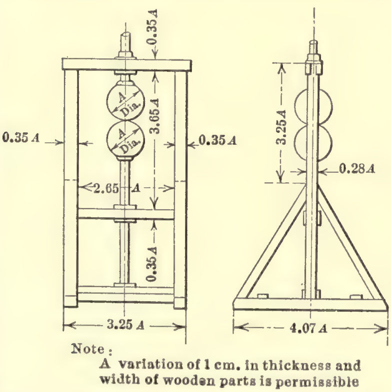


FIG. 149.—Spark gap with spherical electrodes.

The sphere gap is especially adapted for measuring

extra high voltages, above 50 kv. It is not so well adapted for low voltages for the spheres must be brought very near together.

The spheres should be at least twice the gap distance from surrounding objects and greater than this if one sphere is grounded.

If the current be limited to less than 1 amp. by water-tube resistances, pitting of the spheres is avoided and they need be repolished only occasionally.

When neither sphere is grounded, it is possible to calculate mathematically the relation between the breakdown voltage and the length of gap. The dielectric stress will be a maximum at the points where the line joining the centers of the spheres cuts their surfaces. If the value of the potential gradient at this point be denoted by g, then it may be shown that

$$g = \left(\frac{V}{x}\right) f \quad \text{kv./cm.} \tag{11}$$

V is the potential difference and x the distance apart of the surfaces. $\frac{V}{x}$ is therefore the average potential gradient; f is a factor which depends on x and the radius of the spheres, a; it is the quantity which must be multiplied into the average gradient to give the maximum gradient and has been expressed in the form of an infinite series by A. Russell[5].

Table II.—Values of f, Neither Sphere Grounded, Computed by A. Russell

$\frac{x}{a}$	f
0.0	1.000
0.1	1.034
0.2	1.068
0.3	1.102
0.4	1.137
0.5	1.173
0.6	1.208
0.7	1.245
0.8	1.283
0.9	1.321
1.0	1.359
1.5	1.559
2.0	1.770
3.0	2.214
4.0	2.677

These values are plotted in Fig. 150.

When the potential between the spheres is gradually raised, it would naturally be expected that the gap would break down when the maximum gradient reached some definite value, g_s, and that if this value be known the voltage corresponding would be

$$V = g_s \frac{x}{f} \tag{12}$$

Experiment shows that g_s, the maximum gradient at spark-over, depends upon the radius of the spheres, being larger as the radius is diminished. F. W. Peek has shown that between the limits $x = 0.54\sqrt{a}$ and $x = 2a$ it may be represented by the empirical equation[4].

$$g_s = 27.2 \left(1 + \frac{0.54}{\sqrt{a}}\right) \tag{13}$$

a is in centimeters.

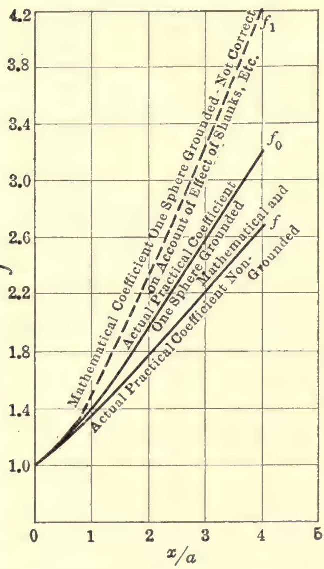


FIG. 150.—Plots of f for spark gap with spherical electrodes.

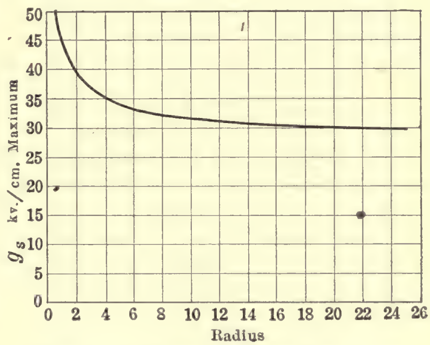


FIG. 151.—Showing relation of maximum potential gradient at spark-over to radius of spheres.

This relation is shown graphically in Fig. 151. If $\frac{x}{a}$ is greater than 4, corona forms before the breakdown and the above equations are no longer applicable.

If one sphere is grounded, the electrostatic field is so distorted by the supporting stems and surrounding objects that the value of f deduced from purely mathematical considerations is

no longer applicable. In this case, experimentally determined values, f_0 (see Fig. 150), must be used. They are deduced on the assumption that at spark-over, g_s is as given by Fig. 151.

Variations of frequency, at least up to 1,000 cycles, have no influence on the results.

As an example of the use of the foregoing, the voltage necessary to break down a 10-cm. gap between spheres 25 cm. in diameter, neither sphere being grounded, will be calculated. Assuming the barometric pressure to be 760 mm.

$$g_s = 27.2\left(1 + \frac{0.54}{\sqrt{12.5}}\right) = 31.4\,\frac{\text{kv.}}{\text{cm.}}$$

$$\frac{x}{a} = \frac{10}{12.5} = 0.8$$

$$f = 1.283 \text{ by table.}$$

$$\therefore V = \frac{31.4 \cdot 10}{1.283} = 245 \text{ kv. max.}$$

For a sinusoidal wave $V = 173$ kv. effective.

If the spheres are 25 cm. apart and one of them is grounded, the calculation is

$$g_s = 31.4\,\frac{\text{kv.}}{\text{cm.}}$$

$$\frac{x}{a} = \frac{25}{12.5} = 2$$

$$f_0 = 2, \text{ from Fig. 150.}$$

$$\therefore V = \frac{31.4 \cdot 25}{2} = 392 \text{ kv. max.}$$

For a sinusoidal wave $V = 277$ kv. effective.

Reference to Fig. 151 shows that in important work it is best to use large spheres and thus avoid the use of the steep part of the curve and the consequent uncertainty in g_s.

In practical work it is best to take the spark-over voltages from experimentally determined curves, or the table given on page 267, rather than to use the above formulæ.

The use of a spark gap is not without danger to the apparatus under test; for high voltage surges may be set up when the gap breaks down, hence the use of the current-limiting resistances.

In using any form of spark gap it is essential that all chance of accidental circuit variations be eliminated. If this is not done, the observer may be misled by the breaking down of the gap due to high-voltage oscillations. The sphere gap is especially susceptible to circuit variations; consequently all accidental spark discharges from the testing circuit must be avoided.

The following table from the Standardization Rules of the American Institute of Electrical Engineers is based on the experimental work of Peek, Chubb and Fortescue.

Sphere-gap Spark-over Voltages

(At 25°C. and 760 mm. barometric pressure)

Kilo-volts effective	Sparking distance in millimeters							
	62.5-mm. spheres		125-mm. spheres		250-mm. spheres		500-mm. spheres	
	One sphere grounded	Both spheres insulated	One sphere grounded	Both spheres insulated	One sphere grounded	Both spheres insulated	One sphere grounded	Both spheres insulated
10	4.2	4.2						
20	8.6	8.6						
30	13.5	13.5	14.1	14.1				
40	19.2	19.2	19.1	19.1				
50	25.5	25.0	24.4	24.4				
60	34.5	32.0	30.0	30.0	29	29		
70	46.0	39.5	36.0	36.0	35	35		
80	62.0	49.0	42.0	42.0	41	41	41	41
90	.....	60.5	49.0	49.0	46	45	46	45
100	.....	....	56.0	55.0	52	51	52	51
120	.....	....	79.7	71.0	64	63	63	62
140	.....	....	108.0	88.0	78	77	74	73
160	.....	....	150.0	110.0	92	90	85	83
180	.....	....	.....	138.0	109	106	97	95
200	.....	....	.....	....	128	123	108	106
220	.....	....	.....	....	150	141	120	117
240	.....	....	.....	....	177	160	133	130
260	.....	....	.....	....	210	180	148	144
280	.....	....	.....	....	250	203	163	158
300	.....	....	.....	....	.....	231	177	171
320	.....	....	.....	....	.....	265	194	187
340	.....	....	.....	....	.....	....	214	204
360	.....	....	.....	....	.....	....	234	221
380	.....	....	.....	....	.....	....	255	239
400	.....	....	.....	....	.....	....	276	257

The values in the above tables are corrected for temperature and barometric pressure as follows. To find the spacing at which it is necessary to set a gap to spark over at some re-

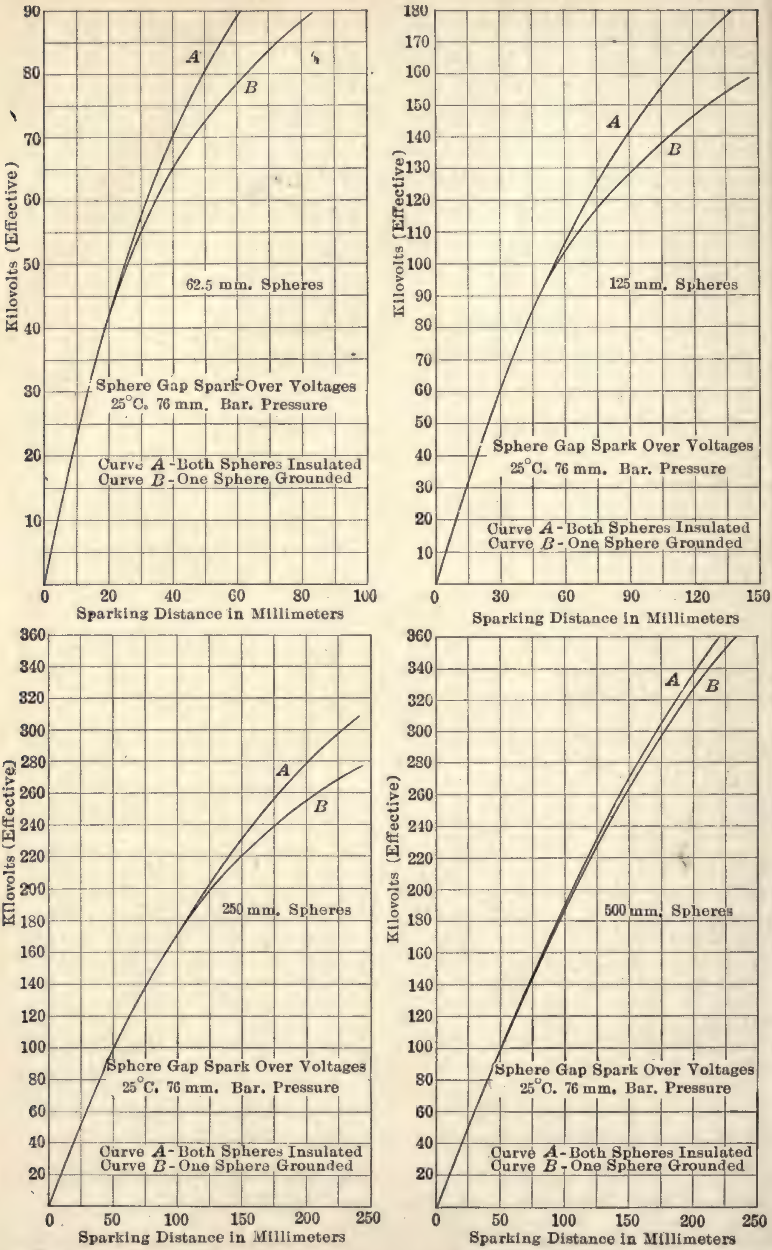


FIG. 152.—Plots of A. I. E. E. table for spark gap with spherical electrodes.

quired voltage divide the required voltage by the correction factor

$$\delta = \frac{0.392b}{273 + t}.$$

b is the barometric pressure in millimeters and t the temperature in degrees C. A new voltage is thus obtained. The spacing corresponding to this new voltage as obtained from the table is that required.

The voltage at which a given gap sparks over is found by taking the voltage corresponding to the spacing from the table and multiplying by the above correction factor.

POTENTIOMETER ARRANGEMENTS

Poggendorf Method of Comparing a Potential Difference and an Electromotive Force.—All of the various methods now used for the rapid standardization of direct-current instruments depend on the ability to compare a potential difference with an e.m.f. This may be accomplished by Poggendorf's method, which is shown diagrammatically in Fig. 153. E_1 and E_2 are the e.m.fs. of the batteries at E_1 and E_2. The cell at E_1 will of necessity have the higher e.m.f. R_1 and R_2 are two variable resistances, K a key which is normally open, and G a suitable galvanometer. The circuit of E_1 is closed through the resistance $R_1 + R_2$, across which there will be established a potential difference, P.D. Suppose the key K to be open. Then the current in R_1 is the same as that in R_2, and its value is

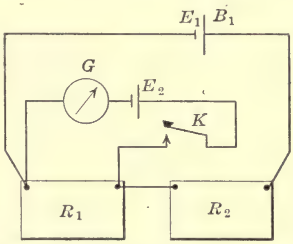


FIG. 153.—Connections for Poggendorf method of comparing a potential difference with an electromotive force.

$$I = \frac{\text{P.D.}}{R_1 + R_2}.$$

The potential difference between the ends of R_1 will consequently be

$$\frac{R_1(\text{P.D.})}{R_1 + R_2}.$$

This may be varied by altering either R_1 or R_2. The battery E_2 is so inserted that, when the key is depressed, its e.m.f. opposes the potential difference due to the passage of the current through R_1. If by varying R_1 or R_2 this potential difference is made equal to E_2, no current will flow through the galvanometer and the battery E_2 when the key is closed. Consequently the absence of a deflection of the galvanometer when the circuit is closed shows that

$$E_2 = \frac{R_1(\text{P.D.})}{R_1 + R_2}$$

or

$$\frac{E_2}{\text{P.D.}} = \frac{R_1}{R_1 + R_2}.$$

If $R_1 + R_2$ be so high that very little current flows through the battery E_2, the fall of potential in E_1, which is given by IB, where B is the battery resistance, will be so small that the P.D., which is equal to $E_1 - IB$, may be taken as equal to E_1, and

$$\frac{E_2}{E_1} = \frac{R_1}{R_1 + R_2}, \text{ very nearly,}$$

or

$$E_1 = E_2 \frac{R_1 + R_2}{R_1}.$$

The larger $R_1 + R_2$, the better the approximation.

It will be noticed that current can flow through E_2 only when the key is depressed, and that when the adjustment is perfect, there can be no current through the battery E_2. This is of importance, for if care be exercised it allows cells to be used at E_2 without danger of altering their e.m.fs. by polarization.

Much labor has been expended in the development and study of galvanic cells, suitable for use at E_2, which shall have perfectly definite e.m.fs., and consequently can be used as standard cells with which P.D. or E_1 may be compared. On account of their high degree of reproducibility the Clark and the Weston cells are now universally used. These cells are of the open-circuit type and no appreciable current can be drawn from them without temporary alteration of their e.m.f.

As it is important to avoid short-circuiting E_1 or connecting

it through a small coil which might be overheated, it is well to unplug large resistances (1,000 ohms) in R_1 and R_2 before making the final connections.

The key should not be left depressed, but released as soon as the galvanometer needle begins to move. The direction of the motion of the needle must be noted and another trial made with a different resistance in R_1. The direction of the deflection depends on whether R_1 is too large or too small.

After repeated trials such a resistance will be found that the galvanometer will not deflect.

Then

$$E_1 = E_2 \frac{R_1 + R_2}{R_1}.$$

To make the final balancing, either R_1 or R_2 may be adjusted. One must be sure that all plugs are firmly inserted and that all connections are perfect. The key is to be depressed for as short a time as possible.

If in any particular case the standard cell is higher in e.m.f. than the cell to be compared with it, the procedure must be changed, for standard cells cannot be used in closed circuits. In this case, an auxiliary battery having a higher e.m.f. than either E_1 or E_2 is used in position E_1. The cells are compared with it in succession, or else the circuit is so arranged that both cells can be compared with the auxiliary battery *at the same time*. The latter is a very accurate method, for it obviates difficulties arising from variations of the current through R_1 and R_2.

The Potentiometer.—In the section on "Calibration of Instruments" some methods of employing standard cells are discussed, the apparatus for which may be assembled in any well-appointed laboratory for electrical measurements. In general, it is much more convenient to use for the purpose pieces of commercial apparatus called potentiometers. These instruments are all more or less convenient arrangements for projecting potentials, so designed as to be direct-reading for voltages up to about 1.5 volts. The principle involved is illustrated by Fig. 154.

ab is a definite resistance along which the slider *c* can be displaced. *Rh* is a rheostat by which the current in *ab* can be

adjusted. Let r_1 be the resistance from b to c when the galvanometer deflection is zero and the standard cell is in circuit, and r_2 the reading of the slider when the galvanometer deflection is zero and the switch is on P.D., *the current in ab being as before.* It is necessary that the current I_{ab} be constant. To ascertain if this is so necessitates the throwing of the switch to E and the resetting of the slider. The potentiometer current will be

$$I_{ab} = \frac{E}{r_1}$$

and

$$\text{P.D.} = \frac{Er_2}{r_1}.$$

An obvious improvement is to tap in the standard cell at a fixed point on ab; if the e.m.fs. of all cells which are commonly

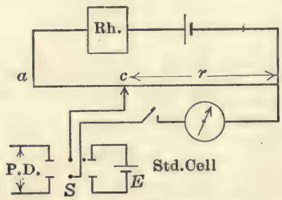


FIG. 154.—Illustrating the principle of the potentiometer.

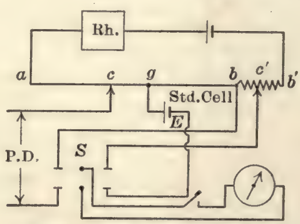


FIG. 155.—Illustrating principle of potentiometer with adjustment for standard cells of different e.m.fs.

employed at E were the same this would fix the value of I_{ab} at which the galvanometer would be in balance, but some forms of standard cells have temperature coefficients and the Weston cell in its commercial form, which is the one most frequently used, is a secondary standard, the e.m.f. of which varies slightly with different cells. The resistance between the cell terminals must therefore have a slight adjustment if I_{ab} is always to be brought to a definite value. The arrangement then becomes that shown in Fig. 155.

Each step of the resistance bb' is marked with the standard-cell e.m.f. to which it corresponds.

I_{ab} is always adjusted to some predetermined value. Therefore $E = I_{ab}r_1$ where r_1 is the resistance between c' and g. The

process is then to set c' so that the drop between the terminals of the standard-cell circuit due to the predetermined potentiometer current will be equal to the e.m.f. of the particular standard cell used. The proper value of I_{ab} is obtained by manipulating the rheostat, *Rh*. With the switch *S* thrown to the right, the correct adjustment is shown by the galvanometer remaining in balance. After this, P.D. is measured as before by adjusting *c*, the switch being thrown to the left. To test the value of I_{ab}, it is necessary to insert the galvanometer in the standard-cell circuit by the double-throw switch. This test is necessary because the battery may not be constant.

The usual range of a potentiometer is from 0 to about 1.5 volts; for higher voltages it is necessary to use it in conjunction with a volt box. This consists of a high resistance which is connected across the potential difference to be measured. It is divided by taps so that the potentiometer measures a definite fraction, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1,000}$ of P.D.

Practical Arrangement of the Potentiometer.—In order that a potentiometer may attain its highest usefulness, it must be so arranged that the value of P.D. may be read directly from the position of the slider *c*, no calculation being necessary. That this is possible is seen from the fact that whenever the instrument is used, the current I_{ab} has a definite value and therefore the drop from *b* to any point on the wire *ab* is always the same. Consequently the scale from which the position of *c* is read may be so graduated that it gives the drop in volts between *b* and the various positions of *c* directly.

Much ingenuity has been expended in arranging the resistance *ab* so that while it is brought into a small compass it is accessible at practically all points. The simplest method of doing this is shown in Fig. 156. Fifteen equal-resistance coils are used in series with a slide wire whose resistance is slightly greater than that of a single coil. The scale of the slide wire is divided into 1,100 equal parts and the resistance of the whole affair may be about 75 ohms.

The slide wire, represented by *DB*, is wound in a screw-thread on a marble cylinder 6 in. in diameter; it consists of 11 turns with a total resistance of 5.5 ohms, and is protected from dirt and mechanical injury by a movable hood mounted on a screw-thread

of the same pitch as the winding on the cylinder. The slider, which is always in contact with the wire, is carried by the hood, which at its lower edge has 100 graduations; fractions of a turn can then be read. The whole turns correspond to the divisions on the vertical scale seen at the front in the upper figure.

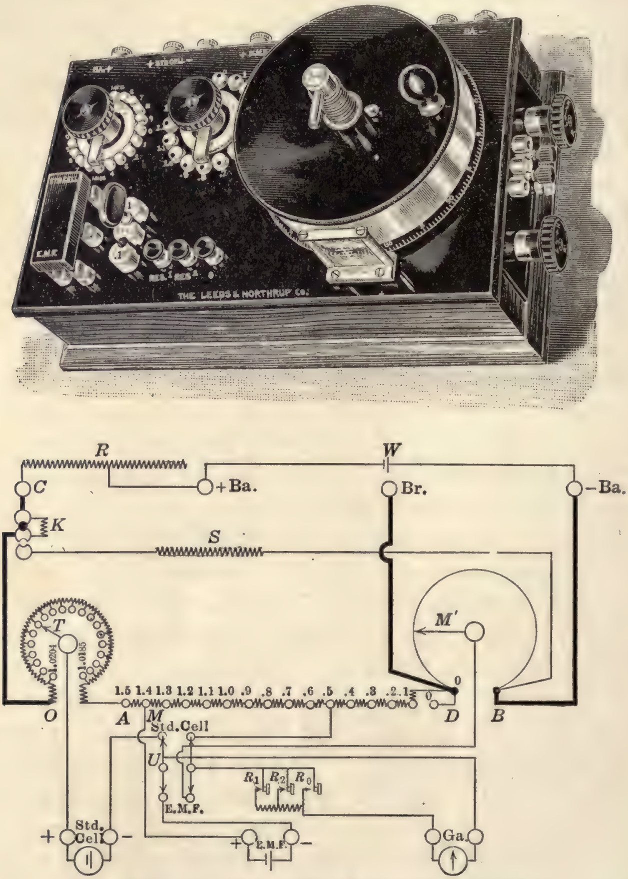


FIG. 156.—Leeds and Northrup potentiometer.

The resistance of each of the 15 coils which are in series with the slide wire, shown between D and A, is 5 ohms. The standard potentiometer current is $\frac{1}{50}$ amp., thus making the drop in each coil and in the slide wire 0.1 volt and the total drop from

B to A, 1.61 volts. One division on the vertical scale, corresponding to one turn, is therefore equivalent to 0.01 volt, and one division on the hood to 0.0001 volt.

The instrument is designed for use with the commercial Weston cell, which has no appreciable temperature coefficient, so no temperature adjustment is provided. Such cells may, however, differ from one another slightly in e.m.f. (a few ten thousandths of a volt), and each is accompanied by a certificate giving its true voltage. In order that the instrument may be conveniently used with different cells, the rheostat between O and A is provided. The standard cell is connected between the movable arm T and the point $0^v.5$. The resistance of each coil of OA is 0.005 ohm; so with the standard current of $\frac{1}{50}$ amp. flowing, the drop through each is 0.0001 volt. There are 19 of these coils; consequently, cells of e.m.fs. from 1.0170 to 1.0189 volts can be used, the arm T being set to correspond to the particular cell used.

By means of a double-throw switch the galvanometer may be quickly transferred from the standard-cell circuit to that marked E.M.F; as the two circuits are entirely distinct, no resetting of the instrument is necessary when checking the potentiometer current. This is a very great convenience.

The process of making a measurement is to set the double-throw switch on the point marked "Standard Cell," and vary the rheostat to get zero deflection; this adjusts the potentiometer current to its standard value. Then throw the switch to "E.M.F." and balance, using the voltage slides without altering the rheostats, and read off V directly in volts. To check the potentiometer current, simply throw the switch to "Standard Cell" and press the key. No resetting is necessary.

For satisfactory action it is necessary to apply a little vaseline to the slide wire occasionally.

Low-scale Arrangement.—If the current through BA be reduced to $\frac{1}{500}$ amp., the drop through each coil and through the slide wire becomes $\frac{1}{100}$ volt, and the entire range of the potentiometer is 0.16 volt. By removing the plug at K from socket 1 to socket 0.1 the resistance OB is shunted by S, which is of such a value that one-tenth the total current flows through OB.

The resistance K is so adjusted that this total current is kept at 0.02 amp.

In using any form of potentiometer it is absolutely necessary to check the potentiometer current before taking a reading.

Wolff Potentiometer.—Referring to Fig. 157, the battery current from B flows through all the coils marked $\times 100$, then to

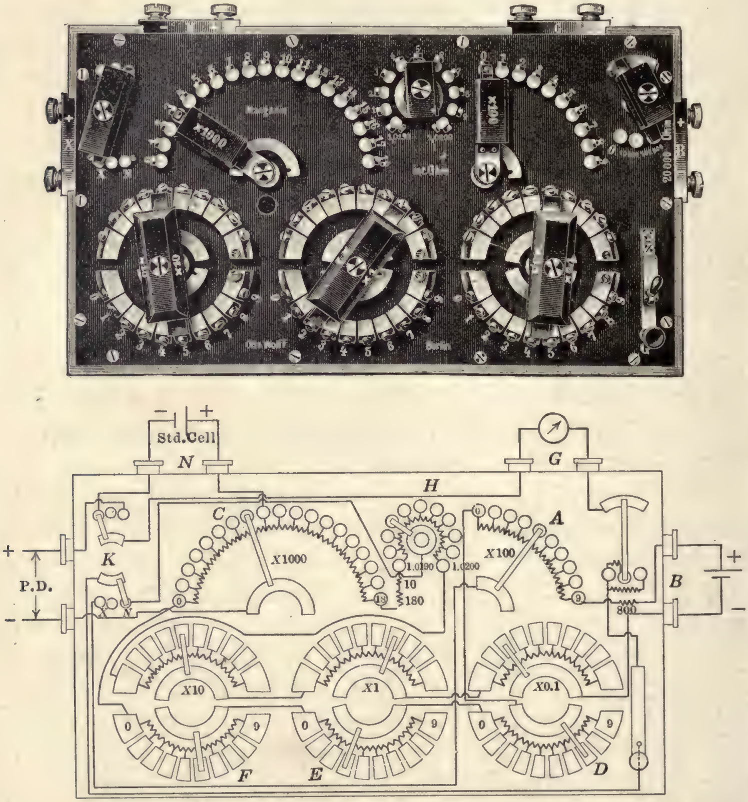


FIG. 157.—Wolff potentiometer.

the lower group marked $\times 0.1$ where it traverses the coils to the left of the contact D and on to the group $\times 1$, traversing the coils to the left of the contact E, thence to the coils to the left of the contact F in the group $\times 10$, and on through all the coils in the group $\times 1{,}000$.

The function of the upper sets of coils in the groups marked $\times 10$, $\times 1$ and $\times 0.1$, which are connected in series with those in the lower or measuring sets, is to maintain the potentiometer current at a fixed value irrespective of the position of the contacts D, E, F. The contacts on the upper and lower sets of these resistances are rigidly connected so that if a coil is removed from the lower set an equal coil is added in the upper set.

When the switch K is on X, the derived circuit containing the unknown P.D. is connected between A and C *via* the galvanometer. By manipulating the switches the resistance between A and C may be varied from 0 to 18,999.9 so if the current be kept constant at 0.0001 amp., any P.D. between 0 and 1.89999 volts may be balanced.

The standard-cell circuit is connected between the eighth and ninth coils in group $\times 1{,}000$ and the contact at H; by moving this contact the resistance between the terminals of the standard-cell circuit may be varied from 10,190 ohms to 10,200 ohms in 1-ohm steps. The instrument may thus be adjusted so that standard cells having e.m.fs. from 1.0190 to 1.0200 volts may be employed. To check the potentiometer current it is necessary merely to throw the switch K to the position marked N and to depress the key.

The instrument is also made in a low-resistance form ($14\frac{1}{2}$ ohms) which is suitable for thermo-electromotive force determinations such as are necessary in pyrometry.

The Brooks Deflection Potentiometer[9].—The potentiometers thus far described are read by the null method, an exact balance being obtained between the potential difference in the instrument, due to the potentiometer current, and the potential difference to be measured. The objection to this method, is that while it gives results of the highest precision, the P.D. to be measured must be steady, and repeated trials have to be made before the balance point is obtained. When the unknown potential difference is not steady, many trials must be made before the null point is hit upon by mere chance, and the expenditure of time and patience becomes so great as to be almost prohibitive in much commercial work.

In the case of a large electrical engineering laboratory, where many instruments must be checked and kept in adjustment, it

is imperative that the work be done with great speed, combined with the accuracy necessary in engineering work.

The Brooks potentiometer was designed with this in mind. By it, results may be obtained even though the P.D. under measurement is not perfectly steady. In this instrument no

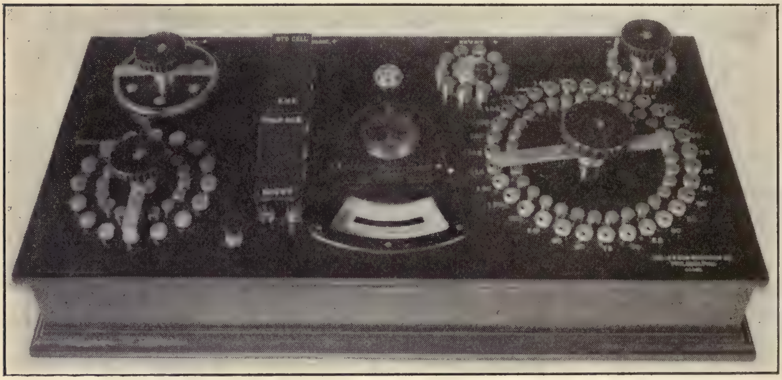


FIG. 158.—Brooks deflectional potentiometer.

attempt is made to obtain an exact balance; the slides are set so near to the null point that the galvanometer deflection is small. The galvanometer is so graduated that it gives the amount that must be added to the reading of the slides in order to obtain the unknown P.D.

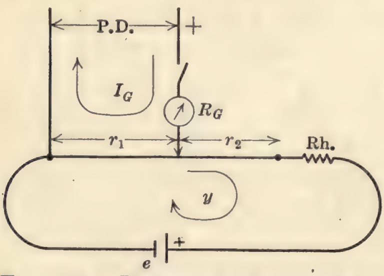


FIG. 159.—Diagram for Brooks deflectional potentiometer, Case I.

That certain conditions must be fulfilled may be seen from the following discussion.

In general the potentiometer is used:—

I. To determine potential differences which are within the normal range of the instrument.

II. To determine, by the use of a volt box, potential differences which are above the normal range of the instrument.

III. To measure currents by the use of shunts.

CASE I. DIRECT MEASUREMENT OF P.D.

A storage cell is used at e (Fig. 159) so its resistance may be neglected.

R_G is the resistance of the galvanometer plus any resistance placed directly in series with the instrument.

The mesh equations are:

$$I_G(R_G + r_1) - yr_1 - \text{P.D.} = 0;$$

$$y(r_1 + r_2 + Rh) - I_G r_1 + e = 0.$$

$$\therefore\ I_G = \frac{\text{P.D.} - \dfrac{er_1}{r_1 + r_2 + Rh}}{R_G + \dfrac{r_1(r_2 + Rh)}{r_1 + r_2 + Rh}}.$$

The standard potentiometer current when it has been adjusted, as in the ordinary potentiometer, by bringing the galvanometer to zero with the standard cell in circuit, is $\dfrac{e}{r_1 + r_2 + Rh}$; therefore $\dfrac{er_1}{r_1 + r_2 + Rh}$ is the graduation marked on the slides of the instrument.

If the null method be employed,

$$\frac{er_1}{r_1 + r_2 + Rh} = \text{P.D.} = \text{reading on slides.}$$

The instrument being in exact balance, suppose that the applied potential difference, P.D., is increased by a small amount $\delta[\text{P.D.}]$, so that it becomes P.D. + $\delta[\text{P.D.}]$, the slides being kept as they were. A current will now flow through the galvanometer and its value will be

$$I_G = \frac{\delta[\text{P.D.}]}{R_G + \dfrac{r_1(r_2 + Rh)}{r_1 + r_2 + Rh}} \tag{14}$$

This shows that in measuring any potential difference, the major portion of it may be read from the slides as usual, and to this may be added the voltage $I_G\left(R_G + \dfrac{r_1(r_2 + Rh)}{r_1 + r_2 + Rh}\right)$ in order to obtain the total value.

The quantity $\left(R_G + \dfrac{r_1(r_2 + Rh)}{r_1 + r_2 + Rh}\right)$ is the total resistance of the galvanometer circuit, with P.D. and e short-circuited. If *this resistance be kept constant for all positions of the slides and*

the adjusting rheostat, and a D'Arsonval galvanometer be used, the galvanometer scale may be graduated to read δ[P.D.] directly in volts. In order that this graduation, as a voltmeter, may be correct for all three of the cases mentioned above, the resistance of the galvanometer circuit must always be kept the same, irrespective of the use to which the potentiometer is put. This is the cardinal point in the design of the deflection potentiometer, and is attained by an ingenious arrangement of coils and switches.

Case II. Potential Difference by Use of Volt Box

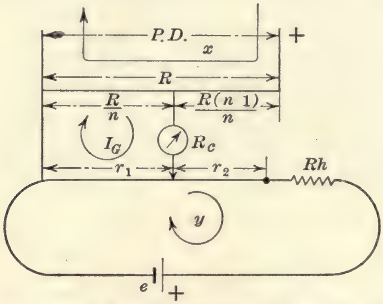


Fig. 160.—Diagram for Brooks deflectional potentiometer Case II.

The mesh equations are:

$$xR - I_G\frac{R}{n} - \text{P.D.} = 0;$$

$$I_G\left(\frac{R}{n} + R_G + r_1\right) - x\frac{R}{n} - yr_1 = 0;$$

$$y(r_1 + r_2 + Rh) - I_G r_1 + e = 0.$$

$$\therefore\ I_G = \frac{\dfrac{\text{P.D.}}{n} - \dfrac{er_1}{r_1 + r_2 + Rh}}{\dfrac{R(n-1)}{n^2} + R_G + \dfrac{r_1(r_2 + Rh)}{r_1 + r_2 + Rh}}.$$

If the slides be set so that I_G is zero, then

$$\frac{er_1}{r_1 + r_2 + Rh} = \frac{\text{P.D.}}{n} = \text{reading on slides.}$$

If now the potential difference be increased by a small amount,

δ[P.D.], the slides remaining fixed, a current will flow through the galvanometer

$$I_G = \frac{\dfrac{\delta[\text{P.D.}]}{n}}{\dfrac{R(n-1)}{n^2} + R_G + \dfrac{r_1(r_2 + Rh)}{r_1 + r_2 + Rh}}.$$

The total potential difference is, therefore, n times the sum of the reading of the slides and the reading of the galvanometer in volts, *provided* the coils be so arranged that

$$\left(\frac{R(n-1)}{n^2} + R_G + \frac{r_1(r_2 + Rh)}{r_1 + r_2 + Rh}\right)$$

is a constant for all settings of the slides, and the galvanometer has been calibrated as a voltmeter with this resistance. The expression in parenthesis is the total resistance of the galvanometer circuit when e and the volt box are short-circuited.

CASE III. CURRENT MEASUREMENT

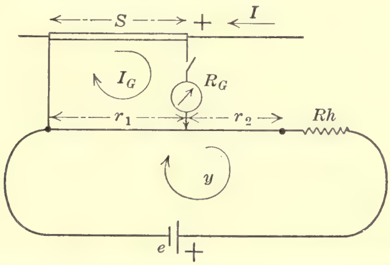


FIG. 161.—Diagram for Brooks deflectional potentiometer Case III.

The mesh equations are:

$$I_G(S + R_G + r_1) - yr_1 - IS = 0;$$
$$y(r_1 + r_2 + Rh) - I_G r_1 + e = 0.$$

$$\therefore I_G = \frac{IS - \dfrac{er_1}{r_1 + r_2 + Rh}}{S + R_G + \dfrac{r_1(r_2 + Rh)}{r_1 + r_2 + Rh}}.$$

If the current be increased by a small amount $\delta[I]$ from that

necessary for an exact balance, the slides remaining fixed, the galvanometer current will be

$$I_G = \frac{\delta[I]S}{S + R_G + \dfrac{r_1(r + R_2h)}{r_1 + r_2 + Rh}}$$

or

$$\delta[I]S = I_G \left(S + R_G + \frac{r_1(r_2 + Rh)}{r_1 + r_2 + Rh}\right).$$

The expression in parenthesis is the resistance of the galvanometer circuit with e short-circuited and the main current circuit open beyond the shunt. The quantity to be added to the reading of the slides before dividing by S is seen to be the galvanometer reading reduced to volts, *provided* the switches and coils are so arranged that

$$\left(S + R_G + \frac{r_1(r_2 + Rh)}{r_1 + r_2 + Rh}\right)$$

has a fixed value.

To obtain the greatest speed of working, the resistance of the galvanometer circuit should be such that the galvanometer is critically damped. The free period of the instrument should be about 1 or 2 sec. The arrangement of the potentiometer is shown diagrammatically in Fig. 162.

To assist in attaining the necessary constant resistance in the galvanometer circuit, the rheostat for controlling the potentiometer current is arranged in two parts, r_3 and r_6, the potentiometer wire being connected to the slider which joins the two sections; the coils are so chosen that the parallel resistance of the active portions of r_3 and r_6 is fixed, thus keeping the rheostat resistance between A and B (with e short-circuited) constant. The rheostat marked $0.5\,\Omega$ is for the fine adjustment of the potentiometer current; the corresponding ballast coils in the galvanometer circuit are marked $0.3\,\Omega$. To correct for changes in resistance due to displacing the contact point along the potentiometer wire, the ballast resistance r_4 is added, the coils in it being given the proper values. As some of the coils at the upper and lower ends of the series have the same values, the number of coils required is reduced by cross-connecting, as shown in the lower figure. The volt box employed, shown in Fig. 163, has ballast

coils at C, which are in series with the connection to the potentiometer proper. When currents are measured, ballast coils are

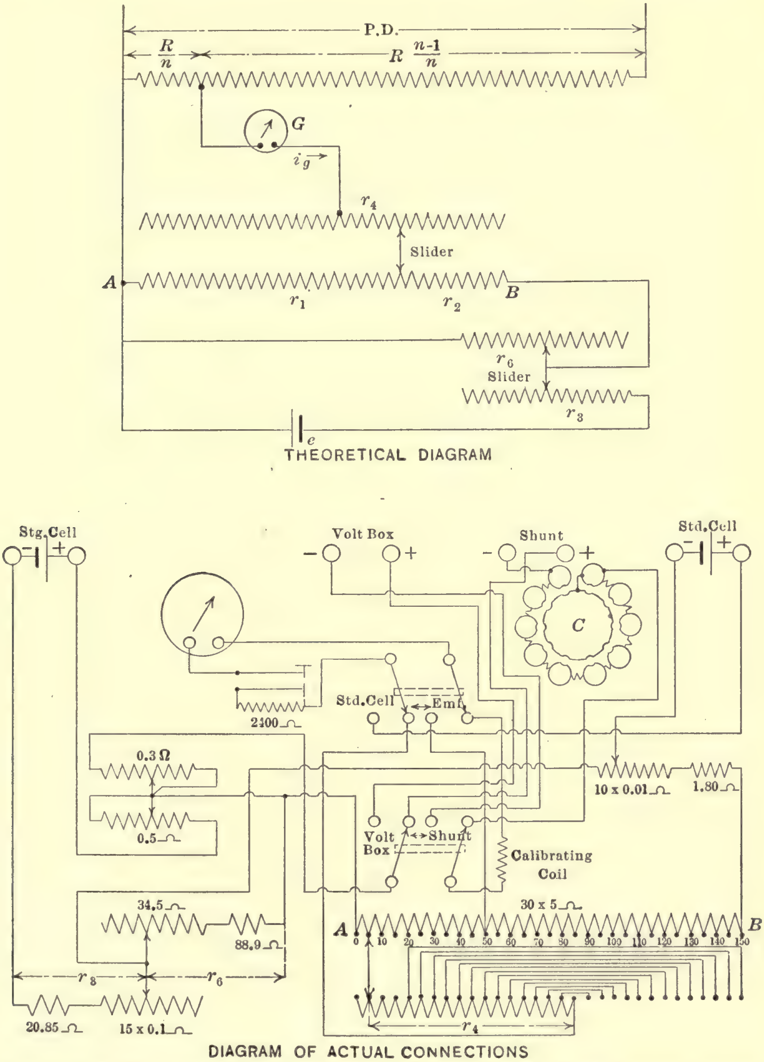


FIG. 162.—Diagram of connections of Brooks deflectional potentiometer.

also used. They are mounted in the case of the instrument, and are shown at C in Fig. 162.

It is obvious that a deflection potentiometer must be used with the particular volt box and shunts for which it was designed.

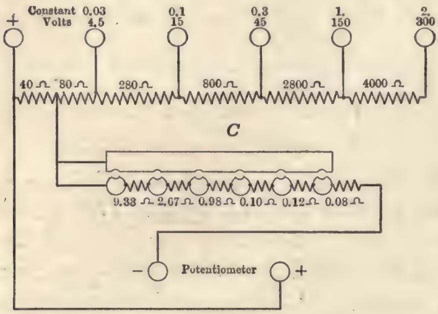


FIG. 163.—Volt box for Brooks deflectional potentiometer.

The "Thermokraftfrei" Potentiometer[10].—In the measurement of small potential differences such as those of thermocouples a low resistance potentiometer is used, and it is necessary to eliminate all thermo-electric disturbances due to contact of dissimiliar materials and inequalities of temperature in the potentiometer itself; therefore the metal employed in the coils and in their terminals must be carefully selected with this result in view. For the same reason, the design should be such that the effect of thermo-electromotive forces introduced by the manipulation of the necessary sliding contacts and switches will be reduced to a minimum. This is accomplished in the instrument under discussion, which includes elements of design due to H. Hausrath, W. P. White and H. Diesselhorst. The potentiometers previously discussed have been series arrangements, that is, the compensating potential difference is the *sum* of the potential differences existing between the terminals of various groups of coils of which the circuit is composed. Hausrath suggested the use of a divided circuit in place of coils in series. In that case the compensating potential difference is due to the *difference* of the potential drops along the two branches measured from the point where the current enters the apparatus. Fig. 164 shows in diagram the arrangement of the coils.

The potentiometer current is kept at its standard value, 0.001 amp., in the usual manner by equating the drop in cg to

the e.m.f. of the standard cell; Rh is the regulating rheostat; S_1 is a reversing switch in the main circuit.

The current entering at T_{I} divides, the resistances being so arranged that ten-elevenths of the total current flows to the left

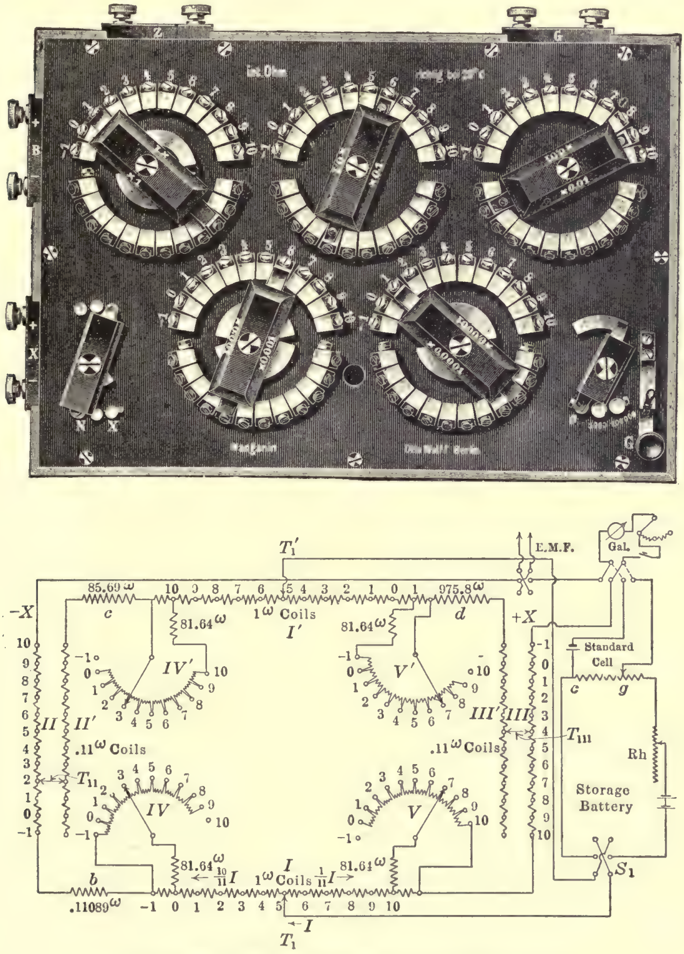


FIG. 164.—Thermokraftfrei potentiometer.

and one-eleventh to the right. The design is such that these relative values are maintained for all positions of the switches.

Each coil in the decade I and in the group of compensating

coils I′ has a resistance of 1 ohm, and the sliding terminals T_{I} and T'_{I} are mechanically connected so that if T_{I} is shifted to the right, T'_{I} is shifted an equal number of coils to the left. The resistances of both paths between the terminals T_{I} and T'_{I} are thus kept constant.

As all the coils in decades II and III and all the compensating coils in II′ and III′ are alike, the resistances of both the left- and the right-hand branches of the circuit are independent of the positions of T_{II} and T_{III}.

The two sliding contacts on decades IV and the compensating coils IV′ are mechanically connected so that they must move together, as are the two on decade V and its compensating coils V′. If the contact in decade IV is set on a terminal having a certain number, then automatically the contact in group IV′ is set on the terminal of the same number, and similarly with V and V′. The group of coils IV, consists of a 1-ohm coil (−1.0 in group I), shunted by a variable resistance which consists of a fixed portion, 81.64ω, and a variable part included between −1 and the position of the sliding contact. The coils in the variable portion of IV are

Between	Ohms	Between	Ohms
−1 and 0	8.264	4 and 5	30.30
0 and 1	10.101	5 and 6	45.41
1 and 2	12.626	6 and 7	75.80
2 and 3	16.234	7 and 8	151.51
3 and 4	21.645	8 and 9	454.54
		9 and 10	∞

In the group IV′ the order is reversed, that is, the coil having a resistance of 8.264 ohms is between contacts 9 and 10. The resistances are given these particular values in order that, when the contact in IV is moved one number, the resistance between T_{I} and T_{II} may always be altered by a definite amount, 0.0011 ohm, which is 1/100 of the alteration which would be obtained by moving T_{II} one number. For instance, if the decade contact is on number 4, the resistance of group IV is

$$R_{\text{IV}} = \frac{1 \times 150.51}{1 + 150.51} = 0.99342.$$

When it is set on number 5, this becomes

$$R_{\text{IV}} = \frac{1 \times 180.81}{1 + 180.81} = 0.99452.$$

The difference of these two values is 0.0011 ohm while the value of a single step in II is 0.11 ohm.

The minimum value of R_{IV} is

$$R_{IV} = \frac{1 \times 89.904}{1 + 89.904} = 0.9890$$

so the general value is

$$R_{IV} = 0.9890 + 0.0011 \times n_{IV}$$

where n_{IV} is the number of the contact on dial IV.

Similarly the general value of $R_{IV'}$ is

$$R_{IV'} = 0.9989 - 0.0011 \times n_{IV'}.$$

In decade V the coil of 8.264 ohms is between contacts 9 and 10; in the group of compensating coils, V', it is between -1 and 0, so, in general,

$$R_{V} = 0.9989 - 0.0011 n_{V}$$

$$R_{V'} = 0.9890 + 0.0011 n_{V'}$$

The coils b, c, d, have such values that taken in conjunction with the other resistances, they divide the current flowing in at T_{I} in the ratio 10 to 1.

The voltage between T_{II} and T_{III} is the difference of the ohmic drops measured from T_{I}, so for any setting (potentiometer current 0.001 amp.)

$$\begin{aligned}\text{P.D.} = \frac{10}{11} \times 0.001 \Big[n_{I} \times 1 + 0.9890 + 0.0011 n_{IV} \cdot \\ + 0.11089 + 0.11(n_{II} + 1) \Big] \\ - \frac{1}{11} \times 0.001 \Big[(10 - n_{I})1 + 0.9989 - 0.0011 n_{V} \\ + 0.11(10 - n_{III}) \Big] \\ = 0.001 \left[n_{I} + \frac{n_{II}}{10} + \frac{n_{III}}{100} + \frac{n_{IV}}{1{,}000} + \frac{n_{V}}{10{,}000} \right].\end{aligned}$$

Therefore when the dials are properly graduated the unknown P.D. is measured by the sum of the dial readings, as in the usual instruments.

A study of the network shows that if the battery circuit is open, the resistance between the galvanometer terminals $+X$

and $-X$ is, to a good degree of approximation, 14.35 ohms, and when the battery circuit is closed through a series resistance, B, which is external to the potentiometer the resistance becomes, using a second approximation,

$$14.35 - \frac{n_1^2}{B + R} \tag{18}$$

where R is the resistance of the potentiometer between T_{I} and T_1' with the galvanometer circuit open.

The resistance of the right-hand path between T_{I} and T'_{I} is 990 ohms; that of the left-hand path, 99 ohms; the resistance of the whole apparatus is therefore $R = 90$ ohms.

If a storage cell is used and the potentiometer current is 0.001 amp., $R + B$ must be approximately 2,000ω; therefore, the maximum variation in the resistance of the galvanometer circuit will be only 0.05 ohm or about 0.3 per cent. This constancy of the resistance allows one to obtain the last figure in the P.D. under measurement, by the deflection method, the reading of the last decade, n_{V}, being kept at zero. By properly setting up the apparatus, the full reading of the last decade, $n_{\text{V}} = 10$, may be made to correspond to 1, 10, or 100 divisions on the galvanometer scale and the necessity for exact balancing may thus be obviated. Where the P.D. to be measured is fluctuating slightly, this is a decided advantage (see page 277, "Brooks Deflection Potentiometer").

Another advantage, if a moving-coil galvanometer is used, is that the damping remains constant, irrespective of the setting of the potentiometer.

Effect of Thermo-electromotive Forces.—The magnitude of the e.m.f. set up by manipulating any switch is less than 10^{-6} of a volt.

Any e.m.fs. arising from manipulating I and I' are added to the battery e.m.f. (2 volts) and will be negligible. The effect of a thermo-electromotive force of magnitude ϵ, due to moving contact II, will be very small.

Referring to Fig. 165, by Kirchoff's laws the current in the left-hand branch, if I is the total battery current coming to the potentiometer, is

$$I'_L = \frac{I\,(\beta + \delta) \mp \epsilon}{\alpha + \beta + \gamma + \delta}$$

and in the right-hand branch

$$I'_R = \frac{I(\alpha + \gamma) \pm \epsilon}{\alpha + \beta + \gamma + \delta}.$$

Therefore, the change in the P.D. between the terminals $+X$ and $-X$ due to $\pm\ \epsilon$ will be

$$\frac{\pm\ \epsilon\,(\gamma + \delta)}{\alpha + \beta + \gamma + \delta}.$$

The maximum value of $\gamma + \delta$ is 14.42 ohms and the value of $\alpha + \beta + \gamma + \delta$ is 1,089 ohms, so

$$\frac{\epsilon\,(\gamma + \delta)}{\alpha + \beta + \gamma + \delta} = \epsilon \times 0.013$$

∴ the error introduced is only 1.3 per cent of ϵ.

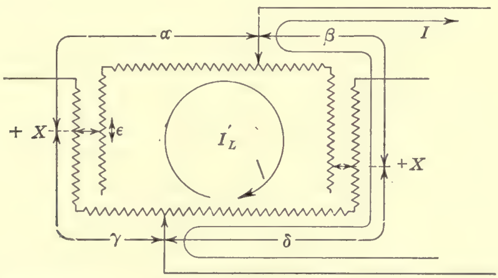


FIG. 165.—Pertaining to effect of thermal e.m.f. in thermokraftfrei potentiometer.

Similar considerations show that the error introduced by manipulating IV and V is only about 1.2 per cent. of ϵ. These errors having been reduced to negligible amounts, the apparatus is said to be free from thermo-electromotive forces.

Application of the Potentiometer to Alternating-current Measurements.—The principle involved in the potentiometer was applied to alternating-current measurements many years ago, but at that time the necessary adjunct of a convenient phase-shifting device had not been developed by Drysdale, and recourse was had to an arrangement of two small dynamos on the same shaft, one of which could be displaced in phase with reference to the other. The arrangement, while satisfactory in many ways, is much less convenient and more costly than the phase-shafting transformer now used. The development of the alternating-

current potentiometer as a distinct instrument is due to C. V. Drysdale, whose instrument is shown diagrammatically in Fig. 167.

In applying the potentiometer principle to alternating-current measurements it is obvious that to balance two potential differences at every instant, they must be of the same frequency, the same wave form, and in the same time phase. The first two conditions demand that the potentiometer current be derived, through a suitable transformer, from the same source as the current to be measured. The third implies the use of some form of phase-shifting device. In addition there must be some means of insuring that the potentiometer current, when it is alternating, is of such a magnitude that the r.m.s. value of the potential difference between the terminals of each of the coils of the instrument is given by the potentiometer scale. As the coils are wound non-inductively, this may be accomplished if the potentiometer current be measured by an electrodynamometer of the astatic form. Such an instrument gives r.m.s. values and is equally accurate on direct and alternating current circuits; therefore it is very readily calibrated.

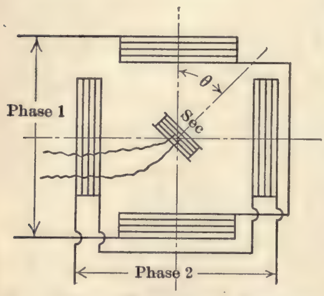


Fig. 166.—Theoretical diagram for Drysdale phase shifter.

The Drysdale Phase Shifter.—The principle involved in the Drysdale phase-shifting transformer may be illustrated by the following ideal arrangement of the apparatus (Fig. 166). The two sets of coils are of equal magnetic strength and may have their axes at right angles, in which case they are energized by currents in quadrature, as from the two phases of a two-phase circuit. The secondary is so mounted that it may be turned by hand and clamped in position; thus θ may be given any desired value. Let the coils in phase 2 be traversed by a current $I \sin \omega t$, and the coils in phase 1 by a current 90° out of phase with the first, or $I \cos \omega t$. The rectangular components of the resultant field at the center are

$$x = H \sin \omega t;$$
$$y = H \cos \omega t.$$

The resultant is

$$R = \sqrt{x^2 + y^2} = H\sqrt{\sin^2 \omega t + \cos^2 \omega t} = H, \text{ a constant.}$$

At any instant the tangent of the inclination of this resultant to the vertical axis is $\frac{x}{y}$;

$$\tan \gamma = \frac{x}{y} = \tan \omega t;$$

$$\therefore \gamma = \omega t.$$

Therefore the resultant field at the center is of constant magnitude and revolves with a constant angular velocity.

The flux threading the secondary coil is proportional to cos $(\omega t - \theta)$ and the induced e.m.f. to sin $(\omega t - \theta)$, so the time-phase displacement of the induced e.m.f. is equal to the angular displacement of the secondary from its zero position.

In the actual construction of the phase shifter the four stationary coils are replaced by stator windings of the induction-motor type; the secondary is wound on an iron core. As built by some makers the apparatus has the fault, serious in some methods of measurement, that change of phase is accompanied by an alteration of secondary e.m.f. This is obviated in Dr Drysdale's own design by a proper arrangement of the windings. If the currents supplied to the phase shifter are not sinusoidal the wave form in the secondary will depend on the value of θ. The device may be wound so as to operate on either two- or three-phase circuits, or it may be arranged to be operated on a single-phase circuit by means of a phase-splitting device. The phase shifter or its equivalent is a necessary adjunct of the alternating-current potentiometer.

The Drysdale-Tinsley Alternating-current Potentiometer.—The Drysdale-Tinsley alternating-current potentiometer is shown diagrammatically in Fig. 167. It consists of a regular Tinsley potentiometer, such as is used for direct-current work (included in the dotted rectangle), supplemented by the electrodynamometer necessary for the measurement of the potentiometer current; a selector switch, for quickly transferring the instrument from one set of terminals to another; a connection bo ard, by which

the potentiometer is attached to the outside circuits; a change-over switch, for quickly substituting alternating for direct current; and the phase shifter; this last device is best operated from a single-phase circuit by means of a phase-splitting condenser and resistance. This arrangement is best because any single-phase supply, of good wave form, can be used and all doubt as to the exact quadrature of the two phases eliminated.

The phases can be adjusted to within 0°.1, the procedure being as follows: Join the 100-volt supply to the two terminals marked Phase 1 on the phase-shifting transformer. The condenser and

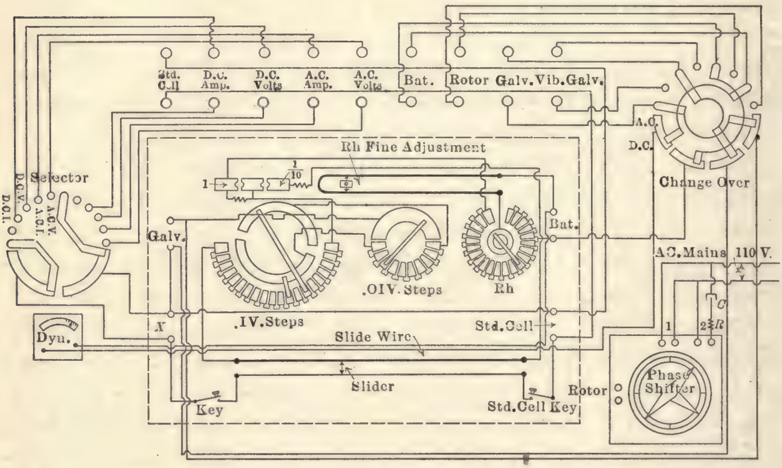


FIG. 167.—Diagram for Drysdale-Tinsley alternating- and direct-current potentiometer.

resistance, in series with the terminals marked Phase 2, are also connected across the 100-volt supply. The secondary is then turned by the tangent screw until the pointer marked AXIS, is at 0° on the dial. By means of the rheostat of the potentiometer, the current is adjusted until the dynamometer reads exactly 50. Then the tangent screw is turned until the AXIS pointer is at 45° *leading*. If the dynamometer reads higher or lower than 50 the *capacity* is altered until the reading is correct. The AXIS pointer is then turned to 90° and the *resistance* altered

until 50 is again registered. After this the dynamometer should remain exactly at 50, while the secondary is turned through 180°; if it does not so remain the process is repeated until it does remain at 50 for all positions of the secondary.

In measuring currents or e.m.fs. it is not necessary to split the phase with great accuracy, although it is more convenient to do so; but the utmost care must be taken in doing this when vector diagrams are being constructed or when an accurate knowledge of phase angles is required.

Above all things, it is necessary that the supply voltage and frequency remain perfectly steady.

After the adjustment of the phase splitter, the first step in using the instrument is to calibrate the electrodynamometer at the reading corresponding to the standard potentiometer current. To do this the battery is used as a source and the adjustment made as with the ordinary potentiometer. The Tinsley instrument has no separate standard-cell tap, so it is necessary to set the slides at the voltage of the cell and then adjust the rheostat; this process must be repeated whenever the standard potentiometer current is checked. When the galvanometer is in balance the reading of the dynamometer is taken; if the instrument is not astatic, reversed readings must be taken and the two results averaged. By means of the change-over switch, alternating is substituted for direct current and the reading brought to the same value and held there. Then the graduation in volts on the potentiometer scale gives r.m.s. values of the P.D. First the phase of the potentiometer current is roughly adjusted; then the unknown P.D. is balanced as nearly as possible by the potentiometer slides. The balance is then improved by shifting the phase of the potentiometer current and still further improved by resetting the slides; thus, by a process of double adjustment, the vibration galvanometer which is used as the detector is brought to rest. As the vibration galvanometer is a tuned instrument which responds freely to currents of only one frequency, *the periodicity of the supply current must be kept constant* if the sensitivity of the potentiometer is to be maintained. With any potentiometer the deflection of the detector is dependent on the difference of the two potential differences which are being balanced. As the vibration galvanometer which

is used as a detector is tuned to the fundamental frequency of the circuit a balance indicates that the values of the fundamentals and not the mean square values of the P.D.'s are equal. If the wave forms are very bad the vibration galvanometer may be forced to vibrate in other than its natural period, in which case an exact balance cannot be obtained. It is seen that sinusoidal currents are necessary for the successful operation of the alternating-current potentiometer.

STANDARD CELLS

In order to realize and maintain the international volt in such a manner that it will be a practical unit, easily applied for purposes of measurement, recourse must be had to some form of galvanic cell. Reference to the section on the legal definitions of the electrical units will show that in the Act of Congress approved July 12, 1894, the Clark normal cell was mentioned and this act is still in force. At that time this cell was the only one that had been carefully investigated and shown to have the necessary characteristics of *reproducibility* and *permanence.* It has since been shown that the Weston normal cell is very nearly as reproducible and more permanent; it possesses the practical advantage of having a much smaller temperature coefficient, only about one-twentieth of that of the Clark cell.

The Weston normal cell was recommended by the London Conference on Electrical Units and Standards, 1908, for use in voltage and current measurements and was adopted by the Bureau of Standards as the working standard for the United States on Jan. 1, 1911.[14]

The Clark Cell.—This cell, the invention of Latimer Clark, was described by him and recommended as a standard of electromotive force in a paper read before the Royal Society.[12] The cell consists of a zinc electrode in a neutral, saturated zinc sulphate solution opposed to a mercury electrode covered with a paste consisting of mercurous sulphate and zinc sulphate in saturated zinc sulphate solution and containing finely divided mercury. The function of the mercurous sulphate is that of a depolarizer.

Board of Trade Cell.—Little careful work was done on the Clark cell until 1885, when Lord Rayleigh investigated a form of cell practically similar to that shown in Fig. 168, which is the Board of Trade cell of 1894. Rayleigh found that the e.m.f. of the cell at any temperature could be expressed by the formula,

$$E_{t^\circ} = E_{15^\circ}(1 - 0.00077[t^\circ - 15^\circ]);$$

and gave 1.434 as the value of the e.m.f. at 15°; subsequent investigation has shown that this figure is too high by nearly 0.1 per cent.

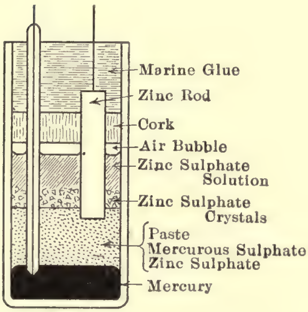


FIG. 168.—Board of Trade standard Clark cell, 1894.

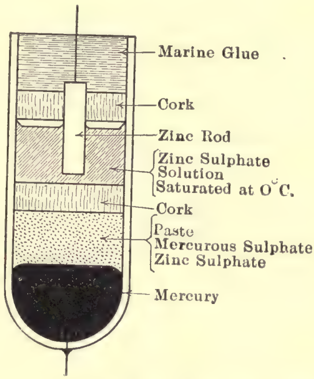


FIG. 169.—Carhart-Clark standard cell. Used as a working standard before the introduction of the Weston cell.

Rayleigh showed that variations in the e.m.f. were due to impurities in the materials, and, in this form of cell, to the fact that the zinc was so placed that it was not covered by zinc sulphate solution of uniform density; this greatly retarded the response of the e.m.f. to a change of temperature. The first effect of a decrease in temperature would be to cause crystallization; this requires time, consequently the density in the neighborhood of the crystals lags behind the temperature change and a still longer time must elapse before the solution becomes uniform by diffusion. This accounts for the lack of concordance in the early values of the temperature coefficient. This lag in the e.m.f. becomes greater as the cells grow older, especially if they are

kept at a uniform temperature and not disturbed, for then the zinc sulphate crystals gradually unite in a compact mass.

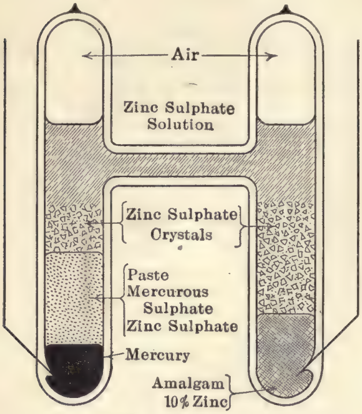


FIG. 170.—H form of Clark standard cell.

The H Cell.—A much more satisfactory cell, designed by Lord Rayleigh and known as the H form, is shown in Fig. 170 which shows an hermetically sealed cell; the zinc is used in the form of an amalgam, and the platinum terminals are amalgamated to prevent accidental contact with the electrolyte.

This cell has been thoroughly studied by Kahle, and later in this country by Wolff and Waters. The latest results show that with skilled manipulation

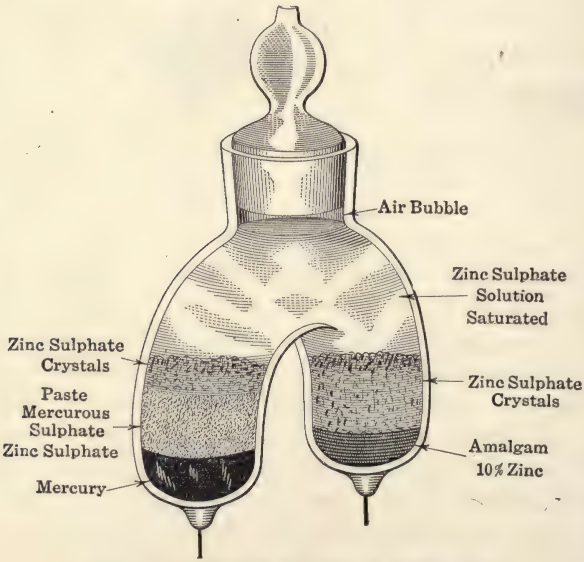


FIG. 171.—Kahle H form of Clark cell.

it is reproducible to within a few parts in 100,000. This cell is

much more permanent than the Board of Trade form and has a much smaller lag error—only about one-fourth as great. Local actions arising from differences in density are also avoided.

The form of cell mentioned in the specifications prepared by the National Academy of Sciences in compliance with the Act of Congress dated July 12, 1894, was developed at the Reichsanstalt, by Kahle, and is shown in Fig. 171.

The value of the e.m.f. at 15°C. stated in the Act of 1894 is 1.434, but subsequent investigation has shown that the value really is 1.4328 international volts.

The H form of Clark cell has the disadvantage of short life, for the glass is likely to crack where the platinum electrode is fused in at the amalgam terminal.

Materials Used in Standard Cells.—In order that the e.m.f. of any form of primary standard cell may accord with the stated value, great care must be exercised in the preparation of the materials, the processes for which have been carefully worked out by Kahle and later by Wolff and Waters.[13]

Many of the impurities ordinarily found in the chemicals employed have but a small effect on the e.m.f., so that secondary standards may be set up with the best of c.p. materials, *except the mercurous sulphate,* which must be specially prepared. Cells so set up should not differ by more than 0.01 per cent. from those where the greatest care has been exercised in the preparation of the materials.

The mercury, if badly contaminated, may be subjected to a preliminary purification by electrolysis. The mercury is made the anode, a piece of platinum foil the cathode, and the electrolyte is 2 per cent. nitric acid in water; the mercury is constantly stirred. The more positive metals go almost completely into solution by electrolysis; the less positive metals, which affect the e.m.f. but little, are left in the mercury, which is then distilled twice in a current of air at greatly reduced pressure. This oxidizes the remaining impurities which distil with the mercury; the oxides float on the surface and are removed by passing the mercury through a pinhole in a filter paper.

The zinc is distilled at reduced pressure to remove the small amounts of cadmium, lead, iron, and arsenic, which are the usual impurities.

The zinc sulphate must be treated to remove the sulphates of cadmium, iron, lead, and free sulphuric acid; the last has the greatest effect on the e.m.f. and gives rise to the formation of gas at the amalgam.

The *purity of the mercurous sulphate is of prime importance;* lack of purity of this salt is the chief cause of variation in the e.m.f. The usual impurities are basic mercurous sulphate, basic mercuric sulphate, traces of mercuric nitrate, etc., according to the method of preparation. This salt is subject to the action of light and must be prepared in subdued light and preserved in the dark under dilute sulphuric acid. Great care must be exercised in washing it before use to free it from sulphuric acid, the washing being done with absolute alcohol, which in turn is removed by zinc sulphate solution.

This salt may be made by a number of methods, all of which give satisfactory results, the best one, apparently, being an electrolytic method devised by Wolff and Waters. The mercury anode is at the bottom of a tall jar, the cathode a piece of platinum foil suspended above the anode, and the electrolyte is dilute sulphuric acid. The mercury is violently stirred during the passage of the current and for some time after the circuit is broken.

The Weston or Cadmium Cell.[13]—As early as 1884 Czapski called attention to the low temperature coefficients of cells with cadmium electrodes, but the matter was forgotten until 1892, when attention was recalled to this type of cell by Edward Weston. The cell suggested by him is composed of cadmium in cadmium sulphate solution opposed to mercury in mercurous sulphate paste. The particular advantage of this form of cell is its very small temperature coefficient. This is in consequence of the fact that the solubility of the cadmium salt is only slightly influenced by the temperature, consequently the changes in density of the solution are very small. Also the temperature effects on the two limbs of the cell tend toward compensation. The normal form of this cell has been studied at the Reichsanstalt by Jaegar, Kahle, Wachsmuth, and Lindeck, and in this country by Wolff and Waters. The cadmium is used in the form of an amalgam made by dissolving, with the aid of heat, 1 part of Kahlbaum's best cadmium in 7 parts of mercury. If

necessary, the cadmium may be purified by distillation at reduced pressure.

The effect of variation of the concentration of the amalgam is shown in Fig. 173, which gives the e.m.fs. when various amalgams are tested against one having 14.2 per cent. of cadmium, the electrolyte being saturated cadmium sulphate solution. Variable results were obtained when the percentage of cadmium was over 14.5 per cent. Amalgamated cadmium rods also gave variable results. Identical results (to $0^v.00001$) were given by amalgams containing from 6 to 14.3 per cent. of cadmium.

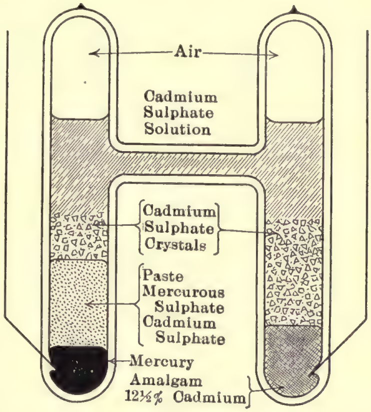


FIG. 172.—Weston normal standard cell.

It may be noted that the stability of the amalgam when subjected to variations of temperature is influenced by its composition; irregularities in the behavior of the cell were noticed at about 15°C. At first it was supposed that these were due to an inversion of the cadmium sulphate, similar to that of zinc sulphate at 39°, but later it was found that they were due to the amalgam, and were obviated by employing a 12.5 per cent. in place of the 14.3 per cent. amalgam at first used.

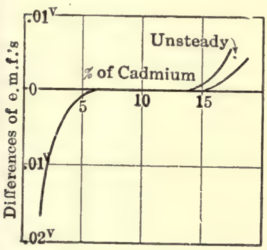


FIG. 173.—Illustrating effect of strength of amalgam in the cadmium cell.

The e.m.f. at 20° of the normal cadmium cell, containing saturated solution, is, when derived from the international ampere and the international ohm, 1.01830 international volts.[14] Its e.m.f. at any temperature, t, is

$$E_t = E_{20} - 0.0000406[t - 20] - 0.00000095[t - 20]^2 + 0.000000001[t - 20]^3.$$

The advantages of the cadmium cell are low temperature

coefficient; exceedingly small lag error; long life, due to freedom from cracking at the negative electrode; continuity of action, due to the fact that gas is not formed at the negative electrode.

The Weston Secondary Standard Cell.—The Weston Instrument Co. has placed on the market the convenient, portable form of cadmium cell shown in Fig. 174. The solution is saturated at 4°C., and therefore does not contain an excess of cadmium sulphate crystals. This type of cell is to be used between the temperatures of 4° and 40°. The temperature coefficient is much smaller than that of the normal cell and is negligible for any ordinary measurements; each cell is accompanied by a certificate stating its e.m.f. The extreme variation among 145

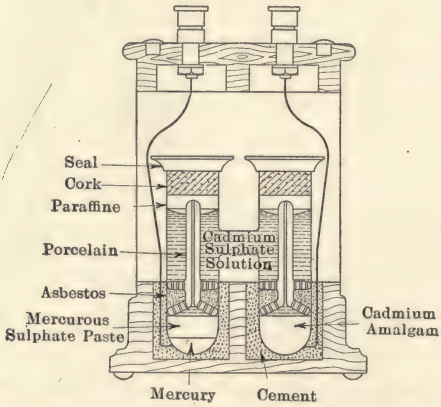


Fig. 174.—Weston secondary standard cell. A working standard of electromotive force.

cells of this type was found to be 0.0009 volt. This is the best form of secondary standard cell, being remarkably permanent. The e.m.f. is very closely 1.0186 international volts; the resistance about 200 ohms.

Precaution in Using Standard Cells.—No appreciable current can be taken from a standard cell without alteration of its e.m.f., due to polarization. It is found that the change is not permanent, for the cell gradually recovers its original e.m.f.; the cell is unreliable until the recovery is complete. This being so, standard cells are used only in compensation methods and must always be protected by a key, and the manipulation must be such that the key is closed for but an instant.

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CHAPTER VI

THE MEASUREMENT OF POWER

In direct-current circuits the measurement of power, or rate of expenditure of electrical energy, is best accomplished by the use of calibrated ammeters and voltmeters. In alternating-current circuits where both the electromotive force (or potential difference) and the current are varying from instant to instant,

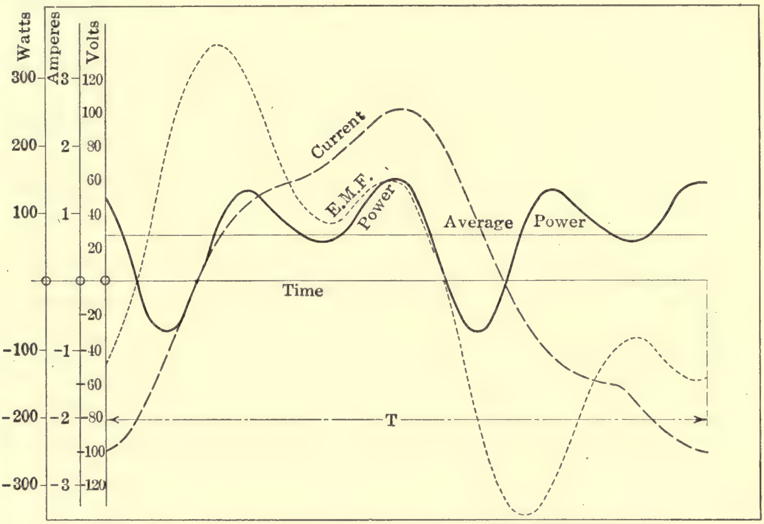


FIG. 175.—Illustrating instantaneous and average power.

passing through a definite cycle of values, the "instantaneous power," or the power being given to the circuit at a particular instant is vi. The power to be measured is the average value of vi during the cycle,

$$\dot{P} = \frac{1}{T}\int_0^T vidt.$$

To illustrate, let the curves of voltage, current, and vi be as shown in Fig. 175.

The net area under the power curve is proportional to $\int_0^T vidt$. T, the time of a complete cycle, is proportional to the length of the base line; $\frac{1}{T}\int_0^T vidt$ is therefore, to the proper scale, the average ordinate of the power curve.

In general, when dealing with alternating-current circuits it is necessary to have methods of evaluating $\frac{1}{T}\int_0^T vidt$. The expression for the power must be left in this general form, for in practical measurements it is not permissible to make any assumptions as to the forms of the voltage and current waves.

For general purposes the simplest and most satisfactory method of measuring the power in an alternating-current circuit is by the use of the electrodynamometer wattmeter. Other methods will be discussed, but their usefulness is restricted to particular cases.

The Electrodynamometer Wattmeter.—It has previously been shown that any electrodynamometer measures the mean product of the currents flowing in the fixed and movable coils; it evaluates

$$\frac{1}{T}\int_0^T i_F i_M dt.$$

If a Siemens dynamometer is connected to a circuit in the manner shown in Fig. 176, the fixed coils being put in series with the load, while the movable coil, in series with a suitable non-reactive resistance, is placed across the line, the current in the fixed coil, i_F, is the instantaneous load current and the current in the movable coil, i_M, is proportional to the instantaneous voltage.

If R is the total resistance of the movable coil circuit, $i_M = \frac{v}{R}$.

In the case of the Siemens dynamometer with a uniformly graduated scale,

$$KD = \frac{1}{T}\int_0^T i_F i_M dt,$$

K is a constant depending on the windings and on the strength of the controlling spring, and D is the angle through which it is

necessary to twist the spring in order to bring the coil back to its initial position. Substituting the above values for the currents in the fixed and movable coils,

$$KD = \frac{1}{R} \cdot \frac{1}{T}\int_0^T vidt$$

or

$$RKD = P \tag{1}$$

If the spring is perfect and the scale is uniform, the power is proportional to the deflection.

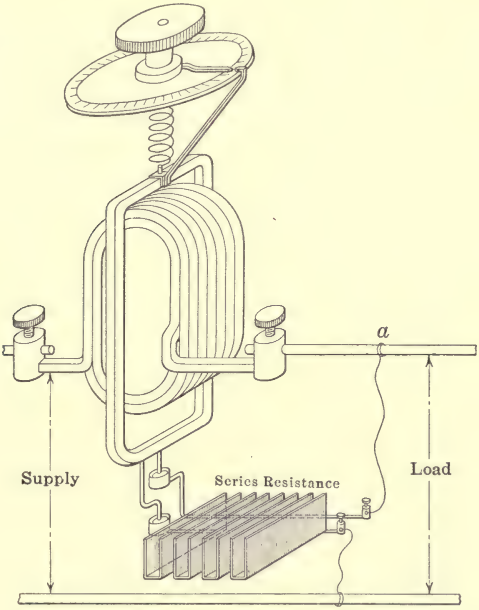


FIG. 176.—Showing connections of wattmeter.

The only assumption involved is that the potential circuit is non-reactive. The influence of reactance in this circuit will be discussed later.

The relation $P = RKD$ will hold for any wattmeter where the movable coil is always brought back to a definite position with respect to the fixed coil. When the movable coil is allowed to deflect against the action of the spring, K is no longer a constant,

for it depends on the geometry of the system of coils, that is, on the angle between the coils, and this angle varies with every change of load. In consequence, the scales of portable wattmeters are generally non-uniform. Proper proportioning of the relative diameters of the fixed and movable coils will do much toward correcting this, see page 80.

Fig. 177 shows, in section, one form of portable wattmeter which is in common use.

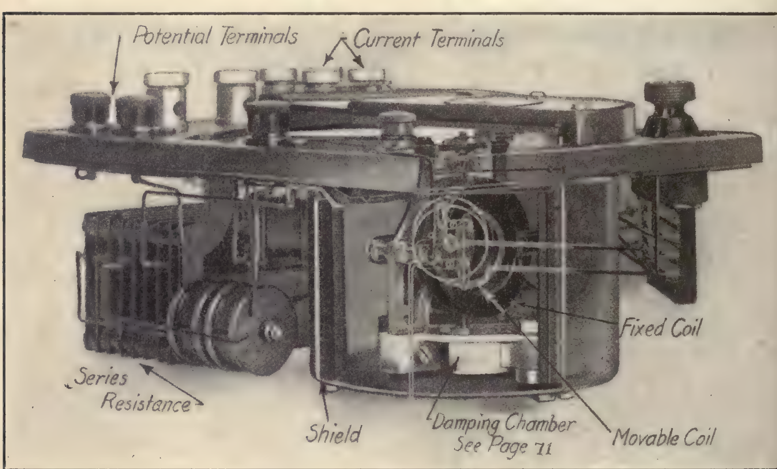


Fig. 177.—Phantom view showing construction of Weston portable wattmeter.

Heating Losses in Wattmeters.—It is important to note just what a wattmeter measures. P in formula (1) is the power given to the circuit by the current which flows through the fixed coils, that power being expended between the two points at which the potential terminals are attached to the circuit. P therefore must include the heating loss either in the current coils or in the potential circuit, depending on whether the point a (Fig. 176) is on the supply or the load side of the current coils.

Where the total amount of power is small, it may be necessary to correct the readings for the loss in the instrument itself. The two possible methods of connection are given in Fig. 178.

With connection I the indication of the wattmeter includes the $\dot{I}^2R$ loss in the current coils; with connection II the loss in

the entire potential circuit is included. The corresponding corrections when a small output is measured are obvious.

Compensation for Energy Loss in the Potential Circuit.[1]—In research work it is frequently necessary to measure small amounts of power, a few watts, and in this case the loss in the potential circuit of the wattmeter may be a large percentage of the power to be determined. To avoid the necessity of making a correction, instruments are sometimes so designed that this error is compensated. In this case connection *II* is used. The current which flows through the fixed coil is made up of two components, one due to the load, the other to the current in the movable coil circuit. The effect of this last must be compensated, for the magnetic field in which the potential coil moves is due to both components. If a second winding, coincident at all points

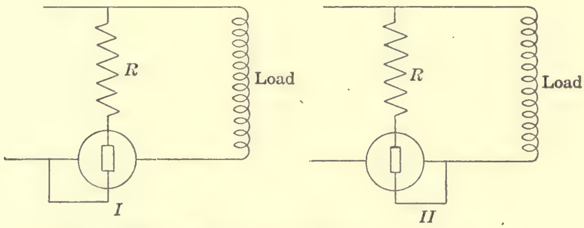


FIG. 178.—Showing wattmeter connections.

with the regular current coil, could be put on the bobbin carrying the fixed coil, and be connected into the potential circuit so that its effect opposed that of the main-current coil, the compensation would be exact; for instance, if the load circuit were broken, the net ampere-turns acting on the movable coil would be zero and there would be no deflection. As the two coils cannot be made coincident, care must be exercised in placing the compensating turns so that when the load circuit is broken, the net magnetic field at the movable coil will be zero for all positions of the movable coil. Otherwise the degree of compensation will vary with the scale reading. There is a small transformer action due to the mutual inductance of the two windings on the fixed coil, but in commercial measurements at the usual voltages this does not cause an appreciable error.

This compensation may be extended so that the power lost

in a voltmeter connected directly across the load may be allowed for, a second compensating winding being added and connected in series with the voltmeter; allowance must be made for the changed resistance of the voltmeter circuit.

Grouping of Instruments.—The heating losses have a bearing on the grouping of the instruments when the voltage, current, power and power factor of a small reactive load are to be determined.

Electrical apparatus is generally sold to be operated at some definite voltage, so the voltmeter is placed across the load as in Fig. 179.

As shown, the wattmeter measures in addition to the load, the power in its own current coils, in the ammeter, and in the voltmeter. The ammeter gives the vector sum of the currents in the load and in the voltmeter.

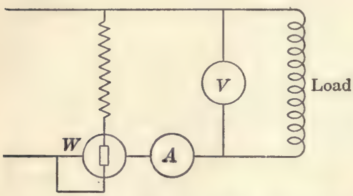


FIG. 179.—Showing grouping of instruments for measuring, power, current and voltage.

Errors due to Local Fields.—In industrial testing, it is not safe to assume that a wattmeter will be uninfluenced by local magnetic fields. In the ordinary portable instruments this error, if present, will depend on the deflection. Its presence or absence may be made obvious by connecting the potential coil alone to the circuit and observing the deflection when the instrument is turned to several different azimuths. If the error is present and it is not feasible to change the location of the observing station, the instrument may be turned to the position where the pointer is undeflected, that is, where the movable coil is threaded by the maximum number of lines of force, due to the stray field. When the proper position has been found, the direction of the pointer may be noted. If in the subsequent use of the wattmeter the pointer is always kept in approximately this direction, the body of the instrument being turned as the load is varied, the error will be rendered negligible, provided the direction of the local field does not change.

Direct-current stray fields have no influence on the readings when alternating currents are being dealt with, and alternating

stray fields, to have any effect, must be of the frequency of the current under measurement.

As high-capacity instruments have comparatively few turns on the fixed coil, special care must be taken that there are no loops in the leads near the instrument and that the current leads occupy the same position with respect to the instrument during its calibration and subsequent use.

To obviate stray field errors, the working parts of the instrument are frequently surrounded by a soft iron shield built up of stampings (see Fig. 177).

Voltage between Current and Potential Coils.—The movable coil of a wattmeter, being in series with a large resistance, may inadvertantly be connected to the circuit so that practically the full line potential exists between the current and the potential coils. This may give rise to errors due to the electrostatic attraction between these two coils; also there is danger that the insulation between the coils may be punctured. The connections should be so made that the current and potential coils are, as nearly as possible, at the same potential. If when so connected, the deflection is in the wrong direction, the current coil should be reversed. The proper position of a multiplier, when one is used, is governed by the same consideration.

It is well to mark the terminals of a wattmeter once for all so that there can be no mistake in making the connections. Such a marking also obviates any question as to the algebraic sign of the readings when measuring polyphase power.

When instrument transformers are used, electrostatic troubles may be avoided if the two coils of the wattmeter be connected by a piece of very fine fuse wire.

The Effect of Reactance in the Potential Circuit.—It has been assumed that the potential circuit is non-reactive; this can never be strictly true since it must contain the movable coil. In commercial instruments, when used at ordinary frequencies and power factors, the resistance of the potential circuit is so high in comparison with its reactance that the effect of the inductance is entirely negligible. In special investigations, however, cases arise where the utmost care must be exercised if reliable results are to be obtained. The presence of reactance has two effects: it cuts down to a certain extent the current

in the potential coil and shifts its phase by an amount dependent on the frequency. Thus the mean product of the currents in the fixed and movable coils is altered from its proper value. If *sinusoidal currents* are assumed, the magnitude of the error thus introduced may readily be computed.

Symbols used in the Discussion of the Theory of the Wattmeter

Maximum values of currents and voltages are denoted by large letters, instantaneous values by small letters.

Referring to Fig. 180

$ab = V =$ voltage at terminals of potential-coil circuit.
$I_P =$ current in potential coil.
$R_P =$ resistance of potential-coil circuit.
$ac = I_PR_P =$ ohmic drop in potential-coil circuit.
$I_L =$ current in load.
$R_L =$ resistance of load.
$R_C =$ resistance of current coils.
$I_C =$ current in current coils.
$ad = I_L(R_L + R_C)$ ohmic drop in load circuit between potential terminals.
$\theta_P =$ phase displacement in potential-coil circuit.
$\theta_L =$ phase displacement in load circuit between potential terminals.
$L_P =$ inductance of potential-coil circuit.
$L_L =$ indutance of load.
$L_C =$ inductance of current coils.
$Z_L =$ impedance of load.
$Z_P =$ impedance of potential-coil circuit.
$Z_C =$ impedance of current coils.
$\omega = 2\pi$ times frequency.
$m =$ mutual inductance of fixed and movable-coil circuits.
$P_L =$ power in load.
$P_W =$ power as read from the wattmeter.
$H_P =$ heating loss in potential circuit.
$H_C =$ heating loss in current coils.
$K =$ constant of dynamometer.
$D =$ deflection of instrument.

Consider case *I* on page 307 where the current coils are traversed by the load current only and the power measured includes the heating loss in the current coils.

On account of the power factor of the reactive load circuit, the current, I_L, will lag behind V by an angle θ_L, and in consequence of the reactance of the potential-coil circuit, the current in it will be out of phase with V by an angle θ_P and will be altered from the value $\frac{V}{R_P}$ to $\frac{V \cos \theta_P}{R_P}$, see Fig. 180.

The deflection of the instrument is proportional to the mean product of the instantaneous values of I_P and I_L, therefore,

$$KD = \frac{1}{T}\int_0^T i_L i_P dt = \frac{I_L I_P}{2} \cos (\theta_L - \theta_P).$$

So the apparent power or the scale reading is

$$R_P KD = I_L \frac{V \cos \theta_P}{2} \cos (\theta_L - \theta_P) = P_W \tag{2}$$

The true power, that is the power which would be read from the scale if there were no reactance in the potential circuit, is given by

$$P_L + H_C = \frac{I_L V}{2} \cos \theta_L \tag{3}$$

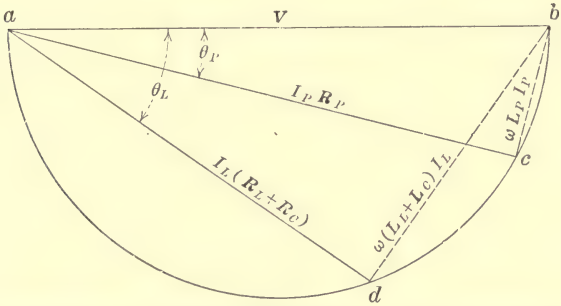


FIG. 180.—Pertaining to effect of inductance in the potential circuit of a wattmeter.

A correction to be subtracted from the apparent power in order to obtain the true power may be determined, for by (2)

$$P_W = \frac{I_L V \cos \theta_P}{2} \cos (\theta_L - \theta_P) = \frac{I_L V}{2}$$

$$(\cos^2 \theta_P \cos \theta_L + \cos \theta_P \sin \theta_P \sin \theta_L)$$

$$\therefore P_L + H_C = \frac{P_W}{\cos^2 \theta_P} - \frac{I_L V}{2} \tan \theta_P \sin \theta_L.$$

In portable instruments the angle θ_P is a very small fraction of a degree but in commutating watt-hour meters, such as were formerly used on alternating-current circuits, it may be as large as 2°. Using the values of the current and voltage given by

indicating instruments, instead of maximum values, and remembering that ordinarily θ_P is very small,

$$P_L + H_C = P_W - I_L V \tan\ \theta_P \sin\ \theta_L = P_W - I_L V\left(\frac{L_P\omega}{R_P}\right)\sin\ \theta_L \quad (4)$$

The effect of the phase displacement in the potential circuit evidently depends on the frequency and on the characteristics of the load as well as on the wattmeter itself. It increases as the power factor of the load is decreased and at extremely low power factors extraordinary precautions must be taken in order to procure accurate results.

If the resistance of the potential circuit is exceedingly high, there may be capacity effects in the series resistance. Some instruments are so designed that a part of the potential circuit is coiled on a metal bobbin and although this is split lengthwise it is not entirely free from eddy currents. Both the capacity and the eddy currents modify the phase displacement.

If the load current is leading and the power factor very low, the wattmeter readings tend toward zero and at some particular value of the power factor the reading will reverse. For an interesting case in point, see *Journal of the Institution of Electrical Engineers*, vol. 30, 1901, p. 467.

The ratio of the true to the apparent power is

$$\frac{P_L + H_C}{P_W} = F = \frac{\cos\ \theta_L}{\cos\ \theta_P \cos\ (\theta_L - \theta_P)} \tag{5}$$

and the total power is obtained by multiplying the apparent power by the correction factor F.

The trigonometrical form of this expression may be changed so that the tangents rather than the cosines of the angles may be used, then

$$F = \frac{1 + \tan^2\ \theta_P}{1 + \tan\ \theta_P \tan\ \theta_L} \tag{6}$$

This is the usual form of the correction factor.

If there are no modifying causes such as capacity or eddy-current effects, $\tan\ \theta_P$ may be expressed in terms of the inductance and resistance of the potential circuit. Then

$$F = \frac{1 + \left(\frac{L_P\omega}{R_P}\right)^2}{1 + \left(\frac{L_P\omega}{R_P}\right)\left(\tan\ \theta_L\right)} \tag{7}$$

It is preferable to use the correction term of equation (4) rather than the factor given in equation (7), for the latter becomes practically indeterminate at low power factors.

Compensation for the Inductance of the Potential Circuit.[3]—It naturally suggests itself that the effect of inductance in the potential circuit may be annulled by the use of capacity. Simple tuning of the circuit is manifestly inadequate, for the arrangement must be such as to be practically independent of the frequency, in order that there may be no errors when dealing with non-sinusoidal waves.

The arrangement used for this purpose by Abraham and Rosa is a condenser inserted in series with the movable coil and shunted by a non-inductive resistance. Such an arrangement may be adjusted so that, for all practical purposes, the net reactance of the circuit is reduced to zero.

The connections of the potential circuit then become those shown in Fig. 181.

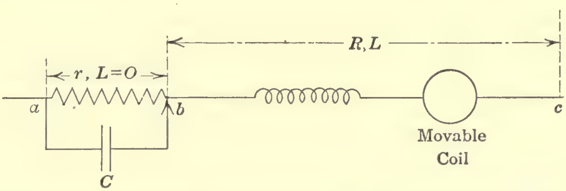


FIG. 181.—Showing method of compensating the inductance of the potential circuit of a wattmeter.

The impedance between a and b is

$$Z_{ab} = \frac{1}{\frac{1}{r} + j\omega C} = \frac{r}{1 + j\omega Cr} = \frac{r - j\omega Cr^2}{1 + \omega^2 C^2 r^2}.$$

The impedance between b and c is

$$Z_{bc} = R + j\omega L.$$

Therefore the total impedance is

$$Z_{ac} = R + j\omega L + \frac{r - j\omega Cr^2}{1 + \omega^2 C^2 r^2} = \left(R + \frac{r}{1 + \omega^2 C^2 r^2}\right) + j\omega\left(L - \frac{Cr^2}{1 + \omega^2 C^2 r^2}\right).$$

If $\omega^2C^2r^2$ can be neglected in comparison with unity

$$Z_{ac} = R + r + j\omega(L - Cr^2) \text{ approx.} \tag{8}$$

If r is adjusted so that

$$L = Cr^2$$

then

$$Z_{ac} = R + r$$

which is the resistance of the circuit as measured by a bridge. As long as the term $\omega^2C^2r^2$ is negligible, the compensation will not be affected by the frequency.

As an example, consider the following data:

$$\begin{aligned} L &= 0.000328 \text{ henry} \\ R &= 20 \text{ ohms} \\ C &= 1 \text{ microfarad} \\ r &= \sqrt{\frac{L}{C}} = 18.11 \text{ ohms.} \end{aligned}$$

Using these values the impedances and phase displacements at 60 and 1,000 cycles per second are:

$$\begin{aligned} Z_{60} &= 38.109 + j0.000016 \\ \theta_{60} &= 0^\circ.000024 \\ Z_{1,000} &= 37.878 + j0.0265 \\ \theta_{1,000} &= 0^\circ.040 \end{aligned}$$

If no compensation is applied the values are

$$\begin{aligned} Z_{60} &= 38.11 + j0.124 \\ \theta_{60} &= 0^\circ.186 \\ Z_{1,000} &= 38.11 + j2.06 \\ \theta_{1,000} &= 3^\circ.1 \end{aligned}$$

To determine whether compensation is needed and to make the necessary adjustments, the potential circuit is short-circuited on itself and a large alternating current sent through the fixed coil.

The instrument will remain undeflected:

1. If there be no mutual induction between the two coils.

2. If the effective inductance of the movable-coil circuit is zero, for then the currents in the movable and fixed coils will be in quadrature.

Having made sure that the mutual induction is not zero, the resistance r may be adjusted until the deflection disappears.

The adjustment is then complete, provided there are no eddy-current effects in the fixed coil or in the frame of the instrument (see page 316). These effects are discussed in the original paper by Rosa.

Effect of Mutual Induction between Fixed and Movable-coil Circuits.—In the previous discussion no reference has been made to the possible effects of mutual induction between the current and potential circuits. In commercial instruments these effects are so small as to be negligible. They might, however, be worthy of consideration in special low-voltage wattmeters where an attempt is made to attain a maximum of sensitivity.

When the instrument is read by a torsion head the mutual inductance can be made zero, but if the movable coil is allowed

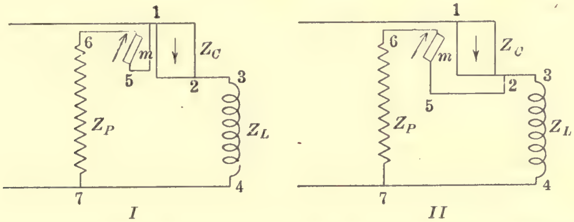


FIG. 182.—Pertaining to effect of mutual induction in wattmeter.

to deflect, the mutual inductance will have an effect dependent on the reading of the instrument, being zero and changing sign when the planes of the coils are perpendicular.

Take the connections shown in Fig. 182*I* which are the same as those assumed in Fig. 180.

Equating the values of the P.D. between 1 and 7 reckoned *via* the load and *via* the potential circuit gives

$$I_{23}(Z_L + Z_C) \mp jm\omega I_{56} = I_{56}Z_P \mp jm\omega I_{23}$$

$$I_{56} = I_{23}\left[\frac{(R_L + R_C) + j\omega(L_L + L_C \pm m)}{R_P + j\omega(L_P \pm m)}\right]$$

$$I_{56} = I_{23}\left[\frac{R_P(R_L + R_C) + \omega^2(L_L + L_C \pm m)\,(L_P \pm m)}{R_P{}^2 + \omega^2(L_P \pm m)^2}\right] + j\omega I_{23}\left[\ \right].$$

The current in the potential coil is seen to consist of two components, one in phase and one in quadrature with the current in the fixed coils, I_{23}, which is also the load current I_L.

The turning moment, and therefore the deflection, are proportional to the mean product of the currents I_{23} and I_{56} and this mean product multiplied by the resistance of the potential circuit is the reading of the wattmeter. Using effective values, power by wattmeter =

$$R_PKD = I_L^2\left[\frac{R_P^2(R_L + R_C) + \omega^2 R_P(L_L + L_C \pm m)(L_P \pm m)}{R_P^2 + \omega^2(L_P \pm m)^2}\right].$$

The instrument should give the power in the load circuit between the points 1 and 4, that is the power in the load, plus the heating in the current coils.

$$P_L + H_C = I_L^2(R_L + R_C)$$

$$\therefore P_L = P_W\left[\frac{R_P^2 + \omega^2(L_P \pm m)^2}{R_P^2}\right] - \frac{I_L^2\omega^2(L_L + L_C \pm m)(L_P \pm m)}{R_P} - H_C.$$

Eddy-current Errors.—Another source of error, one not amenable to calculation, may be found in instruments of faulty design. It is that due to currents induced in masses of metal, such as the frame of the instrument, or, in instruments of large current capacity, in the current coils themselves, for these coils must be made very massive in order to give the requisite carrying capacity.

This error should be reduced to a minimum by the design of the instrument. In high-capacity instruments it will be necessary to wind the current coil with a stranded conductor or its equivalent, the strands being insulated from one another. Great care must be taken in arranging them. The average position of each strand in the cross-section should be the same as that of every other strand, otherwise the heating of the coil may change the current distribution and therefore the calibration of the instrument.

In laboratory instruments intended for heavy currents, the current coil may be made of a small and thin copper tube through which there is a rapid circulation of water.

To detect the presence of eddy-current errors in research instruments where the inductance of the potential-coil circuit is compensated, the zero reading is brought to a noted point on the scale and the compensation for the inductance of the moving-coil circuit made as shown on page 313. The zero is then changed

by use of the torsion head and the compensation tested at various points along the scale. If the eddy-current error is absent, the adjustment of the potential circuit for zero effective inductance will be the same at all points.

OTHER METHODS OF MEASURING POWER

In consequence of the development of the wattmeter, the three methods now to be described are merely of historical interest as methods for power measurement. However, two of them have other applications which are still of practical value.

The Three Dynamometer Method.—The connections are as shown in Fig. 183. At D_1, D_2 and D_3 are three electrodyna-

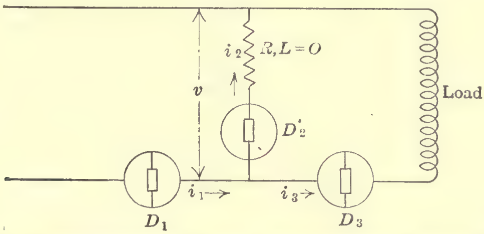


FIG. 183.—Connections for three dynamometer method for power measurement.

mometers each with its fixed and movable coils in series. R is the total resistance of the circuit of D_2. The instantaneous values of the currents are as indicated. In general

$$P = \frac{1}{T}\int_0^T vi_3dt.$$

R is assumed to be non-inductive so,

$$P = R\frac{1}{T}\int_0^T i_2i_3dt.$$

At any instant

$$i_1 = i_2 + i_3$$

and

$$i_1^2 = i_2^2 + 2i_2i_3 + i_3^2$$

so

$$i_2i_3 = \frac{i_1^2 - i_2^2 - i_3^2}{2}$$

and

$$P = \frac{R}{2}\left[\frac{1}{T}\int_0^T i_1^2dt - \frac{1}{T}\int_0^T i_2^2dt - \frac{1}{T}\int_0^T i_3^2dt\right]. \tag{12}$$

The three integrals are the mean square values of the three currents and will be given by the readings of the dynamometers. If Siemens instruments are used,

$$K_1D_1 = \frac{1}{T}\int_0^T i_1^2dt$$

where K_1 is the constant of the instrument and D_1 its deflection, and similarly for the other instruments.

Hence

$$P = \frac{R}{2}[K_1D_1 - K_2D_2 - K_3D_3]. \tag{13}$$

The result involves no assumption as to wave form. It does assume that the resistance, R, is non-inductive. This cannot be exactly true for the potential circuit contains the electro-dynamometer. The use of hot-wire ammeters would render this assumption practically true. P includes the power in D_3.

The results are greatly affected by the errors of reading. To obtain the best precision in the measurement, the power wasted in R must be equal to the load, an obviously impracticable condition.

This method is used in telephone investigations as an aid to determining line constants.

The Three Voltmeter Method.—In this analogous method the three instruments capable of measuring the mean square values of the currents are replaced by three instruments which measure mean square voltages. The connections are as shown in Fig. 184.

The non-inductive resistance R is joined in series with the load and the three voltages read, as indicated.

$$P = \frac{1}{T}\int_0^T iv_3dt = \frac{1}{R}\cdot\frac{1}{T}\int^T v_2v_3dt.$$

At any instant

$$v_1 = v_2 + v_3 \text{ and } v_1^2 = v_2^2 + 2v_2v_3 + v_3^2$$

$$\therefore P = \frac{1}{2R}\left[\frac{1}{T}\int_0^T v_1^2dt - \frac{1}{T}\int_0^T v_2^2dt - \frac{1}{T}\int_0^T v_3^2dt\right]. \tag{14}$$

If the voltmeters are of the electrodynamometer or the electrostatic type,

$$P = \frac{1}{2R}\left[K_1D_1 - K_2D_2 - K_3D_3\right]. \qquad (15)$$

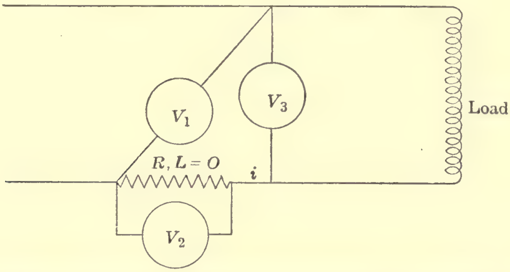


Fig. 184.—Connections for three-voltmeter method for power measurement

For the greatest precision the power wasted in R must be equal to the load. P includes the power in V_3.

The Split-dynamometer Method.—The connections are shown in Fig. 185.

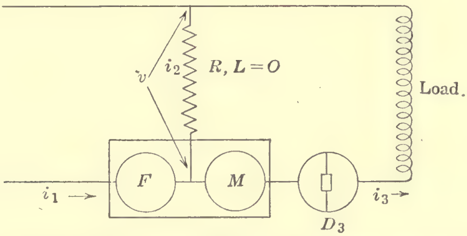


Fig. 185.—Connections for split-dynamometer method for power measurement.

D_3 is an ordinary current electrodynamometer with its coils in series. FM is another dynamometer with a non-inductive resistance, R, tapped in between the fixed and movable coils.

$$P = \frac{1}{T}\int_0^T vi_3dt = R\cdot\frac{1}{T}\int_0^T i_2i_3dt.$$

At any instant

$$i_1 = i_2 + i_3$$

$$\therefore P = R\left[\frac{1}{T}\int_0^T i_1i_3dt_t - \frac{1}{T}\int_0^R i_3^2dt\right]. \qquad (16)$$

The split dynamometer gives the mean product of the currents in its coils, so

$$P = R[K_1D_1 - K_3D_3]. \tag{17}$$

Potier Method for Power Measurement—The Electrostatic Wattmeter.[4]—The electrostatic wattmeter is an instrument of importance in research work, its particular field of usefulness being the measurement of small amounts of power at high voltage and low power factor. It is also used at the National Physical Laboratory, London, in the testing of watt-hour meters.

This method for the measurement of electrical power was first given by A. Potier. The readings are obtained by using a quadrant electrometer. In the original arrangement, a non-reactive resistance is joined in series with the load and the two

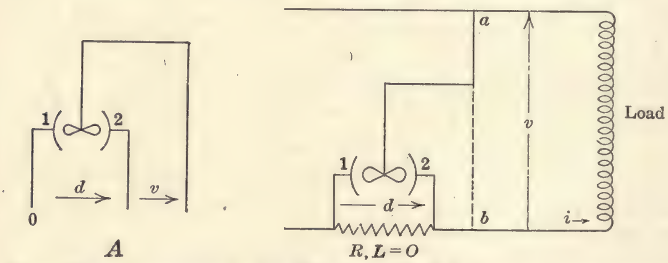


FIG. 186.—Connections for Potier method for power measurement.

pairs of quadrants connected to its terminals. Two readings are then taken, one, with the needle connected to the load terminal on the far side of the line, and another, with the needle connected to the other load terminal. The connections are shown in Fig. 186. It has previously been shown that with the instantaneous potential differences indicated at A, the deflection of the quadrant electrometer may be expressed by

$$D = K\frac{1}{T}\int_0^T \left[2vd + d^2\right]dt.$$

When the connection is at a, if the charging current for the electrometer be neglected,

$$D_a = K\left[2R \cdot \frac{1}{T}\int_0^T vidt + \frac{1}{T}\int_0^T d^2dt\right] = K\left[2RP + \frac{1}{T}\int_0^T d^2dt\right]. \tag{18}$$

The term $\frac{1}{T}\int_0^T d^2dt$ is evaluated by the second reading, for with the connection at b the deflection is

$$D_b = K\frac{1}{T}\int_0^T d^2dt$$

$$\therefore P = \frac{D_a - D_b}{2KR} \tag{19}$$

D_b is dependent on the power wasted in the non-reactive resistance.

The necessity for taking the second reading may be obviated by a method given by Miles Walker[4].

The connections are shown in Fig. 187.

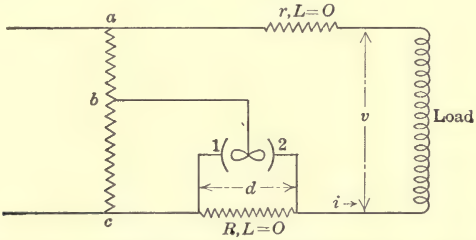


FIG. 187.—Connections for electrostatic wattmeter.

Two non-inductive resistances, R and r, are placed in series with the load and in the positions indicated; the needle is connected to some form of potential divider so that its potential with reference to the quadrants is only a fraction of the line voltage. The potential divider may be a high resistance, but it is better to provide taps on the secondary of the high voltage transformer, for in this case the potential of the point b is not disturbed by the charging current flowing to the needle.

Let

$$\frac{v_{ac}}{v_{bc}} = n$$

then

$$v_{ab} = v_{ac}\left(1 - \frac{1}{n}\right).$$

The voltage between quadrant (2) and the needle is (see Fig. 187),

if the charging current for the electrometer be neglected,

$$v_{2b} = v + ri - (Ri + v + ri)\left(1 - \frac{1}{n}\right) = \frac{v}{n} + Ri\left(\frac{1}{n} - 1\right) + \frac{ri}{n}.$$

Consequently

$$D = K\frac{1}{T}\int_0^T \left[\frac{2dv}{n} + 2R^2i^2\left(\frac{1}{n} - 1\right) + \frac{2Rri^2}{n} + R^2i^2\right]dt \qquad (20)$$

The term $\frac{2dv}{n}$ is proportional to the instantaneous power; the other three terms can be eliminated by properly adjusting r, for if their sum be placed equal to zero,

$$\left(\frac{2 - n}{n}\right)R = -\frac{2r}{n}$$

$$r = \left(\frac{n - 2}{2}\right)R \qquad (21)$$

If r be adjusted to this value,

$$D = \frac{2KR}{n}\frac{1}{T}\int_0^T vidt$$

or

$$P = \frac{nD}{2KR} \qquad (22)$$

and no correction for the power wasted in R is necessary.

If the tap be brought out at the middle of ac, $n = 2$, and no compensating resistance, r, is required.

If the compensating resistance is not used, $(r = 0)$,

$$D = K\frac{1}{T}\int_0^T \left[\frac{2dv}{n} + \left(\frac{2 - n}{n}\right)d^2\right]dt$$

$$= \frac{2KR}{n}\left[P + \left(\frac{2 - n}{2}\right)I^2R\right].$$

$$P = \frac{nD}{2KR} + I^2R\left(\frac{n - 2}{2}\right) \qquad (23)$$

When small amounts of power at low power factor are measured, the term

$$I^2R\,\frac{n - 2}{2}$$

which gives the correction for the power in the resistance R, may become very important. If n is 2 the instrument gives the power without correction.

When the electrostatic wattmeter is used in connection with a fictitious load (see page 500) in the calibration of power and energy meters, the correction term due to the power loss in the series resistance will be eliminated if the current and voltage circuits are electrically connected at the middle of the resistance, R, as shown in Fig. 188.

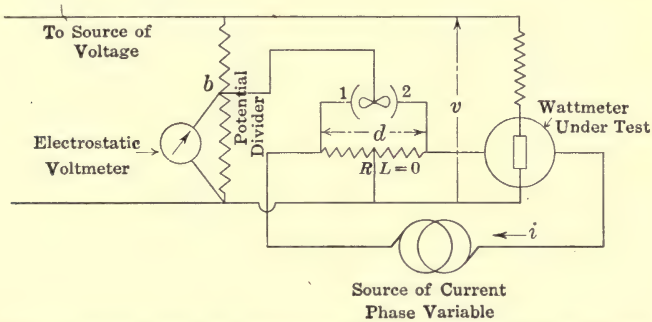


FIG. 188.—Connections for electrostatic wattmeter with fictitious load.

In this case, the potential difference between quadrant 2 and the needle is

$$v_{2b} = -\frac{d}{2} + \frac{v}{n}$$

$$\therefore D = K\frac{1}{T}\int_0^T \left[2\left(-\frac{d}{2} + \frac{v}{n}\right)d + d^2\right]dt = \frac{2KR}{n}\frac{1}{T}\int_0^T ivdt.$$

$$\therefore P = \frac{nD}{2KR} \tag{24}$$

This method of connection is used in a highly developed form at the National Physical Laboratory, for calibrating commercial instruments.

The electrostatic wattmeter has been used in measurements of the power lost in dielectrics at high voltages, as in cables running without load, in condensers and in samples of insulating material.

In testing small samples of insulating materials at high voltages, it is necessary to avoid measuring the energy dissipated in the air around the electrode and on the surface of the sample.

For this reason Rayner used a guard-ring electrode as shown in Fig. 190.

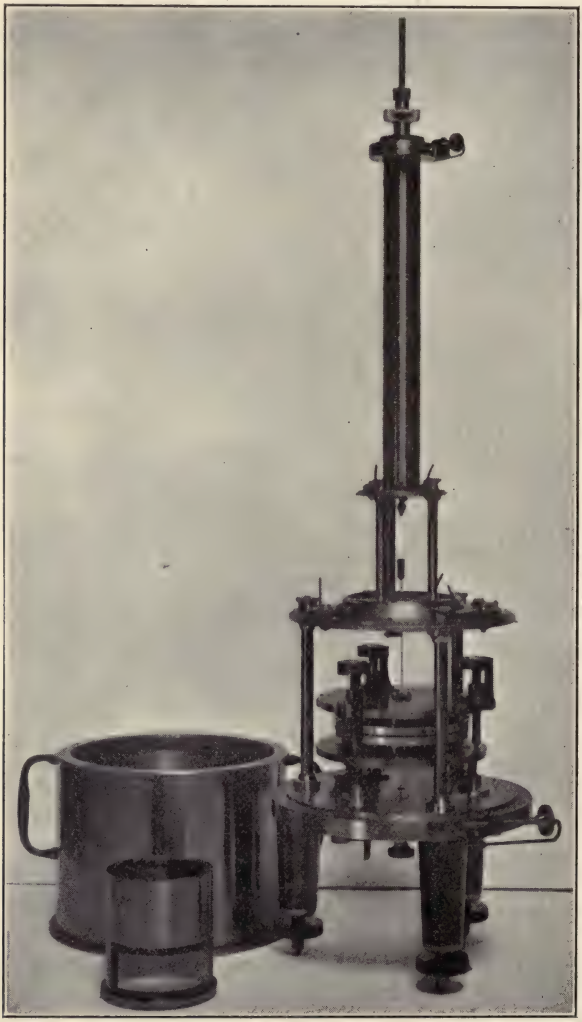


FIG. 189.—Quadrant electrometer used at National Physical Laboratory as an electrostatic wattmeter.

The current for the guard ring is furnished directly from the transformer and is not included in the measurement.

Sources of Error.[4]—When measuring small amounts of power the resistance R must be large. This produces two results; the condensers formed by quadrant 2 and one-half of the needle must be charged through this resistance, while that formed by the quadrant 1 and the other half of the needle is charged directly from the source. Consequently the potential of the condenser formed by the needle and quadrant 2 is a trifle lower than it should be, the time-phase relation of the potentials on the two condensers is not quite correct and the result is that with the main circuit open there will be a small deflection. A high resistance also alters the power factor of the circuit which in testing insulating materials may be very low. The current must be relatively high, and the correction term in (23) becomes large.

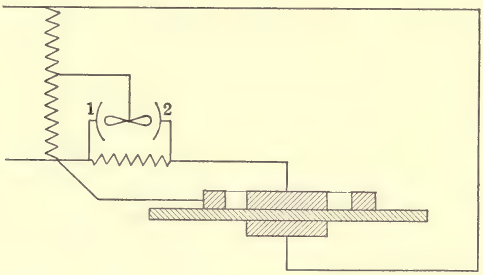


FIG. 190.—Illustrating Rayner guard-ring electrode.

The net error is practically proportional to R. When investigating dielectric losses a correction can be made with sufficient accuracy by observing the deflection with two known values of R and extrapolating for the watts expended when $R = 0$.

If the needle is not tapped into the transformer but is energized from a potential divider, an error may be introduced because of the alteration of the potential of the point b, Fig. 187, due to the charging current necessary for the needle. Errors may also arise from capacity effects in very high resistances of the ordinary construction. In electrostatic wattmeters for use at high voltage it is essential that there be ample separation between the quadrants and the needle so that brush discharges will be avoided.

Ryan Power Diagram Indicator.[5]—The Ryan Power Diagram Indicator is a device in which the electrostatic tube (see page 647) is employed to trace diagrams, the areas of which are proportional to the power supplied to the load. Its particular field of usefulness is the measurement of small amounts of power at high voltage. It has been applied to the measurement of corona losses and to the total losses in samples of insulating materials. Losses as small as 0.03 watt at 9,000 volts have been measured. The accuracy of the original arrangement was about 5 per cent. The application of the device (sometimes called the cyclograph) to the investigation of the behavior of insulating materials has been developed by Minton.[6]

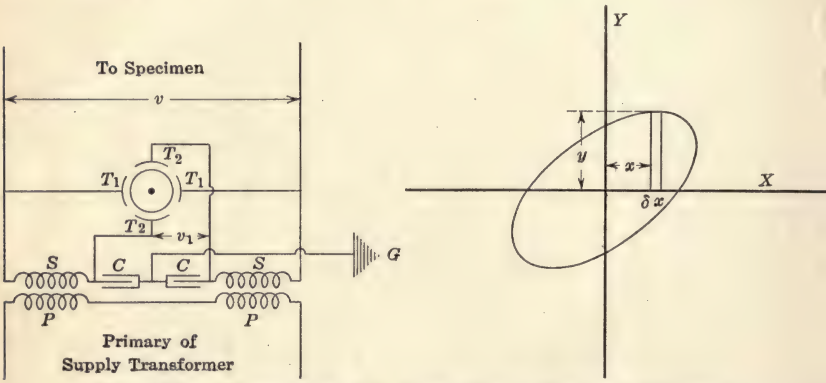


FIG. 191.—Connections for Ryan power diagram indicator.

In order to obtain the diagram, it is necessary to employ two sets of electrodes. One set is arranged to deflect the fluorescent spot along the X-axis on the screen; another, along the Y-axis.

Looking along the axis of the tube, the electrodes are placed as shown in Fig. 191, which also shows the other connections.

To employ this arrangement, one must be able to divide the secondary of the testing transformer into two equal sections so that the two condensers CC may be inserted between them. The junction of the condensers is grounded. By means of the electrodes, T_1, the fluorescent spot is at every instant deflected along one coördinate on the screen proportionally to v. The electrodes, T_2, cause the spot to be deflected along the other coördinate proportionally to the instantaneous values of v_1,

the potential difference between the outer terminals of the condensers C.

Let the displacement along y be

$$y = Kv$$

and that along x be

$$x = K'v_1.$$

K and K' are constants.

Then for an elementary area, $dA = ydx = KK'vdv_1$.

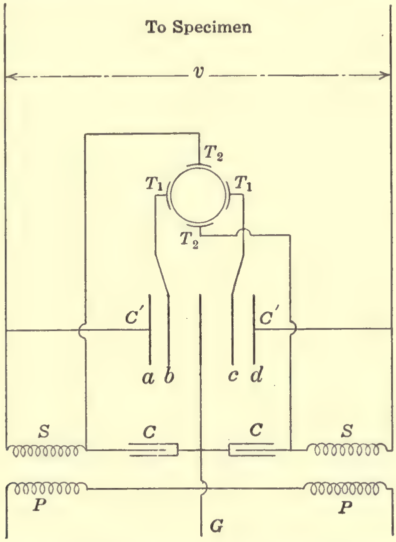


FIG. 192.—Connections for Ryan power diagram indicator with condenser multiplier.

The current i through the condensers is that taken by the specimen, and

$$i = \frac{C}{2}\frac{dv_1}{dt}.$$

Then

$$dA = \frac{2KK'}{C} vidt$$

and the area of the diagram is

$$A = K'' \int_0^T vidt = k'P. \tag{25}$$

The constant k' is determined by calibration.

Any losses in the insulation or oil used in the transformer are included in the measurement, so the oil should be specially dried. The condensers CC may be oil-immersed. Perfectly dry oil must be used so that the dielectric may be as free from losses as possible.

With the arrangement as just described the potential difference to give a full-scale deflection is only a few hundred volts. To adapt the arrangement to high-voltage work, the voltage v must be applied to the electrodes, T_1, by means of a condenser multiplier, as shown in Fig. 192.

The voltage v_1 which is dependent on the current is obtained as before. The plates of the condenser multiplier are at a, b, c, d, and by adjusting the position of the plates a and d, the multiplying power may be varied as desired. Air condensers are used. In the original apparatus the "plates" a and d were long sheet-metal cylinders with hemispherical ends. They were hung by insulating cords so that they could be readily raised or lowered. By this means the range of the instrument could be increased to 250,000 volts.

POWER MEASUREMENT IN POLYPHASE CIRCUITS

Blondel's Theorem.[7]—If energy be supplied to any system of conductors through n wires, the total power in the system is given by the algebraic sum of the readings of n wattmeters, so arranged that each of the n wires contains one current coil, the corresponding potential coil being connected between that wire and some point on the system which is common to all the potential circuits. If this common point is on one of the n wires and coincides with the point of attachment of the potential lead to that wire, the measurement may be effected by the use of $n-1$ wattmeters.

The receiving and generating circuits may be arranged in any desired manner and no assumption is made as to the way in which the e.m.fs. and currents vary.

To prove the theorem, denote by the subscripts 1, 2, 3, . . . n the different supply wires, by v_1, v_2, . . . v_n, the instantaneous *potentials* of the points on the various wires which form the terminals of the absorbing device, and by i, i_2 . . . i_n, the

instantaneous currents at these same points. Then the rate of displacement of electricity through wire No. 1 will be i_1 and the rate of doing work, or the instantaneous power will be i_1v_1, and similarly for all the others. Therefore,

$$p = i_1v_1 + i_2v_2 + \ldots i_nv_n \tag{a}$$

In practice it is necessary to deal with potential differences rather than with potentials. Let v_0 be the *potential* of some particular point on the system. In general,

$$i_1 + i_2 + i_3 \ldots + i_n = 0$$

consequently

$$i_1v_0 + i_2v_0 \ldots + i_nv_0 = 0. \tag{b}$$

Subtracting (*b*) from (*a*)

$$p = i_1(v_1 - v_0) + i_2(v_2 - v_0) \ldots i_n(v_n - v_0).$$

The average power will be

$$P = \frac{1}{T}\int_0^T i_1(v_1 - v_0)dt + \frac{1}{T}\int_0^T i_2(v_2 - v_0)dt \\ \ldots + \frac{1}{T}\int_0^T i_n(v_n - v_0)dt. \tag{26}$$

But $\frac{1}{T}\int_0^T i_1(v_1 - v_0)dt$, etc., are the readings of the n wattmeters connected as above. If the common point is on one of the n wires at one of the terminal points of the absorbing device, then one of the quantities in parenthesis will be zero, the corresponding wattmeter will read zero, and only $n - 1$ wattmeters will be required.

The above demonstration is perfectly general and therefore applies in all cases that can arise in polyphase power measurements. However, the consideration of the cases which are of frequent occurrence in practice is instructive.

Designation of Wattmeter Terminals.—As some of the mean products in (26) may be negative, it is necessary that the connections be so arranged that a negative deflection of the wattmeter signifies that the reading should be subtracted when computing the power. That there may be no confusion, the potential and current terminals of the wattmeters through which the cur-

rents should enter when flowing from the generator to the load should be determined and marked on the instruments once for all. The proper marking may be determined by putting the instruments in a single-phase circuit. Then, whenever the instruments are used, the currents, as they flow from the generator to the load, must enter both the current and the potential coils at the marked terminals. When the instruments are so connected, if the pointer deflects up the scale, the mean product vi is positive; if the deflection is in the contrary direction, the mean product is negative. To obtain its numerical value the *current* coils must be reversed, and the reading so obtained regarded as negative. This simple procedure avoids all uncertainty as to the algebraic signs of the readings and renders unnecessary any special tests for their determination. The terminals of current and potential transformers should be similarly marked.

Two-phase Three-wire System.—By the theorem, two wattmeters are required, the connections being as in Fig. 193.

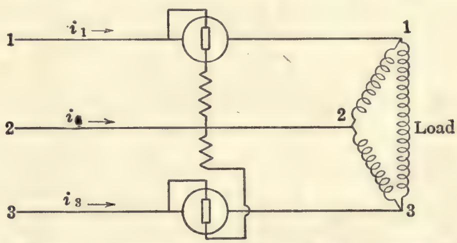


FIG. 193.—Power measurement; two-phase three-wire system.

If the two phases are separately loaded, it is obvious that the power is the sum of the wattmeter readings. A load might, however, be connected between leads 1 and 3 as indicated and then the instantaneous power would be

$$
\begin{aligned}
p &= v_{12}i_{12} + v_{23}i_{23} + v_{31}i_{31} \\
v_{31} &= v_{32} + v_{21} \\
p &= v_{12}i_{12} + v_{23}i_{23} + v_{32}i_{31} + v_{21}i_{31} \\
&= v_{12}(i_{12} - i_{31}) + v_{32}(i_{31} - i_{23}) \\
i_1 &= i_{12} + i_{13} = i_{12} - i_{31} \\
i_3 &= i_{32} + i_{31} = i_{31} - i_{23} \\
\therefore P &= \frac{1}{T}\int_0^T v_{12}i_1 dt + \frac{1}{T}\int_0^T v_{32}i_3 dt \qquad (27)
\end{aligned}
$$

The two wattmeters evaluate the integrals.

Two-wattmeter Method.—Three-phase system, load connected in Y.

By Blondel's theorem two wattmeters are required. Referring to Fig. 194, the instantaneous power is

$$p = v_{10}i_1 + v_{20}i_2 + v_{30}i_3$$
$$v_{12} = v_{10} + v_{02}$$
$$v_{32} = v_{30} + v_{02}$$
$$i_1 + i_2 + i_3 = 0$$
$$p = i_1(v_{12} + v_{20}) + i_2v_{20} + i_3(v_{32} + v_{20})$$
$$= i_1v_{12} + i_3v_{32} + (i_1 + i_2 + i_3)v_{20}$$
$$\therefore P = \frac{1}{T}\int_0^T i_1v_{12}dt + \frac{1}{T}\int_0^T i_3v_{32}dt. \tag{28}$$

The two integrals are given by the wattmeters.

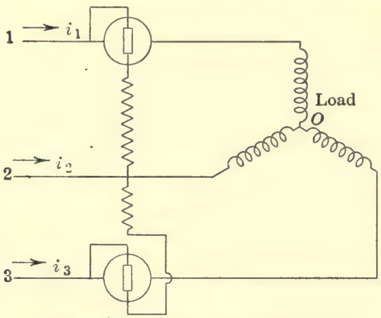


FIG. 194.—Power measurement with Y load.

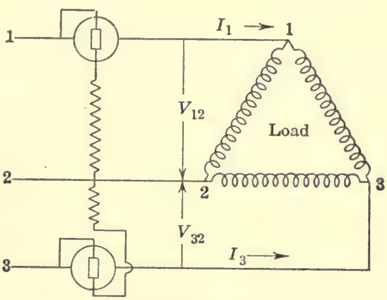


FIG. 195.—Power measurement with Δ load.

A Δ-connected load is similarly dealt with.

To draw the vector diagram corresponding to Fig. 195, assume that the load is balanced. Let V_{12}, V_{23}, V_{31} be the line voltages and θ the phase angle between the potential applied to, and the current in, any branch of the circuit.

From the figure it is evident that instrument No. 1 gives the mean product of the instantaneous value of V_{12} and I_1 while No. 2 gives the same product for V_{32} and I_3. Using the effective values of line voltage and line current (refer to Fig. 196)—

Reading of No. 1 $= VI \cos (30° + \theta) = VI\,(- \sin \theta \sin 30° + \cos \theta \cos 30°)$

Reading of No. 2 $= VI \cos(30° - \theta) = VI(\sin\theta \sin 30° + \cos\theta \cos 30°)$.

$$\text{Sum of No. 1 and No. 2} = VI\,(2 \cos 30° \cos\theta)$$
$$= VI\sqrt{3}\cos\theta \qquad (29)$$

which is the power applied to the circuit.

As the lag angle, θ, increases, the reading of No. 1, which is the smaller of the two, decreases and will become zero when $\theta = 60°$ (corresponding to a power factor of 0.5), for then I_1 and V_{12} are in quadrature. If $\theta > 60°$, No. 1 will reverse and

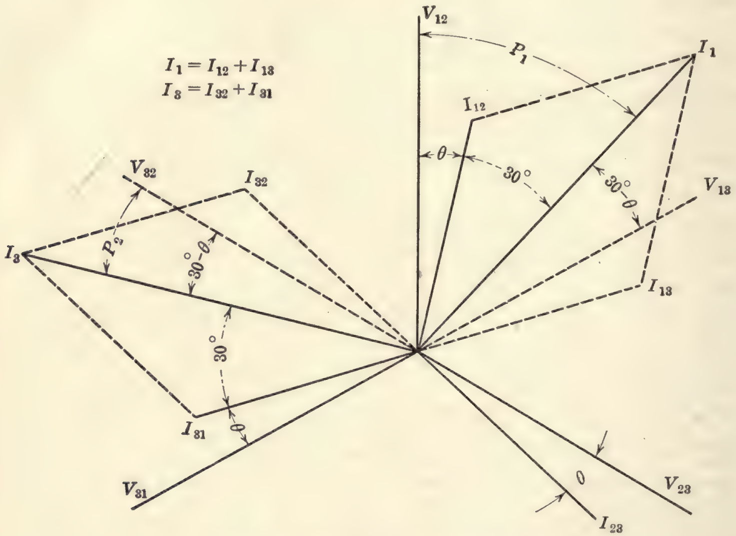


FIG. 196.—Vector diagram for Fig. 195.

its reading is taken as negative when adding the readings of No. 1 and No. 2 to obtain the power.

It will be noticed that the phase difference of V_{32} and I_3 is the same as that of V_{13} and I_1 and as the maximum values of the current and voltage are the same in both cases, one wattmeter will suffice for measurements on a *balanced* load. For example, two readings may be taken with wattmeter No. 1, the first with the potential terminals connected between mains 1 and 2, the second with the potential terminals connected between mains 1

and 3. If it is necessary to reverse the current coils, the smaller of the readings is considered negative.

The Polyphase Wattmeter.—To avoid the necessity of using two separate instruments the polyphase wattmeter has been devised.

The instrument consists of two complete wattmeters mounted in the same case. The two movable coils are attached to the same rigid stem and consequently act against the same spring. The deflection is thus made to depend on the sum of the torques of the two elements, so that the total power is read directly from the scale. The electrical connections are the same as for two single-phase wattmeters, see Fig. 194.

Ample insulation must be provided between the two elements and it is imperative that the stray field from one element have no influence on the torque generated by the other element. Protection from both external and internal stray fields may be obtained by the use of laminated shields.

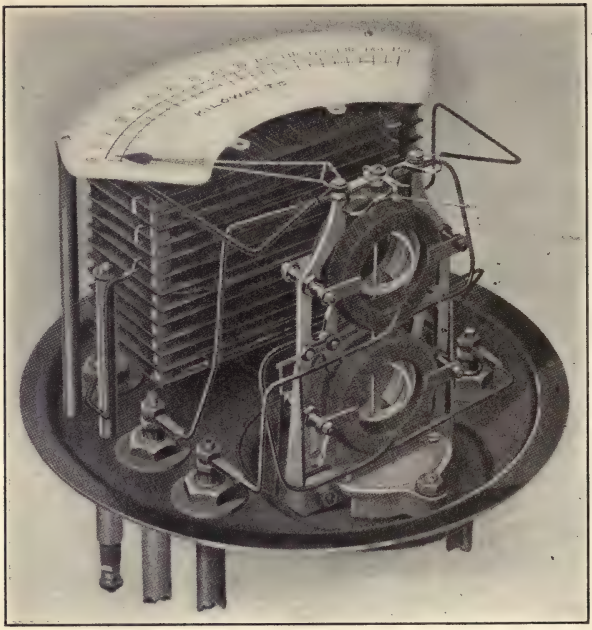
In careful tests at low power factors the use of the polyphase wattmeter in connection with instrument transformers is to be avoided, for the necessary corrections for the ratio and phase angle of the current transformers cannot be made.

Fig. 197 shows two forms of polyphase wattmeter. The laboratory instrument is read by means of a torsion head and the effect of the stray field from the upper element on the torque of the lower element and *vice versa* is minimized by placing the elements with their axes perpendicular.

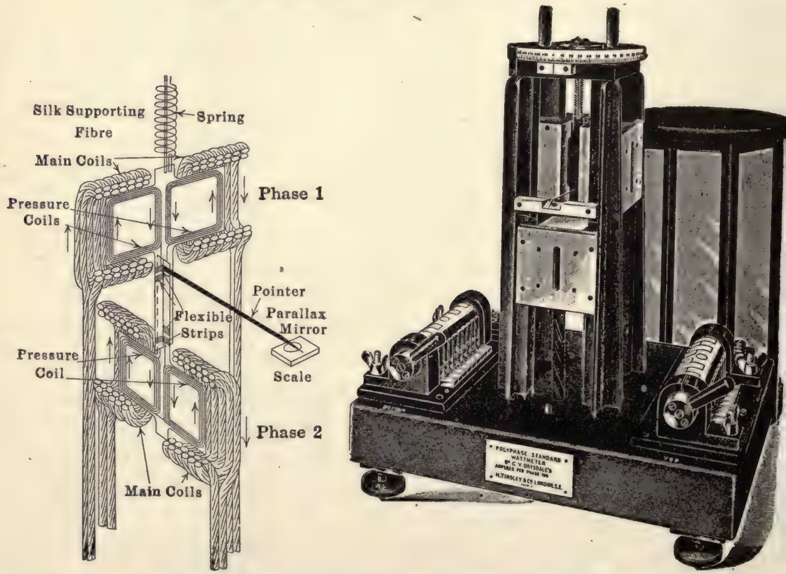
In switchboard instruments the interference between the elements, which would naturally be large, may be compensated by an ingenous device due to Edward Weston. Ordinarily the potential circuits of both elements of a polyphase wattmeter are connected directly to lead number 2 (see Fig. 195). Then, if there is no interference and the resistance of *each* potential circuit is $r_1 + r$ ohms, the turning moment acting on the movable system is, when the potential coil currents are i_{12} and i_{32},

$$M = \frac{K}{T}\int_0^T i_1 i_{12}dt + \frac{K}{T}\int_0^T i_3 i_{32}dt =$$

$$\frac{K}{r_1 + r}\left[\frac{1}{T}\int_0^T i_1 v_{12}dt + \frac{1}{T}\int_0^T i_3 v_{32}dt\right]. \qquad (30)$$



Weston polyphase switchboard wattmeter.



Drysdale polyphase wattmeter for laboratory work.
FIG. 197.

K is the dynamometer constant for each of the two elements. Equation (30) gives the correct value of the turning moment.

If there be interference between the elements, and K' is the constant of the dynamometer formed by the lower fixed coil and the upper movable coil, or by the upper fixed coil and the lower movable coil, the turning moment is modified and becomes

$$M' = \frac{K}{T}\int_0^T i_1 i_{12}dt + \frac{K'}{T}\int_0^T i_3 i_{12}dt + \frac{K}{T}\int_0^T i_3 i_{32}dt + \frac{K'}{T}\int_0^T i_1 i_{32}dt \tag{31}$$

The electrical connections for Weston's method of compensation are shown in Fig. 198.

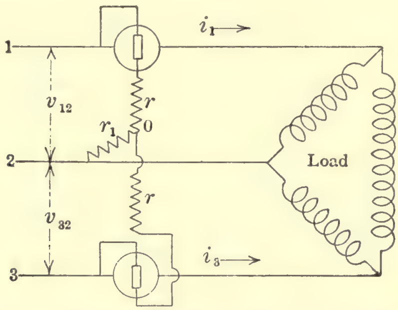


FIG. 198.—Connections for compensating a polyphase wattmeter for interference between elements.

The compensation is effected by altering the potential coil currents.

This is done by causing the potential-coil circuits to have the part r_1 of their resistances in common. The currents through the potential coils then become

$$i'_{10} = \frac{-v_{32}r_1 + v_{12}(r_1 + r)}{r(2r_1 + r)}$$

and

$$i'_{30} = \frac{v_{32}(r_1 + r) - v_{12}r_1}{r(2r_1 + r)}$$

When the potential coil currents in equation (31) are replaced by these new values the turning moment becomes

$$M' = \frac{K(r_1 + r) - K'r_1}{r(2r_1 + r)}\left[\frac{1}{T}\int_0^T i_1v_{12}dt + \frac{1}{T}\int_0^T i_3v_{32}dt\right] + \frac{K'(r_1 + r) - Kr_1}{r(2r_1 + r)}\left[\frac{1}{T}\int_0^T i_1v_{32}dt + \frac{1}{T}\int_0^T i_3v_{12}dt\right].$$

The second term on the right-hand side disappears if r_1 is adjusted so that

$$K' = \frac{Kr_1}{r_1 + r}$$

M' then reduces to

$$M' = \frac{K}{r_1 + r}\left[\frac{1}{T}\int_0^T i_1v_{12}dt + \frac{1}{T}\int_0^T i_3v_{32}dt\right].$$

That is, the turning moment becomes the same as if there were no interference. If the instrument is of the deflectional type the correctness of this adjustment will vary for different points on the scale but the error will be small.

The present practice of the Weston Instrument Co. is to use laminated iron shields between the two elements.

Three-phase Power Measurement by Three Wattmeters.—If the neutral is accessible, the sum of the readings of three wattmeters connected as in Fig. 199 will give the power.

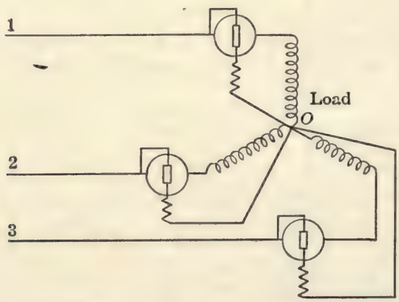


FIG. 199.—Connections for measurement of three-phase power by three wattmeters.

With this scheme of connections, no question can arise as to the algebraic sign of the readings; their arithmetical sum gives the desired result.

It is not necessary that the neutral be accessible, for by the theorem one has only to connect the potential coils at some common point, as O' in Fig. 200.

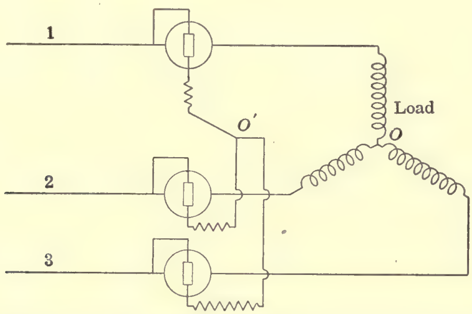


FIG. 200.—Connections for three-phase power measurements with artificial neutral.

As previously shown

$$P = \frac{1}{T}\int_0^T v_{12}i_1dt + \frac{1}{T}\int_0^T v_{32}i_3dt$$

but

$$v_{12} = v_{10'} + v_{0'2}$$
$$v_{32} = v_{30'} + v_{0'2}$$

so

$$P = \frac{1}{T}\int_0^T (v_{10'} + v_{0'2})i_1dt + \frac{1}{T}\int_0^T (v_{30'} + v_{0'2})i_3dt$$
$$= \frac{1}{T}\int_0^T v_{10'}i_1dt + \frac{1}{T}\int_0^T v_{30'}i_3dt + \frac{1}{T}\int_0^T v_{0'2}(i_1 + i_3)dt$$
$$i_1 + i_3 = -i_2$$
$$\therefore P = \frac{1}{T}\int_0^T v_{10'}i_1dt + \frac{1}{T}\int_0^T v_{30'}i_3dt + \frac{1}{T}\int_0^T v_{20'}i_2dt \qquad (32)$$

= sum of wattmeter readings.

No assumptions are made as to the resistances of the potential circuits of the wattmeters.

If the load is balanced, the readings on all three instruments will be the same; this leads to the use of the Y-box for the measurement of balanced three-phase loads.

The Y-box.—The Y-box consists of a small case, like that of an ordinary multiplier, containing two resistors in series, *both of which have a resistance equal to that of the potential circuit of the wattmeter with which the box is to be used.* As a tap is carried to the junction of the two resistors the Y-box has three terminals. It is connected in circuit as indicated in Fig. 201.

If the load is balanced the power is given by $P = 3$ times reading of wattmeter.

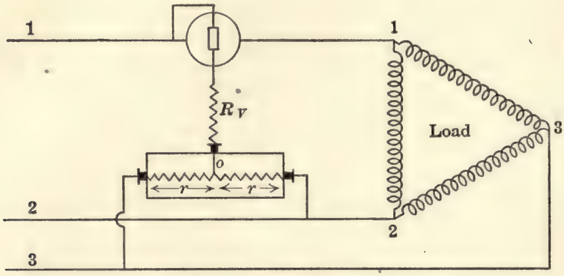


FIG. 201.—Connections of Y-box for measuring three-phase power under balanced loads.

It may be convenient to use a Y-box with some other instrument than that for which it was designed; in this case the factor is no longer 3. Referring to Fig. 201, by Kirchoff's laws the current i_V through the potential coil of the wattmeter is

$$i_V = \frac{v_{12} + v_{13}}{2R_V + r},$$

where R_V is the resistance of the potential circuit of the wattmeter and r is the resistance of each section of the Y-box. If the line current be i_L the reading of the wattmeter is given by

$$R_V \frac{1}{T}\int_0^T i_V i_L dt.$$

Therefore,

$$\text{Reading} = \frac{R_V}{2R_V + r}\left[\frac{1}{T}\int_0^T i_1 v_{12} dt + \frac{1}{T}\int_0^T i_1 v_{13} dt\right]$$

For a *balanced load* (see page 332).

$$\text{power} = \frac{1}{T}\int_0^T i_1 v_{12} dt + \frac{1}{T}\int_0^T i_1 v_{13} dt$$

$$\therefore P = \frac{2R_V + r}{R_V} \text{ times reading of wattmeter} \qquad (33)$$

Four-wire Three-phase System.—The power will be given by the sum of the readings of three wattmeters connected between the leads 1, 2, 3 and the neutral point. Fig. 202 shows another arrangement which also conforms to Blondel's theorem.

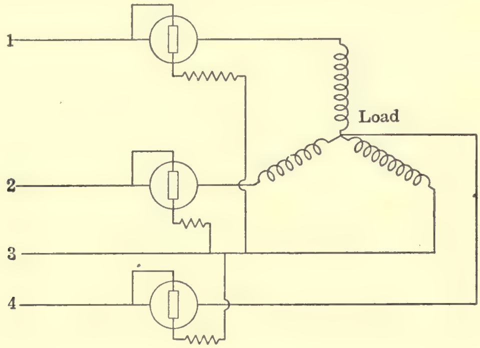


FIG. 202.—Connections for power measurement in a four-wire three-phase circuit.

$$p = v_{14}i_1 + v_{24}i_2 + v_{34}i_3$$

but $v_{13} = v_{14} + v_{43}$

$$v_{23} = v_{24} + v_{43}$$

$$i_1 + i_2 + i_3 + i_4 = 0$$

so
$$\begin{aligned} p &= (v_{13} + v_{34})i_1 + (v_{23} + v_{34})i_2 + v_{34}(-i_1 - i_2 - i_4) \\ &= v_{13}i_1 + v_{23}i_2 + v_{43}i_4 + v_{34}(i_1 + i_2 - i_1 - i_2) \end{aligned}$$

$$\therefore P = \frac{1}{T}\int_0^T v_{13}i_1dt + \frac{1}{T}\int_0^T v_{23}i_2dt + \frac{1}{T}\int_0^T v_{43}i_4dt \qquad (34)$$

which is the sum of the readings of the three wattmeters. The theorem shows that the power in any four-wire combination of loads may be measured by three wattmeters.

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CHAPTER VII

MEASUREMENT OF INDUCTANCE AND CAPACITY

STANDARDS OF INDUCTANCE

For carrying out the methods of measurement described in this chapter, standards of inductance and capacity are required, and convenience dictates that, in many cases, they be made adjustable. These standards may be divided into two classes: primary standards, whose values are calculated from their dimensions; and secondary standards, whose values are determined experimentally.

The subject of the calculation of primary standards of self- and mutual inductance is beyond the scope of this work. Readers are referred to the papers of Rosa and Grover, who have collected and tested all the available formulæ and have published them together with illustrative examples in the *Bulletin* of the Bureau of Standards.[1]

Standards of Mutual Inductance.—Primary standards of mutual inductance which have a single fixed value are useful in the calibration of ballistic galvanometers, also in the calibration of variable working standards of mutual inductance which are used in the laboratory.

The considerations governing the design are:

1. The value must be accurately calculable from the geometrical dimensions.

2. The construction must be such that permanence is assured.

3. The value must be sufficiently large to give high sensitivity when comparisons are made.

4. The resistances of the coils must be kept as low as possible.

5. Eddy-current effects must be eliminated as far as possible.

6. The capacity effect between the primary and the secondary must be a minimum.

7. The bobbins upon which the coils are wound must be free from magnetic materials.

To eliminate eddy-current effects, all conductors which carry large currents must be made of insulated strands, and all metal frames, etc., near the coils must be avoided.

In the past inductance coils have frequently been wound on bobbins of serpentine. It has been found, however, that a coil so wound has an inductance which depends, to a slight extent, upon the strength of current flowing in the conductor, thus showing that the permeability of serpentine is not unity and that it depends on the magnetic field in which the serpentine is placed.

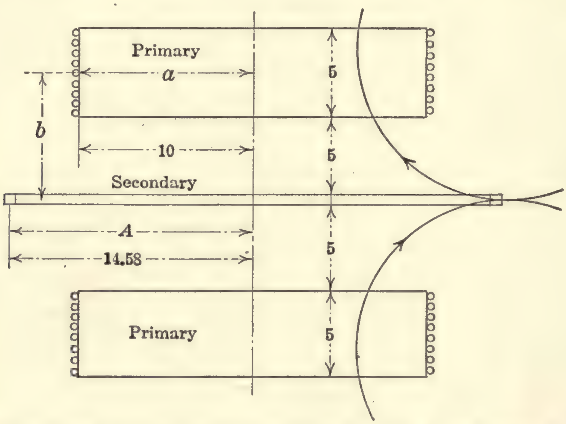


FIG. 203.—Campbell single valued primary standard of mutual inductance.

For primary standards of *self-inductance* it is customary to use single layer coils which are wound on accurately ground cylinders of marble, or of plaster of paris which has been impregnated with paraffin. Such a construction facilitates the exact determination of the geometry of the coil. In a standard of *mutual inductance,* this construction is not admissible for *both* the primary and the secondary coils, since the product of the primary and secondary turns must be large, about 100,000 for a mutual inductance of 0.01 henry.

Campbell Fixed Standard of Mutual Inductance.[2]—It is highly desirable that the arrangement of the primary and secondary coils in a standard of mutual inductance be such that errors arising from slight displacements of the coils from their supposed relative position will be reduced to a minimum. By

dividing the primary into two equal sections, properly separating them, and placing the secondary midway between them, all three coils being coaxial, a satisfactory arrangement may be obtained. If the diameter of the secondary coil is such that the mutual inductance is a maximum, the secondary will be so placed that its circumference is in a zero field. For this reason a small variation in the diameter or a small axial displacement of the coil will produce only a slight variation in the mutual inductance. The construction is indicated in Fig. 203, where the proper relative dimensions are shown.

With the proportions indicated a multiple layer secondary having a considerable cross-section (0.5 cm. by 0.5 cm.) may be used. The effect of variations of the diameter of the secondary coil are shown below. The mutual inductance is a maximum when $A = 14.58$ cm. If the secondary circuit is displaced from the midposition between the primary coils by 0.35 cm., m is reduced by less than 1 part in 10,000.

Campbell Standard of Mutual Inductance

$a = 10$ cm., $b = 7.5$ cm., $n_1n_2 = 100{,}000$

A, in centimeters......	14.1	14.3	14.5	14.7	14.9	15.0
m, in millihenrys......	9.1630	9.1728	9.1759	9.1759	9.1696	9.1567

Variable Mutual Inductances.—Variable mutual inductances, in other words, air core transformers of variable ratio, are extremely useful in alternating-current measurements, as, for example, in determining self-inductances and in measuring the ratios of instrument transformers.

For the highest utility the coils should be wound astatically, expecially if the apparatus is to be used in an electrical engineering laboratory, and even then one should satisfy himself by tests that stray field effects, due to non-uniform fields, are absent. If an astatic arrangement is not used, great care must be taken that the standard is not set up where there are alternating stray fields or where its field will influence other instruments.

It is desirable that the scales of variable mutual inductances be as uniform as possible. By the use of Lord Rayleigh's ar-

rangement of two concentric circular coils, the ratio of the radii being 0.548 (see page 80), a practically uniform scale extending over about 60° may be obtained; if the coils are used in conjunction with a fixed mutual inductance the scale may be extended to about 120°.

Ayrton and Perry Inductor.—The Ayrton and Perry variable standard of self-inductance has long been used for general laboratory purposes. This standard consists of two coils of slightly different diameters, the smaller pivoted within the larger in such

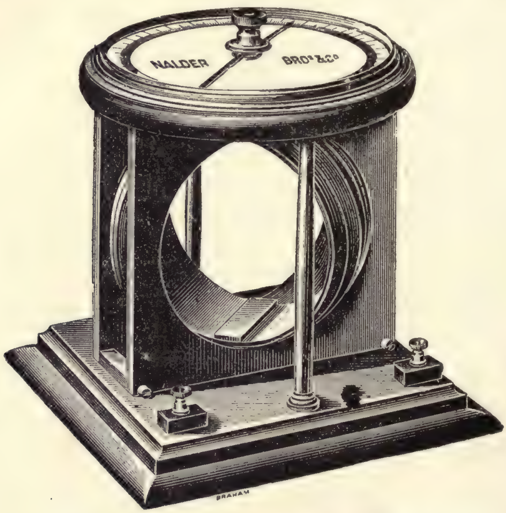


FIG. 204.—Ayrton and Perry variable inductor.

a manner that it can be rotated about a vertical diameter. The coils are connected in series and a pointer shows their relative position; as each position corresponds to a particular value of the self-inductance of the combination, the dial may be graduated in henrys. When the index stands at the lower end of the scale, the inductance of the combination is nearly zero, owing to the fact that the currents in the two coils are circulating in opposite directions. When the movable coil is turned through 90°, the inductance becomes the sum of the self-inductances of the two coils, as there is no mutual induction in this position. When turned through 180° the currents in the coils are in the same di-

rection and the total inductance becomes the sum of the self-inductances of the coils and twice their mutual inductance. Thus the inductance of the standard may be varied continuously from a minimum, which is nearly zero, to a maximum, usually about 5 millihenrys. The scale is irregular and the instrument is not astatic.

Brooks Variable Inductor.[3]—The Brooks Variable Inductor very closely fulfills the requirements for a variable standard of mutual and self-inductance. It was developed for use in testing current transformers (see page 581) but is applicable in any measurement where such a variable standard, having a constant resistance, is necessary.

The particular advantages of the instrument are its large carrying capacity, low resistance and practically uniform scale. The range of the instrument, as described, is from 125 to 1,225 microhenrys.

Fig. 205 shows the instrument complete and in section, as well as the form of the coils. Referring to the diagram the four coils, F, are fixed; by means of the handle, H, the two movable coils, M, can be displaced in their own plane about the axis, A. The number of flux linkages between F and M can thus be varied. The coils of stranded wire are arranged astatically. Current is carried to the movable coils through heavy copper spirals, thus eliminating all contact resistances. The cross hatching in the diagram indicates the numbers of turns in the coils. The four fixed coils are permanently connected in series and provided with binding post terminals; likewise the two movable coils.

When the fixed and movable elements are connected in series the instrument may be used as a standard of self-inductance, and when they are separated, as a standard of mutual inductance.

The self-inductance, or scale reading, is

$$L = L_1 + L_2 \pm 2m$$

where L_1 and L_2 are the inductances of the fixed and movable elements and m is the mutual inductance of the elements. L_1 and L_2 are constants, and for the instrument as described, $L_1 + L_2 = 669$ microhenrys. The expression for the mutual inductance is therefore

$$\pm m = \frac{L - (L_1 + L_2)}{2} = \frac{L - 669}{2}.$$

The whole device is about 14 in. in diameter. The interleaving of the fixed and movable coils is important for if M recedes axially from one fixed coil it approaches the other, so that the net effect on the inductance of a slight axial displacement of M is zero.

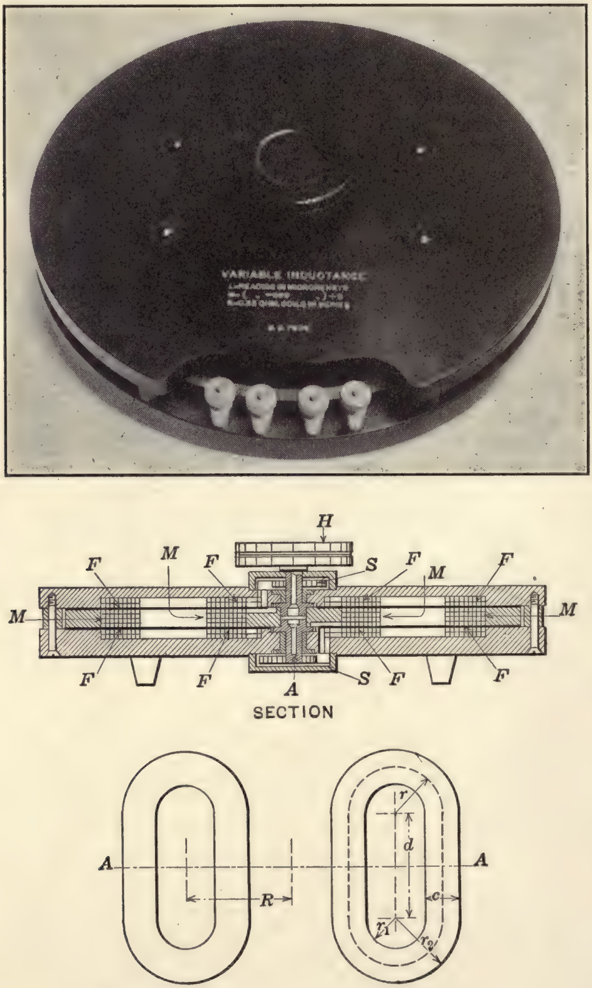


Fig. 205.—Brooks variable inductor.

The distinctive feature of this inductor is that the scale divisions are of equal length throughout the greater part of the use-

ful range. For instance, in an instrument having a useful range of from 125 to 1,225 microhenrys the scale is uniformly divided between 325 and 1,025 microhenrys; outside of these limits the length of the divisions gradually decreases, but there are no sudden changes.

The uniform scale is attained by using link-shaped coils; the proper proportions were determined experimentally and are given below.

Referring to Fig. 205

$$
\begin{aligned}
r &= \text{mean radius of semicircular end of coil}\\
c &= 0.78r\\
d &= 2.2r\\
R &= 2.26r\\
r_1 &= 0.61r\\
r_2 &= 1.39r
\end{aligned}
$$

The net cross-section of the fixed and movable coils is a square having a side c units long.

STANDARDS OF CAPACITY

As examples of primary standards of capacity, that is, condensers whose capacities in electrostatic units are calculated from their dimensions, those used by Rosa and Dorsey[4] in their determination of v, the ratio of the electromagnetic to the electrostatic unit of quantity, may be taken. It is in connection with the determination of v that the possible sources of error in such primary standards have been most carefully studied.

In order to be able to calculate the capacity of a condenser with a high degree of accuracy, the dielectric coefficient of the medium between the plates must be definitely known and the medium must be free from absorption and from dielectric losses. For these reasons air is always used as the dielectric in primary condensers. Also, the distance between the plates must be so great that the thickness of the dielectric may be determined with accuracy. In consequence of these facts the capacities of primary condensers are very small.

Three forms of primary condensers have been developed, viz., those with spherical, cylindrical and plate electrodes. A section

of the spherical condenser used by Rowland in the determination of v in 1879, by Rosa in 1889, and by Rosa and Dorsey in 1905, is shown in Fig. 206.

The capacity in electrostatic units of such a spherical air condenser is

$$C = \frac{Rr}{R-r}$$

where R and r are the radii bounding the dielectric.

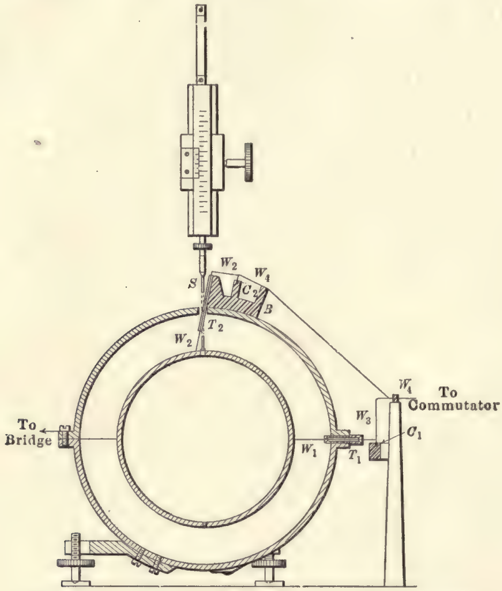


FIG. 206.—Section of spherical air condenser.

The internal radius of the spherical shell in the Rowland condenser is 12.67158 cm., and the radius of the ball is 10.11806 cm. (at 20°). The capacity is, therefore, 50.2095 electrostatic units.

When the condenser is used, the ball must be carefully centered and corrections made for the holes in the shell, the bushings, and the cord by which the ball is suspended. The error in the final calculated value of the capacity was estimated at about 2 parts in 100,000. When measured by Maxwell's method (see page 362) the capacity was found to be 5.59328×10^{-20} absolute electromagnetic units or 0.0000559328 microfarad.

In any experimental work with such a very small capacity, it is necessary to make allowances for the capacity of the wire by which the charge is imparted to the ball, for the capacity of all leads and of the commutator by which the charging and discharging is effected.

The capacity of an air condenser with coaxial cylindrical electrodes, if the charge be uniformly distributed, is given in electrostatic units by

$$C = \frac{l}{2 \log_\epsilon \frac{R}{r}}$$

where l is the length of the cylinder and R and r are the radii bounding the dielectric. For precision work, on account of the effect of the ends of the condenser, the assumption of a uniform density of charge is not tenable, so recourse is had to guard cylinders, shown in Fig. 207 at G. The effect of the ends is thus removed from the central section, which is the one connected to the measuring apparatus, to the guard cylinders where it does no harm. In order to make practically all the lines of force radial the air gaps between the main section and the guard cylinders must be made as small as possible, and the measuring apparatus so arranged that the guard cylinders are always at the same potential as the main or central section.

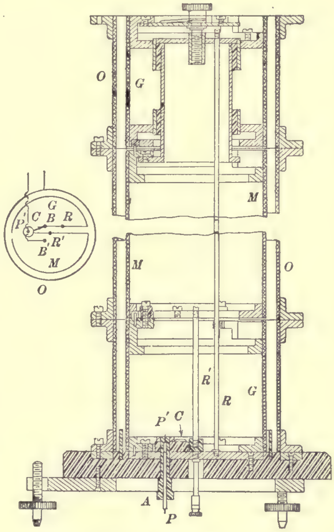


FIG. 207.—Section of cylindrical air condenser with guard cylinders.

For one of the condensers used by Rosa and Dorsey,

$$l = 20.00768 \text{ cm.}$$
$$R = 7.23831 \text{ cm.}$$
$$r = 6.25740 \text{ cm.}$$

Then, as a first approximation, $C = \dfrac{20.00769}{2 \log_\epsilon \dfrac{7.3831}{6.25740}} = 68.696$

electrostatic units.

This result must be corrected for the effects of the gaps between the main cylinder and the guard cylinders and for the different thicknesses of the dielectric on the two sides of the gaps due to differences in the diameters of the cylinders. The deduction of these corrections involves the use of the higher forms of analysis. The numerical value of the net correction for the condenser just mentioned, when the lower gap is 0.6 mm. and the upper gap 0.5 mm., is +0.182 electrostatic units. Thus, in the case of this short condenser, the corrections with even these small air gaps amount to over one-fourth of 1 per cent. The longer the central section, the smaller is the percentage correction.

For the most refined work the parallel plate condenser is inferior to these just referred to, for comparatively large errors may be introduced if the adjustments are not perfect.

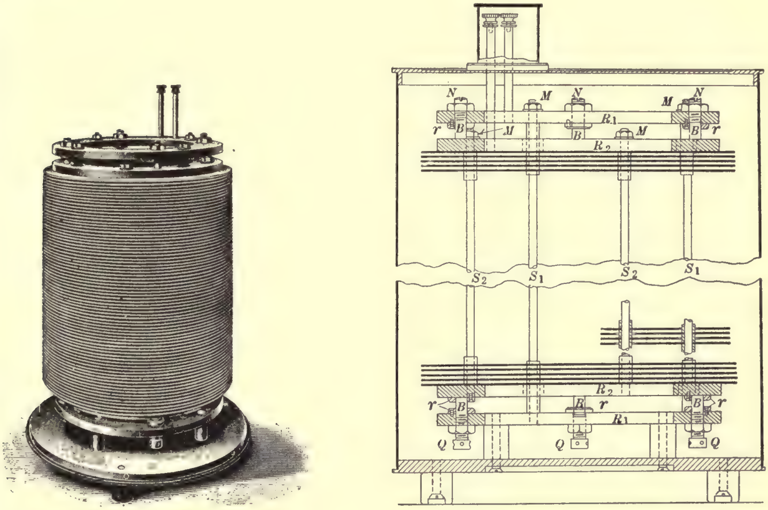
Secondary Air Condensers.—Secondary air condensers are useful in experimental work in those cases where the dielectric losses must be reduced to zero; as in determining the phase angles of mica condensers or in the investigation of the dielectric losses occurring in insulating materials. As such condensers must be calibrated, it is possible to use for each electrode a number of plates in parallel and to make the distance between the electrodes only a few millimeters. By this means capacities of a few hundredths of a microfarad may be obtained without undue bulk.

In any air condenser, the ohmic resistance between the terminals and the condenser proper must be kept low in order that there may be no appreciable internal I^2R losses which would cause an alternating current to lead the applied voltage by less than 90°.

A secondary air condenser,[5] having a capacity of 0.01 microfarad, is shown (with the case removed) in Fig. 208, *A*. The plates (of magnalium) are 20 cm. in diameter, 1 mm. thick, and so spaced that the thickness of the dielectric is 2 mm.; 35 plates are used in one electrode and 36 in the other. In order to insure permanence a very solid construction must be employed.

The plates are supported as shown in Fig. 208, *B*. The bronze ring, R_1, is firmly screwed to the base of the instrument; through it pass four adjusting screws of fine pitch, *Q*, which support a second ring, R_2, by means of the little amber cylinders, *B*, which

move in the guides, r. Four equally spaced brass rods, 5 mm. in diameter and having screw threads cut on their upper ends, are firmly attached to ring R_1, and pass upward, with ample clearances, through holes in the ring R_2. Four equally spaced vertical rods are attached to R_2. Each plate is pierced with eight holes, four of them 5 mm. and four 12 mm. in diameter. The holes are so placed that they accommodate the eight vertical rods.



A B

FIG. 208.—Secondary air condenser.

The condenser is built up by first putting on a member of electrode 2. The smaller holes in this plate just fit the rods S_2 and the larger holes allow ample clearances for the rods S_1. Distance pieces of the proper diameter and of a length sufficient to give a 2-mm. air space between the plates are then slipped over the rods S_1. They rest on the ring R_1 and pass with ample clearances through the holes in R_2; on these four distance pieces rests the first plate of electrode 1. Distance pieces 8 mm. in diameter and 5 mm. long are then slipped over the rods S_2 and rest on top of the first plate of electrode 2; the second plate of electrode 2 is then slipped on and rests on the top of the distance pieces, and so on.

When the pile has been completed, electrode 1 is firmly clamped between the rings R_1R_1 by means of the nuts M. Electrode 2 is clamped between the rings R_2R_2. The spaces between the two electrodes are finally adjusted by raising or lowering electrode 2 by means of the adjusting screws Q, which are then locked. The top of 2 is firmly held by tightening and then locking the screws N which bear on the ring R_2 by means of the amber cylinders B.

The assembled condenser is about 30 cm. high and weighs approximately 37½ lb.

By making the air space 1 mm. instead of 2 mm. and employing 107 plates, condensers having a capacity of 0.03 microfarad have been constructed. With this extremely small thickness of the dielectric, trouble was experienced in insulating the two sets of plates, for when voltage was applied fine particles of dust from the air bridged the space between the plates, thus reducing the insulation resistance. It is not possible to remove the dust after the condenser is assembled but the insulation may be improved by placing a drying material in the case of the instrument.

The breakdown voltage of the condenser with 2 mm. air space is 3,000 volts, and with 1 mm. air space, 900 volts. All sharp edges on the plates and internal fittings must be avoided, in order to prevent brush discharges.

To render the condenser independent of the surroundings, one set of plates is connected to the case, the other set being connected directly to the measuring apparatus.

Variable capacities are necessary for general laboratory purposes, but a difficulty presents itself when one attempts to put a number of very small condensers in parallel by the ordinary means, since the connections possess an unknown capacity which may be enough to introduce serious errors. For this reason it is necessary that the design of the small sections from which the larger capacities are built up be such that this uncertainty is eliminated.[6]

In the most refined work the temperature coefficient of an air condenser, due to the change of dimensions and change in the dielectric coefficient of the air, must be considered. It may amount per degree to 2 or 3 parts in 100,000.

The dielectric strength of air condensers may be greatly increased if the dielectric be dry compressed air, at a pressure of 60 lb. per square inch or greater.[7] This eliminates brush discharges and energy losses at high voltages. For example, with an air pressure of 175 lb. per square inch, a condenser with plates 2.1 mm. apart showed no appreciable energy loss at 27,500 volts. It broke down at 28,500 volts.

In another case with the plates 3.2 mm. apart the breakdown voltage at atmospheric pressure was 6,000 volts; when the

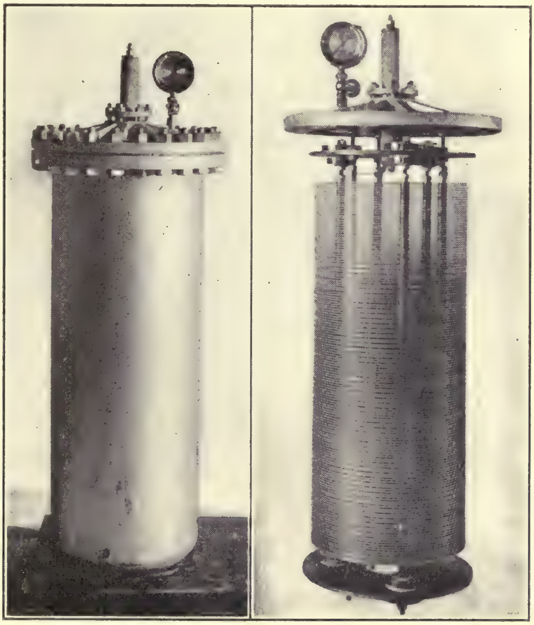


Fig. 209.—Compressed gas condenser for use in radio-telegraphy.

air pressure was raised to 260 lb. per square inch the breakdown voltage became 30,000.

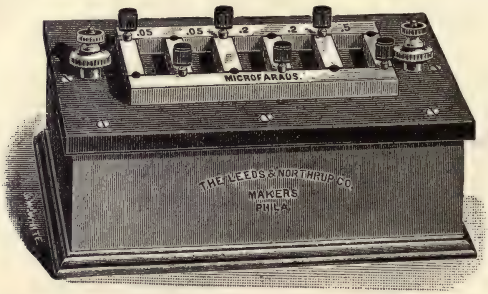
It is necessary to use a drying material in the condenser cases. The use of compressed air, of course, necessitates a strong and, therefore, a very heavy metal case. A practical difficulty arises in the introduction of the lead to the insulated set of plates; it must be perfectly insulated and all joints must be air-tight as well. Practically, the casing cannot be made absolutely

tight, so a pressure gage must be supplied and the air pressure renewed from time to time.

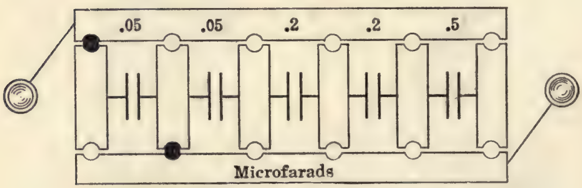
Experience has shown that with careful construction the pressure may not fall more than 20 lb. during a year from an initial value of 220 lb. per square inch.

Compressed gas condensers are used in connection with radio-telegraphic apparatus. Fig. 209 shows one of those installed at Arlington, Va., by the United States Government.

Working Standards of Capacity.—Though the determination of the capacity and phase angle of working standards of capa-



A



B

Fig. 210.—Subdivided condenser.

city depends ultimately upon the use of secondary air condensers, such instruments are not suitable for general use in the laboratory on account of their size, and more convenient arrangements must be employed. Ordinarily the capacity of laboratory standards is of the order of magnitude 1 microfarad. Such standards should be subdivided and made adjustable by arrangements for placing the various sections in parallel. When this is done the net capacity in terms of the capacities of the various sections is

$$C = C_1 + C_2 + C_3 + \dots .$$

The usual construction of the terminal blocks for putting the sections in parallel is shown in Fig. 210. By the proper insertion of the plugs, the sections may be put in parallel as desired, any section may be discharged, or the whole condenser may be short-circuited.

For precision standards of small capacity, this scheme of connections is open to the objection that the capacity of the top itself, which depends upon the position of the plugs, is included between the terminals. For precision work, it is better to have each section provided with small and well-insulated terminal

FIG. 211.—Subdivided precision condenser.

posts, spaced as far apart as practicable. This also allows the sections to be connected in series as well as in parallel.

With the series connection the capacity will be

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots}.$$

The capacity in electrostatic units of a condenser with parallel plates separated by a medium having a dielectric coefficient K is

$$C = \frac{KA}{4\pi t}.$$

A is the total active area of one set of plates and t is the thickness of the dielectric.

To transfer the value of the capacity from the electrostatic to the electromagnetic system of units,

$$v = 2.996 \times 10^{10}$$
$$v^2 = 8.976 \times 10^{20}.$$

One c.g.s. electromagnetic unit of capacity $= v^2$ c.g.s. electrostatic units of capacity.

One c.g.s electromagnetic unit of capacity $= 10^{15}$ microfarads.

Consequently the capacity in microfarads of a parallel plate condenser is

$$C = \frac{KA10^{15}}{(2.996 \times 10^{10})^2\, 4\pi t} = \frac{0.886KA}{10^7 t}.$$

Obviously, to increase the capacity without an undue increase of bulk, a dielectric having a high dielectric coefficient and capable of being used in very thin sheets must be employed. The material chosen should have an exceedingly high specific resistance and high dielectric strength.

The above formula for capacity is convenient for rough preliminary calculations; as t cannot be known with any great certainty all condensers with dielectrics of small thickness must be calibrated.

All materials used in the construction of condensers must be clean and perfectly dry and the finished instrument must be sealed in some manner so that the access of moisture is prevented. Mica and paraffined paper are the materials commonly used for the dielectric. Air pockets in the dielectric must be entirely eliminated.

Temperature has an appreciable effect on the behavior of condensers having solid dielectrics. It is not possible to give a definite statement as to the temperature coefficient of the capacity of a particular condenser, for the temperature effects are dependent on the particular cycle of operations to which the condenser is subjected. This is illustrated in Figs. 212A and 212B, which are typical of good and poor mica condensers. The best mica condensers when subject to the ordinary fluctuations of room temperature may show variations in the capacity of 2 or 3 parts in 10,000.

The active portion of any condenser intended for use as a standard must be firmly confined between clamps, so that its geometry and, consequently, the capacity of the condenser may be definite. Condensers *without clamps* are greatly affected by temperature, and when taken through a cyclic variation of temperature (for instance, 17°, 30°, 17°), do not return to their initial capacities. This permanent alteration may be as much as 3 or 4 parts in 10,000.

Condensers on Direct-current Circuits.—In the use of condensers with direct currents, difficulties arise from "absorption" and its related effects. It is found that the discharge of any condenser having a solid dielectric consists of two portions—a sudden rush of current at the instant of closing the circuit, due to the free charge, and a small, gradually decreasing current, due to the liberation of the "absorbed" charge. This latter current complicates the various methods of measurement when direct currents are employed (see "Direct-deflection Method," page 369). If high voltages be used, the absorbed charge continues to be given up for a long time.

When the condenser is charged, the first rush of current consists of two portions—one furnishing the free charge, the second a diminishing current furnishing the absorbed charge. This latter current, for a short time, about 0.01 sec., may be relatively large. If the charging circuit be broken too soon, before the dielectric is "saturated," the absorption goes on, and if there is a delay in discharging the condenser, the free charge will be diminished below its proper value. Thus the apparent capacity depends on the previous history of the condenser, on the time of charging, on the length of time between disconnecting from the battery and discharging and on the time of discharge.

An arbitrary measure of the absorption may be obtained by subjecting the condenser to a definite series of operations, for example, by charging for 1 sec., insulating for 30 sec., discharging instantaneously through a ballistic galvanometer, insulating for 30 sec., discharging again, and so on, until five residual deflections have been obtained. The measure of the absorption is the total quantity in the five residuals expressed as a fraction of the free charge. The absorption curves in Fig. 212 were obtained in this manner.

As the absorption increases very markedly with the increase in temperature, while the insulation resistance decreases, condensers are preferably used at low temperatures, about 20°.

The measured capacities of condensers which have large absorption are greatly affected by the time of discharge.

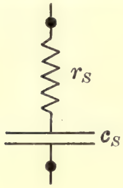
Condensers on Alternating-current Circuits.—If an air condenser, which is perfectly insulated and the resistance of whose leads is zero, is subjected to an alternating potential difference, the current flowing into the condenser will lead the potential difference across its terminals by 90°, there being no expenditure of energy; if the dielectric is solid, energy is expended in the condenser as is shown by its rise of temperature under continuous operation. If energy is expended, the current flowing into the condenser must have an energy component, or, in other words, the current and potential difference will no longer have a phase difference of 90°. The amount of departure from the 90° relation will be denoted by ϕ: the power factor of the condenser is then sin ϕ. The angle ϕ, called the phase angle, is dependent on the quality of the condenser. For a first-class instrument with mica as the dielectric, it may not be more than a few minutes of arc, possibly 5, and may be much below this. If the condenser is of poor quality with a paraffined paper dielectric, this angle may in extreme cases be as much as 20°. The power factor of such inferior condensers is very sensitive to changes of frequency. It must not be assumed that mica condensers are of necessity characterized by very small phase angles, for such condensers from well-known makers may occasionally show phase angles of several degrees. Such abnormal values are found most frequently in the small sections ($1/1000$ microfarad) and show the condenser to be of poor quality. In a divided condenser, the different sections may have very different phase angles. The measured capacities of condensers which have large phase angles will be found to be very dependent on the frequency.

Various methods for the measurement of electrostatic capacity by means of alternating currents are to be given and it is of practical importance to be able to apply them to condensers with imperfect dielectrics such as are met with in practice, that is, to find the equivalent capacity of such condensers. As there is a dissipation of energy in the condenser, its equivalent arrangement

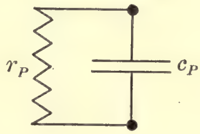
should be a perfect condenser in connection with such a resistance that the energy dissipated in the combination is the same as that in the actual condenser. The energy loss may be duplicated by assuming the resistance to be either in parallel or in series with the perfect condenser, as is indicated below.

Given V, I, P, ω, and assuming sinusoidal currents,

Series Arrangement



Parallel Arrangement



$P = I^2 r_S$	$P = \dfrac{V^2}{r_P}$
$\therefore r_S = \dfrac{P}{I^2}$	$\therefore r_P = \dfrac{V^2}{P}$
$Z_S = r_S - \dfrac{j}{\omega C_S}$	$Z_P = \dfrac{1}{\dfrac{1}{r_P} + j\omega C_P}$
$\tan\theta = \dfrac{1}{\omega r_S C_S}$ leading	$\tan\theta = \omega r_P C_P$ leading

$$\text{Power factor} = \text{P.F.} = \cos\theta$$

Phase angle, $\phi = \tan^{-1} \omega r_S C_S$	Phase angle, $\phi = \tan^{-1} \dfrac{1}{\omega r_P C_P}$
$C_S = \dfrac{I}{\omega V \sqrt{1 - \text{P.F.}^2}}$	$C_P = \dfrac{I}{\omega V} \sqrt{1 - \text{P.F.}^2}$

To illustrate the foregoing the following measurements of a 5-mile length of impregnated paper cable used in power transmission may be taken.

Applied P.D.	30,000 volts.
Current	10 amp.
Power supplied to cable	12,000 watts.
Cycles per second	60

From these data:

$$\text{Power factor} = \frac{12{,}000}{30{,}000 \times 10} = 0.04$$

Power-factor angle = 87°71, denoted by θ.
Phase angle = 90° − 87°71 = 2°29, denoted by ϕ.

Then

$r_S = 120$ ohms $\qquad r_P = 75{,}000$ ohms

leakance $= 0.0000133+$

$C_S = 0.8848$ microfarad $\qquad C_P = 0.8835$ microfarad

In cases where the dielectric losses are large, the equivalent capacities for the series and the parallel arrangements are slightly different.

The equivalent parallel resistance r_P has no relation to the insulation resistance as measured with direct current.

Mica Condensers.—Mica is used as the dielectric in the best working standards of capacity. Its specific resistance is about 1×10^{10} (megohm, centimeter). Its dielectric coefficient varies between 6 and 8. The puncturing voltage of selected specimens 0.1 mm. thick, when tested between plates may be as high as 12,000 volts (r.m.s.). The average strength is much lower.

Mica condensers are not ideally perfect and vary greatly in their properties, so that in careful work the characteristics of the particular condenser employed as a standard must be known. However, a mica condenser always behaves in the same manner if the same conditions be maintained. For this reason, in work of high precision, the cycle of operations to which the condenser is subjected, both when its capacity is determined and in subsequent use, should be the same. The capacity of a good mica condenser is independent of the voltage.

To be useful as a standard a mica condenser must be firmly clamped.

Experiments show that the capacity of a good mica condenser, when determined at higher and higher frequencies by a method of rapid charge and discharge using direct currents, approaches the same value as that obtained by the use of alternating currents, the period with alternating currents and the time of discharge with direct currents being the same. The two curves connecting the reciprocal of the frequency and the capacity and the time of discharge and the capacity, when extrapolated for infinite frequency and zero time of discharge apparently cut the capacity axis at the same point. The capacity determined by this process of extrapolation is called the "instantaneous," or by some writers, the "geometric" capacity, being independent

of absorption and depending only on the dielectric coefficient and on the dimensions of the condenser.

Changes of atmospheric pressure cause minute changes of capacity in mica condensers which may be detected by the most refined methods of measurement. The changes are subject to a considerable time lag and may be of the order of magnitude, 1 or 2 parts in 100,000 for 1 cm. change of pressure. Usually if the pressure be reduced, the condenser expands and as the increase in the distance between the plates produces more effect

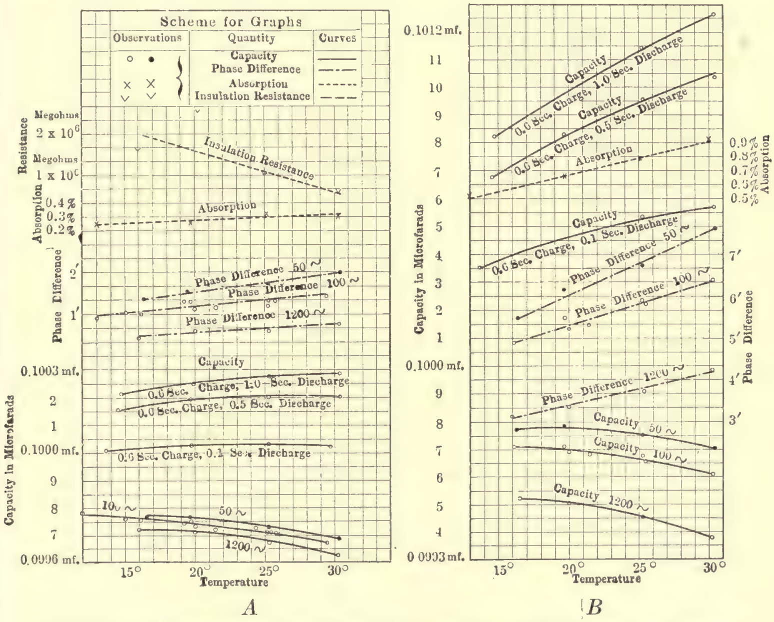


FIG. 212.—Characteristics of good (*A*) and poor (*B*) mica condensers.

than their increase of size, the capacity is decreased. Firmly clamped condensers are but very slightly affected.

The characteristics of good and poor mica condensers are illustrated by Fig. 212.

Condensers of silvered mica are sometimes used, but are inferior to those of the ordinary construction, being more unstable and having a capacity dependent upon the voltage. This unstable character is probably due to deposits of silver, under flakes of mica, which are imperfectly attached to the main deposit.

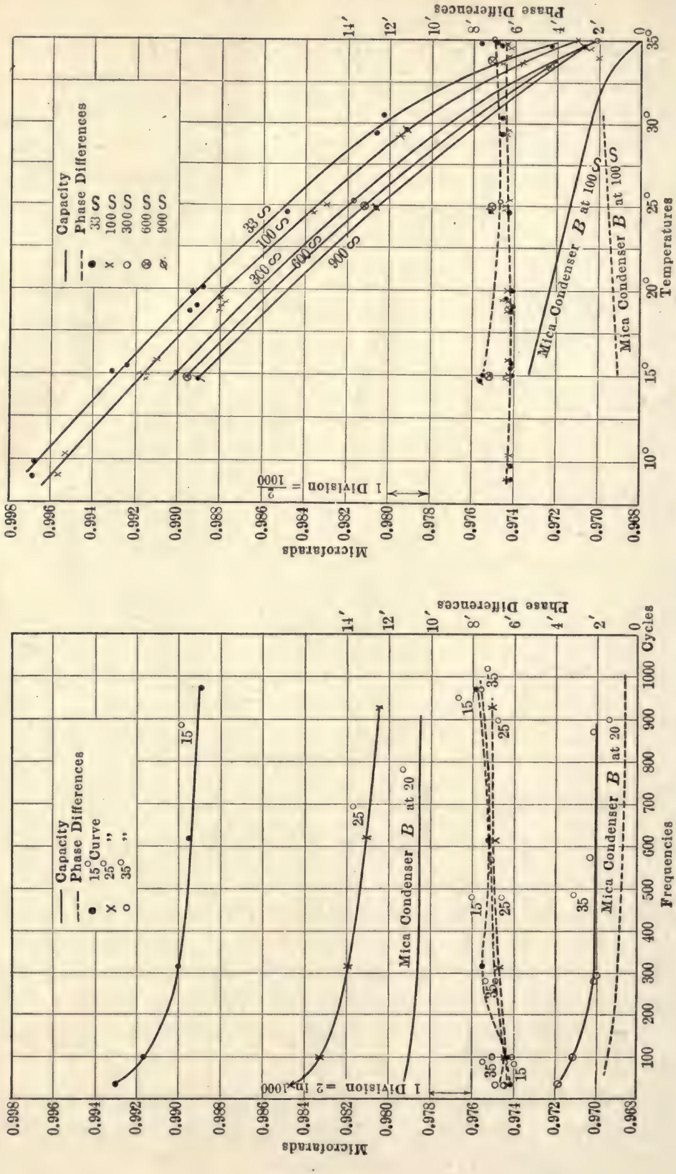


FIG. 213.—Characteristics of a good paraffined paper condenser.

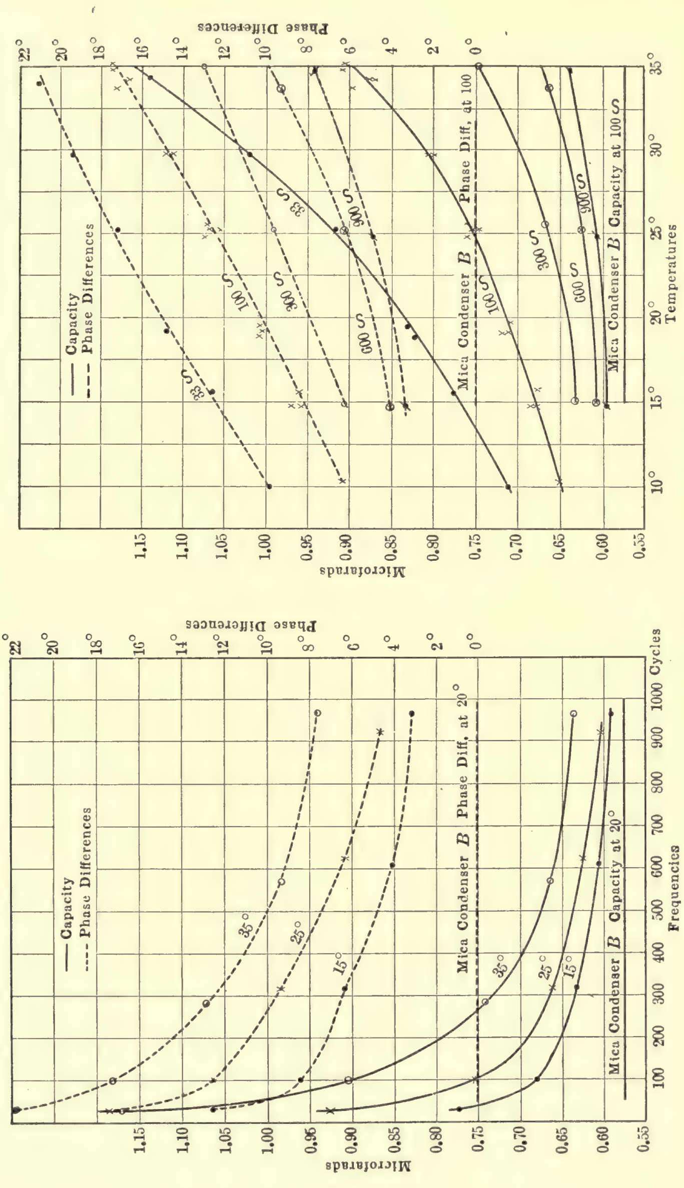


FIG. 214.—Characteristics of a poor paraffined paper condenser.

Paraffined Paper Condensers.[9]—On account of the possibility of large absorption effects and frequency errors, paraffined paper condensers should not be employed as standards. No general rule concerning their behavior can be formulated. When used with alternating current, the capacity of a paraffined paper condenser decreases with an increase of frequency and very markedly if the phase angle be large. An increase of temperature usually causes an increase in the capacity; for an exception see Fig. 213. The phase angle is much larger than that of a good mica condenser. It generally increases with a rise of temperature and more and more rapidly as the temperature becomes higher. The phase angle is very susceptible to changes in frequency. Usually an increase of frequency causes a decrease in the angle. Fig. 213 shows the characteristics of a good paraffined paper condenser. Fig. 214 applies to a rolled condenser such as is frequently used in telephony, and an inspection of the curves will show that this is a poor instrument and that in some respects its behavior is the reverse of that of the better condenser.

The internal resistance of a condenser, that is, the resistance of the connections from the binding posts to the plates and of the plates themselves, may cause an abnormal phase angle. This is the case in that form of telephone condenser which is made by rolling up long strips of tin foil together with the paper dielectric, the electrical connections being made at the *ends* of the strips. High internal resistance causes excessive heating and an increase of phase angle with an increase of frequency.

METHODS OF MEASUREMENT

Determination of Capacity in Absolute Measure.—Maxwell in his "Treatise on Electricity and Magnetism"* gives a bridge method for determining the electrostatic capacity of a condenser in electromagnetic measure. This method has been employed in many determinations of "v" and is probably the best yet devised for determining pure capacities in absolute measure.[4] The connections are shown in Fig. 215.

The condenser to be measured is at C; M, N, and P are the resistances of the bridges arms, B is the battery resistance and

*Art. 776, third edition.

R_G and L_G the resistance and self-inductance of the galvanometer. The resistances *ab* and *cd* are negligibly small. The condenser is rapidly charged and discharged by the switch *e*, which is usually a commutator driven at a constant and known speed.

When contact is made at *c*, the condenser is discharged and so remains until the tongue *e* touches *b*. Until *e* touches *b* the currents in the network are determined by the e.m.f. of the battery and the resistances, and a steady current flows through the galvanometer as indicated by the arrow. When contact is made with *b*, a varying current i_C will flow into the condenser until it is fully charged. A part of this varying current flows through *M*

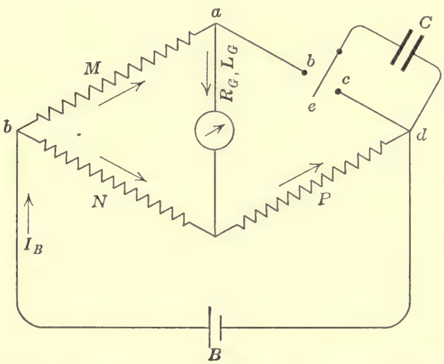


FIG. 215.—Connections for Maxwell method for determining a capacity in absolute measure.

and a part flows upward through the galvanometer, tending to deflect it in a direction opposite to that produced by the steady current. When C is fully charged, the currents return again to their steady values. By properly adjusting the arms of the bridge and the number of times per second the condenser is charged and discharged, the galvanometer deflection can be reduced to zero, which means that the net quantity displaced through the instrument in a second is zero.

When *e* and *b* are not in contact, the currents have the steady values I_M, I_N, I_P, I_G.

Call the quantity of electricity on the condenser when it is fully charged, Q_C. Then

$$Q_C = C(\text{P.D.})_{ad}.$$

As the currents have arrived at the steady state,

$$\frac{I_G}{I_N} = \frac{N}{M + R_G}.$$

Consequently

$$\frac{I_G}{I_G + I_N} = \frac{I_G}{I_P} = \frac{N}{M + R_G + N}$$

$$\therefore (\text{P.D.})_{ad} = I_G R_G + I_P P = I_G\left(R_G + \frac{P(M + R_G + N)}{N}\right)$$

and

$$Q_C = C\left(R_G + \frac{P(M + R_G + N)}{N}\right) I_G \qquad (1)$$

where I_G is the galvanometer current when the circuit is in the steady state.

When e touches b, a varying current, i_C, which finally becomes zero, flows into the condenser and all the other currents are temporarily altered.

Let the alteration in I_M be δI_M
in I_N be δI_N
in I_P be δI_P
in I_B be δI_B
in I_G be δI_G

If Q_G is the quantity of electricity displaced through the galvanometer by the current δI_G at each contact of e and b and there are n such contacts per second, the quantity per second so displaced will be nQ_G.

Therefore, if the galvanometer stands at zero under the combined action of I_G and nQ_G,

$$I_G + nQ_G = 0$$

and

$$Q_C = -Cn\left(R_G + \frac{P(M + R_G + N)}{N}\right) Q_G. \qquad (2)$$

To find Q_G, the quantity displaced through the galvanometer by the variable current δI_G when the condenser is charged, suppose C to be discharged and the steady currents to be flowing as indicated in Fig. 215. The potential differences between the terminals of all the resistances will have definite values. As soon

as e makes contact with b, the varying current i_C flows into the condenser, causing temporary alterations in the currents through M, N, R_G, B, and P. The change in P.D. between the terminals of any one of the resistances is the change in the current multiplied by the resistance. The currents δI_B, δI_G, etc., are variable and become zero when the condenser is fully charged.

Referring to Fig. 215 it will be seen that

$$\begin{aligned}
\delta I_M &= i_C + \delta I_G \\
\delta I_B &= \delta I_M + \delta I_N \\
\delta I_P &= \delta I_B - i_C \\
\delta I_P &= \delta I_G + \delta I_N \\
\delta I_N &= \delta I_B - i_C - \delta I_G.
\end{aligned}$$

Using the changes in the currents, Kirchhoff's laws may be applied to the meshes $M\ R_G\ N$ and $B\ N\ P$ and by using the above relations the resulting equations may be expressed in terms of the resistances and the variable currents δI_B, δI_G and i_C.

For the mesh $M\ R_G\ N$ at any instant during charging,

$$M(\delta I_M) + R_G(\delta I_G) + \frac{L_G\, d(\delta I_G)}{dt} - N(\delta I_N) = 0$$

or

$$M(i_C + \delta I_G) + R_G(\delta I_G) + \frac{L\, d(\delta I_G)}{dt} - N(\delta I_B - i_C - \delta I_G) = 0.$$

Uniting terms gives

$$i_C(M + N) + \delta I_G(M + R_G + N) - N(\delta I_B) + \frac{L\, d(\delta I_G)}{dt} = 0. \quad (3)$$

For the mesh $B\ N\ P$ at any instant during charging,

$$B(\delta I_B) + N(\delta I_B - i_C - \delta I_G) + P(\delta I_B - i_C) = 0$$

or uniting terms

$$-i_C(N + P) - \delta I_G(N) + \delta I_B(N + P + B) = 0. \quad (4)$$

Equations (3) and (4) may be integrated to obtain the total quantities displaced by the variable currents during the charging of the condenser; δI_G is zero at both limits. Therefore

$$Q_C(M + N) + Q_G(M + R_G + N) - Q_B(N) = 0 \quad (3a)$$

and

$$-Q_C(N + P) - Q_G(N) + Q_B(N + P + B) = 0. \quad (4a)$$

Eliminating Q_B gives

$$Q_C = \left[\frac{M + R_G + N - \dfrac{N^2}{N + P + B}}{\dfrac{N(N + P)}{N + P + B} - (M + N)}\right] Q_G. \tag{5}$$

The values of Q_C from (2) and (5) may be equated, thus eliminating Q_G, and the resulting equation solved for C.

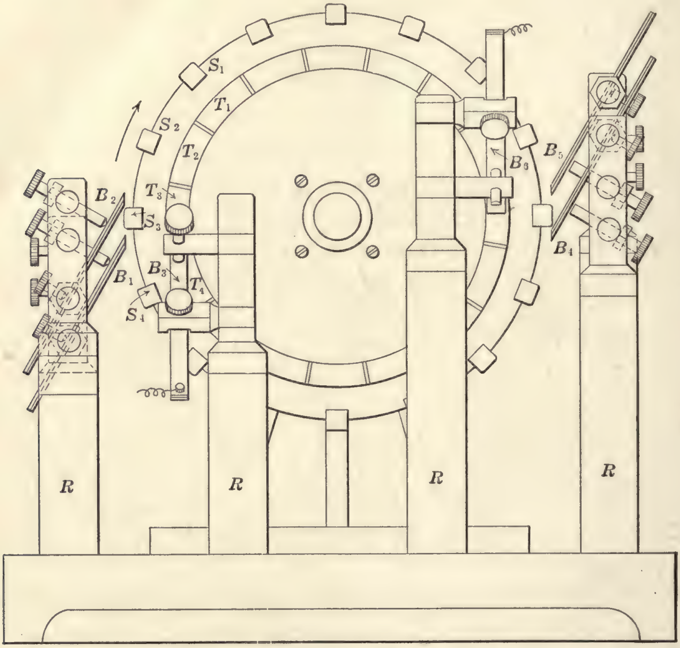


FIG. 216.—Commutator used by Rosa and Dorsey in applying Maxwell's method for the absolute measurement of electrostatic capacity.

$$C = \frac{N}{nPM}\left[\frac{1 - \dfrac{N^2}{(B + N + P)(M + R_G + N)}}{\left(1 + \dfrac{NB}{M(B + N + P)}\right)\left(1 + \dfrac{NR_G}{P(M + R_G + N)}\right)}\right] \tag{6}$$

With very small capacities it is necessary to use a high value of n (500 cycles per second), and high voltages (100 to 200 volts) in order to obtain sufficient sensitiveness.

The capacity as determined includes that of the commutator and the leads to the condenser under measurement. The correction due to these capacities is determined by a separate measurement, the leads being disconnected from the condenser without altering their position more than is absolutely necessary.

The commutator used by Rosa and Dorsey is shown in Fig. 216. The 16 phosphor-bronze contact pieces, S_1, S_2, etc., are carried by an ebonite disc and to each one of the pieces is connected its corresponding section of the brush ring, T_1, T_2, etc. This ring is sectionalized to reduce the capacity of the commutator and to allow the guard ring to be charged and discharged synchronously with the main condenser. One terminal of the condenser is connected to the copper brush B_3. The brushes B_1 and B_2 correspond to b and c in Fig. 215.

The condenser is charged when S_3 touches B_1, is discharged when it touches B_2, and so on for each contact piece. The guard ring is charged and discharged in a similar manner by the other side of the commutator (B_6, B_5, B_4). It will be noted that the brushes are air-insulated between contacts, thus avoiding the possibility of leakage across the commutator from brush to brush. The usual speed of the commutator is from 1,200 to 1,500 revolutions per minute.

Direct-deflection Method for Comparing Capacities.—The most obvious method for comparing the capacities of two condensers is to charge the condensers from the same battery and then to determine the relative quantities which they have accumulated by discharging them in turn through the same ballistic galvanometer. To carry out this test in its simplest form, the connections shown in Fig. 217 may be used.

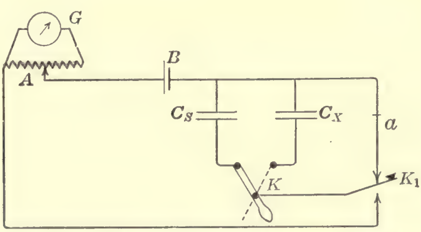


Fig. 217.—Connections for direct deflection method for comparing capacities.

When K is thrown to the right-hand stop and K_1 is depressed, the condenser C_X is charged through the ballistic galvanometer, giving rise to a deflection θ_{1X}; this must be corrected for the multiplying power of the shunt, m_X. A similar observation with K thrown to the left gives θ_{1S}, the multiplying power of the shunt now being m_S. If the damping of the galvanometer be the same in the two cases,

$$\frac{C_X}{C_S} = \frac{Q_X}{Q_S} = \frac{\theta_{1X} m_X}{\theta_{1S} m_S}$$

or

$$C_X = \frac{\theta_{1X} m_X}{\theta_{1S} m_S} C_S.$$

If the damping is not the same in the two cases, the deflections must be corrected (see page 116). Constant damping may be attained by use of the Ayrton universal shunt, A.

If the standard and the unknown are of very different capacities, instead of shunting the galvanometer, different known voltages may be used in the two tests, and an allowance made. This procedure is convenient if an Ayrton shunt is not at hand.

The proper value of the standard is one which will give a deflection about equal to that due to the unknown capacity. Enough battery should be used so that the deflections may be read with good precision.

If the galvanometer be placed at a, the capacities may be compared by discharging the condensers.

In arranging the apparatus, care must be taken that the capacities to earth of long leads and of the instruments, as well as the capacities between the leads to the condensers, do not introduce errors. For instance, the leads from K_1 to the condensers should be short, of small wire and well separated from the leads to the other side of the condenser, otherwise, a separate test must be made to determine their capacity. To prevent errors from leakage, the battery and all the wiring should be well insulated, especially the keys K and K_1 and the leads from K_1 to the condensers via K.

Sources of Error.—In reality this test is not so simple as might appear, for the assumption has been tacitly made that the condenser is entirely charged or discharged before the needle of the ballistic galvanometer has moved appreciably. Reference to the

section on condensers (page 357) will show that this assumption is strictly true only for air condensers charged or discharged through a negligible resistance. If the capacity is determined from the discharge deflection, both the first rush of current due to the free charge, and the gradually decreasing current due to the liberation of the absorbed charge, are active in producing the deflection, and the galvanometer needle is acted on by two forces—a sudden blow due to the passage of the free charge and a long-continued and gradually diminishing push due to the absorbed charge. Therefore, when there is considerable absorption, the apparent capacity as determined by this simple method

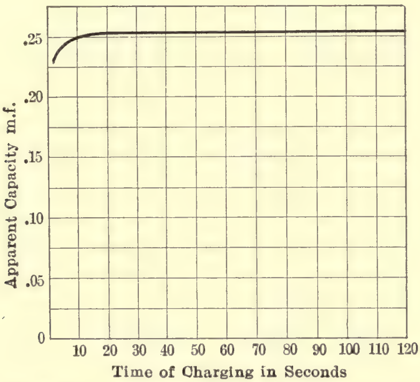


FIG. 218.—Showing effect of time of charging on the apparent capacity of sample of rubber-covered wire.

is dependent on the period of the galvanometer employed. In working with the discharge deflection, any delay after the condenser is disconnected from the battery and before it is connected to the galvanometer will cause an error if the time of charging has not been sufficiently long for the dielectric to become saturated, for absorption goes on during this delay, thus reducing the free charge.

In industrial testing the direct deflection method is very commonly applied to cables, but it is obvious that to obtain results which are of value as a basis of comparison between samples which are nominally the same, some definite procedure must be adopted.

This point is emphasized by Fig. 218, which shows the apparent

capacity, on discharge, of a piece of rubber-covered wire when subjected to different times of charging. Specifications for rubber-covered wires commonly call for a charging period of 10 seconds.

The Zeleny Discharge Key.—In order to obtain the free charge capacity of a condenser with an imperfect dielectric, the condenser must be disconnected from the galvanometer after it has parted with its free charge and before any appreciable portion of the absorbed charge has been given up. This may be accomplished by the Zeleny discharge key,[8] shown diagrammatically in Fig. 219. In this key there are three flexible leaves, L_1, L_2, and L_3. As shown, the condenser C is being charged from the battery B. When the key is depressed the battery circuit is broken

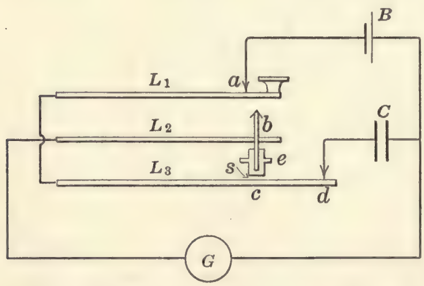


FIG. 219.—Diagram for Zeleny discharge key.

at a, and the discharge circuit completed at b. On continuing the depression, mechanical contact is made at c, and the discharge circuit broken at d. The period of delay before the galvanometer is taken out of circuit is controlled by varying the distance s, by turning the milled head e. The key must be kept depressed until the first elongation has been completed.

In using this key one starts with the distance s large (5 mm.) and, maintaining as nearly as may be a constant velocity of tapping, successive throws of the galvanometer needle are observed as s is diminished. The result will be as shown in Fig. 220. The deflections fall off regularly until the point is reached where sufficient time has not been allowed for the condenser to part with its free charge. After this, the deflections rapidly decrease and the ordinary variations in the velocity of tapping

begin to cause irregularities. The results are independent of the period of the galvanometer.

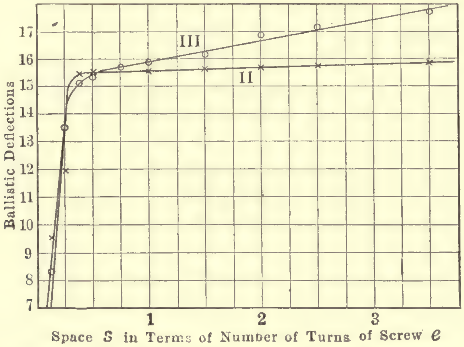


FIG. 220.—Illustrating results attained by use of Zeleny discharge key.

Deflection Method Using a Commutator.—By means of a rotary commutator the condenser may be rapidly charged and discharged, possibly 100 times a second. The galvanometer will then take up a steady deflection, and after the instrument has been properly calibrated the capacity may be calculated. This method is adapted to the measurement of small capacities which are without absorption and leakage; for example, air condensers, or the capacities between wires arranged as in a transmission line.

The commutator by which the charging and discharging are effected should be substantial in design and directly driven by a motor of considerable power which is provided with a flywheel and supplied with current at a fixed voltage so that the number of discharges per second may be constant. Fig. 221 shows the device as used by Fleming and Clinton.[10]

In order to obtain good contacts copper brushes should be employed. It is essential that surface leakage at the commutator be eliminated. For this reason the brushes as they pass from one active segment to the next must not be supported by substances like mica or agate, for after a time these become covered with a conducting coating. Air must be used as the insulation between the segments.

In the form of commutator shown in Fig. 221, A and B are two rown wheels of composition, about 4 in. in diameter. They are

thoroughly insulated from the shaft and from each other by bushings and washers of hard rubber. *I* is a toothed wheel insulated from *A* and *B* and from the shaft and serves merely as a support for the brush as it passes from *A* to *B*, thus preventing mechanical shock and undue wear. The brushes 1 and 3 make contact with the continuous portions of *A* and *B*; as the commutator revolves 2 is alternately connected to *A* and to *B* and the condenser is rapidly charged and discharged. At their

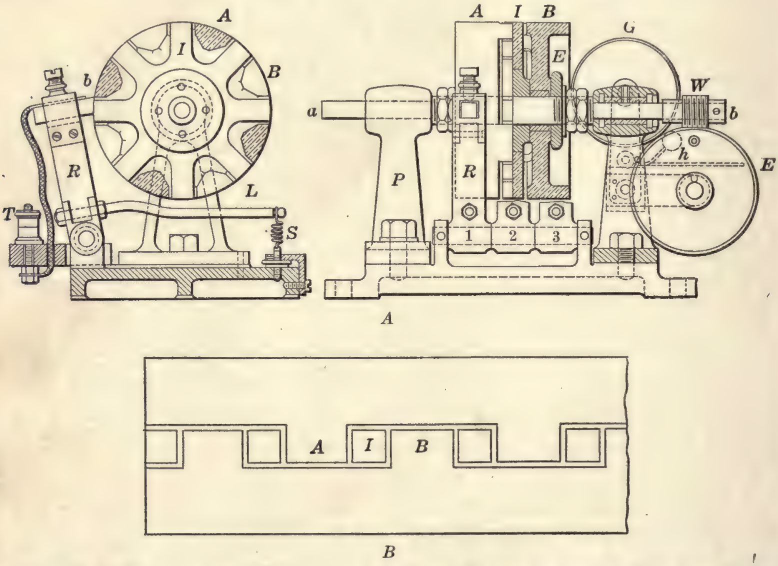


FIG. 221.—Fleming and Clinton commutator for use in comparing capacities.

peripheries the wheels *A*, *B*, and *I* are separated by air spaces. Means for accurately determining the speed must be provided.

If the condenser has a capacity of *C* microfarads and is discharged through the galvanometer *n* times per second, the voltage of the battery being *V*, the quantity displaced through the galvanometer in 1 sec., or the average current, will be

$$I = \frac{nCV}{10^6} = KD_1$$

K is the constant of the galvanometer and D_1 the deflection. To determine *K* a steady current from the same battery may be sent through the instrument and regulated by the insertion of a

series resistance, R, and a shunt, S, on the galvanometer. Call the deflection so obtained D_2; then

$$C = \frac{S10^6}{n\left[(R_G + S)R + R_G S\right]}\left(\frac{D_1}{D_2}\right)\text{microfarads.}$$

The two deflections should be approximately equal.

A better procedure was adopted by Fleming and Clinton, who used a special differential galvanometer of the D'Arsonval type. The two movable coils were rigidly attached to the same vertical insulating stem, one above the other, each coil having its own permanent magnet. The adjustment for obtaining an exactly differential instrument was by means of a magnetic shunt. This construction allows the two coils to be highly insulated, which is essential in order that the results may not be vitiated by cross-leakages, since the voltage employed on the condensers may be considerable—100 volts or more. One coil is traversed by the discharges from the condenser, while the other carries a steady current derived from the battery. When the instrument stands at zero,

$$C = \frac{S10^6}{n\left[(R_G + S)R + R_G S\right]}\text{ microfarads.}$$

A separate experiment must be made to determine the capacity of the leads.

Thomson Method for Comparing Capacities.[11]—If two condensers be charged with equal quantities of electricity the voltages required will be inversely as the capacities. To take advantage of this relation some ready means must be provided for indicating when the charges are equal and for showing the relative voltages applied to the condensers. The arrangement shown in Fig. 222 is that necessary for carrying out the measurements according to Thomson's method. The battery current flows through R_1 and R_2 in series. If K_1 and K_2 are both depressed at the same time, C_X is charged to a voltage IR_1 while C_P is charged to a voltage IR_2. When K_1 and K_2 are released and make contact with their back stops, the condensers are connected in series + to − so that their charges tend to neutralize each other or "mix." To see if the neutralization is perfect, that is, to see if there were equal quantities on the two condensers,

K_3 is depressed and the unneutralized portion of the charge sent through the galvanometer, the deflection of which will be to the right or to the left according to which charge preponderates. By successive trials, altering $\frac{R_1}{R_2}$, the galvanometer deflection may be reduced to zero. Then

$$IR_1C_X = IR_2C_P$$

$$C_X = C_P \frac{R_2}{R_1}.$$

To carry out the test a special key which combines K_1, K_2 and K_3 on a single base is usually employed. In manipulating this key

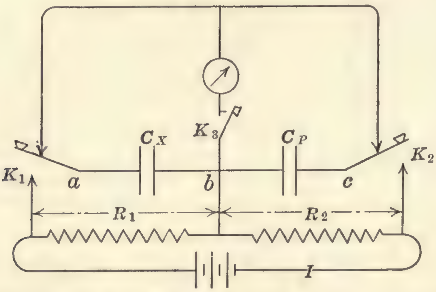


FIG. 222.—Connection for Thomson method of comparing capacities.

care must be taken that it performs its functions properly and that cross-contacts by the fingers of the observer are avoided.

To obtain a good precision the variable resistances R_1 and R_2 should be high so that their adjustment may be sufficiently flexible. If resistance boxes are used the smallest step is usually 1 ohm, so when comparing ordinary condensers the resistance which is adjusted should be at least 1,000 ohms. In submarine cable work much higher resistances are employed; $R_1 + R_2$ may be as much as 100,000 ohms. The galvanometer must be very sensitive and the battery voltage as high as is consistent with safety of the apparatus. With high voltages, to avoid throwing an unduly large potential on either condenser, the known and unknown capacities should be about the same.

When cables are tested, the core is connected to a, and b then becomes the common "ground"; in this case perfect insulation of the battery is essential.

The resistances R_1 and R_2 and their leads must be thoroughly insulated.

The effect of any considerable leakage in C_X during the period of charging, is practically to shunt R_1 by the leakage resistance, so, to get equal quantities on the condensers, R_1 as unplugged in the resistance box must be made larger than if the condenser had been of the same capacity and devoid of leakage. Thus the apparent capacity will be too small.

In this case, the procedure recommended is to use instead of the resistance unplugged in the box, the parallel resistance of R_1 and the condenser under test. This necessitates a measurement of the insulation resistance of the condenser. The period between charging and mixing must be made as short as possible, since leakage during this time will cause the charge on the unknown to be too small, which will necessitate an increase in R_1 above the proper value. Leakage during mixing will reduce, by shunting, the quantity to be discharged through the galvanometer and will thus diminish the sensitiveness of the method.

If absorption be present, it is necessary to adopt some definite cycle of operations in order to obtain comparable results, for the behavior of an imperfect condenser depends to a certain extent on its previous history, that is, on the voltage to which it has been subjected, the time of electrification and the duration of the charging and mixing periods, together with the completeness of the discharge.

As the absorbed charges reappear gradually and as it is the free charges which are to be neutralized, the key K_3 must be closed for only an instant, when observing the galvanometer. The condensers should be completely discharged between the observations.

Gott Method for Comparing Capacities.[12]—The connections for Gott's method are shown in Fig. 223. If condensers are being compared, *acbd* including the galvanometer circuit is a perfectly insulated system. The condensers being discharged, the key K_1 is depressed, thus sending a current through the circuit *cad* and charging the condensers C_X and C_P in series. Then

$$\frac{V_{da}}{V_{ac}} = \frac{R_N}{R_M}$$

and

$$C_X = C_P \frac{V_{db}}{V_{bc}}.$$

If K_2 be depressed, still keeping K_1 closed, there will, in general, be a deflection of the galvanometer, due to the difference in the potentials of a and b. If there is a deflection, K_1 and K_2 are raised and the condensers completely discharged by the use of K_3. After this another test is made with a different value of $\frac{R_N}{R_M}$. By successive trials, adjusting either R_M or R_N, the deflection due to the difference of potential of a and b at the instant of closing K_2 may be reduced to zero. When the adjustment is complete,

$$C_X = C_P \frac{R_N}{R_M}.$$

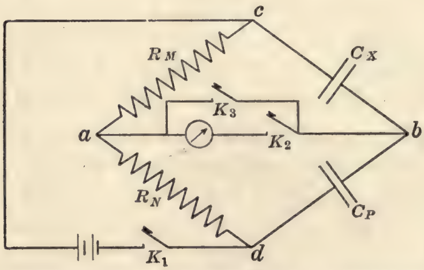


FIG. 223.—Connections for Gott method for comparing capacities.

As anything which gives rise to a false distribution of potentials in the network *acbd* will cause errors, all parts must be carefully insulated, and K_2 especially, as it is handled. Leakage within the battery or between the battery leads is not a source of error, as it merely alters the P.D. applied to the network. Direct leakage from the battery to the condenser circuits must be avoided. Any leakage between the terminals of either condenser is a source of error; this leakage may be through the dielectric of the condenser or between the leads. If C_X be imperfect in this respect and K_1 be kept depressed, the potential of b gradually approaches that of c. The value found for C_X will then be too great and will increase with the time of charging.

When cables are measured the core should be connected to

b, as the sheath is necessarily grounded. Because of the small effect of battery leakage, this method is very commonly employed in submarine cable work.

If the rates of absorption of the two condensers are not the same, the results obtained will be dependent on the time of charging. For condensers, and for cables up to a length of about 1,000 knots, a correction devised by Muirhead and intended to correct for both absorption and leakage is applicable; it fails, however, in the case of cables of greater length, supposedly on account of the retardation due to the resistance of the cable itself.[12]

The methods of Thomson and of Gott are of importance in submarine cable testing. In this work, exceedingly large capacities must be measured, frequently several hundred microfarads. It is in connection with these measurements that the complications due to leakage and absorption become most troublesome.

Elementary Methods of Determining Inductance and Capacity by Alternating Currents.[13]—If a sinusoidal current of known frequency be used, the most obvious method of measuring an inductance is to determine the current and the P.D. between the terminals of the coil. Then if ω is 2π times the frequency, R the resistance, V the applied voltage and I the current, the inductance is given by

$$L = \frac{\sqrt{V^2 - I^2R^2}}{\omega I}$$

or if the resistance is negligible, by

$$L = \frac{V}{\omega I}.$$

When the current wave is non-sinusoidal it is possible to allow for the effect of the harmonics. Suppose that the maximum values of the various components are I_1, I_3, I_5, etc. Then

$$i = I_1 \sin \omega t + I_3 \sin (3\omega t - \theta_3) + I_5 \sin (5\omega t - \theta_5) + \dots$$

The effective value of the current will be

$$I = \sqrt{\frac{I_1^2}{2} + \frac{I_3^2}{2} + \frac{I_5^2}{2} + \cdots}$$

The fundamental equation for the flow of current through an inductive resistance is

$$v = Ri + L\frac{di}{dt}.$$

From the above,

$$v = R[I_1 \sin \omega t + I_3 \sin (3\omega t - \theta_3) + I_5 \sin (5\omega t - \theta_5) + \ldots]$$
$$+ \omega L[I_1 \cos \omega t + 3I_3 \cos (3\omega t - \theta_3) + 5I_5 \sin (5\omega t - \theta_5) + \ldots]$$

The mean square value of the P.D. will be

$$V^2 = I^2R^2 + \omega^2L^2 \left[\frac{I_1^2}{2} + 9\frac{I_3^2}{2} + 25\frac{I_5^2}{2} + \ldots\right]$$

therefore,

$$L = \frac{\sqrt{V^2 - I^2R^2}}{\omega\sqrt{\frac{I_1^2}{2} + 9\frac{I_3^2}{2} + 25\frac{I_5^2}{2} + \ldots}}$$

$$= \frac{\sqrt{V^2 - I^2R^2}}{\omega I}\sqrt{\frac{I_1^2 + I_3^2 + I_5^2 + \ldots}{I_1^2 + 9I_3^2 + 25I_5^2 + \ldots}}. \tag{7}$$

When the resistance is negligible,

$$v = L\frac{di}{dt}.$$

In this case, suppose the P.D. wave has been analyzed. Then at any instant,

$$v = V_1 \sin \omega t + V_3 \sin (3\omega t - \theta_3) + V_5 \sin (5\omega t - \theta_5) + \ldots$$

Therefore

$$-Li = \frac{V_1}{\omega}\cos \omega t + \frac{V_3}{3\omega}\cos (3\omega t - \theta_3) + \frac{V_5}{5\omega}\cos (5\omega t - \theta_5) + \ldots$$

The mean square value of Li will be

$$L^2I^2 = \frac{1}{\omega^2}\cdot\frac{V_1^2}{2} + \frac{1}{9\omega^2}\cdot\frac{V_3^2}{2} + \frac{1}{25\omega^2}\cdot\frac{V_5^2}{2} + \ldots$$

and

$$L = \frac{V}{I\omega V}\sqrt{\frac{V_1^2}{2} + \frac{1}{9}\cdot\frac{V_3^2}{2} + \frac{1}{25}\cdot\frac{V_5^2}{2} + \ldots}$$

or

$$L = \frac{V}{I\omega}\sqrt{\frac{V_1^2 + \frac{1}{9}V_3^2 + \frac{1}{25}V_5^2 + \ldots}{V_1^2 + V_3^2 + V_5^2 +}}. \tag{8}$$

Capacity Measurements.—The current through a condenser with a perfect dielectric is given by

$$i = C\frac{dv}{dt}.$$

Assuming sinusoidal currents the capacity is

$$C = \frac{I}{\omega V}.$$

If the applied e.m.f. is not sinusoidal, the harmonics will be exaggerated in the current wave, for the current is

$$i = \omega C\,[V_1 \cos \omega t + 3V_3 \cos (3\omega t - \theta_3) + 5V_5 \cos (5\omega t - \theta_5) + \ldots]$$

Using root-mean-square values,

$$I = \omega C \sqrt{\frac{V_1^2}{2} + \frac{9V_3^2}{2} + \frac{25V_5^2}{2} + \ldots}$$

Therefore,

$$C = \frac{I}{\omega V}\sqrt{\frac{V_1^2 + V_3^2 + V_5^2 + \ldots}{V_1^2 + 9V_3^2 + 25V_5^2 + \ldots}}.$$

BRIDGE MEASUREMENTS OF CAPACITY AND INDUCTANCE

As originally devised, many of the methods for comparing capacities and for comparing inductances, as well as methods for determining an inductance in terms of a capacity, depended on the employment of variable currents. As the industrial uses of alternating currents have developed, especially in connection with telephony, it has become important that tests be made under conditions which are as nearly as possible those pertaining to the ordinary use of the apparatus. Hence, alternating currents have replaced the variable currents formerly employed and the methods for capacity measurement have been so modified that they give data of value, in addition to determining the capacity of the condenser under measurement.

Condition for Zero Indication of Detector.—When variable currents are used in balance methods for measuring inductance and capacity, a long period galvanometer is employed as the detector. The arrangement of the circuits is such that at balance no permanent current flows through the instrument; this being so—

1. The deflection will certainly be zero if no current flows through the galvanometer at any time during the establishment of the permanent state of the circuit.

2. Presumably the deflection will also be zero when the net

quantity of electricity displaced through the instrument during the establishment of the permanent state of the circuit is zero, or when

$$\int_0^t i_G dt = Q_G = 0.$$

If $i_G = 0$ continuously, then necessarily $Q_G = 0$. The converse is not true, for the net quantity may be made zero by a current which flows through the detector first in one direction and then in the reverse direction. When deducing the conditions which must be fulfilled in order that the galvanometer may remain undeflected, it is best to impose the condition that no current shall flow through the galvanometer at any time, for in some cases the galvanometer needle will be disturbed even though the integral current is zero.[27] The disturbance depends on the alteration of the strength of the galvanometer needle by the transient current. In a general way, the reason for the deflection may be seen by supposing the instrument to be traversed by an alternating current. If the magnetism of the needle is affected by the current, the galvanometer becomes in effect a soft-iron instrument with a magnetic control and there will be a deflecting moment proportional to the square of the current. In the case of the steadily applied alternating current the needle will come to rest in a deflected position depending upon the strength of the current.

Some moving-coil instruments are subject to this same error, when used as detectors for integral currents.

As an example of a method where the phenomenon is of importance, take the comparison of two mutual inductances by the method given on page 416. The integral flow of current through the galvanometer will be zero if

$$\frac{m_X}{m_P} = \frac{r_X}{r_P}.$$

That the current through the galvanometer may be zero continuously it is necessary that

$$\frac{m_X}{m_P} = \frac{r_X}{r_P} \tag{A}$$

and

$$\frac{m_X}{m_P} = \frac{L_X}{L_P}. \tag{B}$$

On trying the experiment it will be found that unless the relation (B) is approximately fulfilled the needle will be slightly disturbed.

In the following proofs, except in those for De Sauty's method for comparing capacities where the application of both conditions for balance will be illustrated, the condition $i_G = 0$ continuously will be imposed.

De Sauty Method for Comparing Capacities.—This method is adapted to the comparison of condensers without leakage, which are either free from absorption or have equal rates of absorption. Fig. 224 shows the connections.

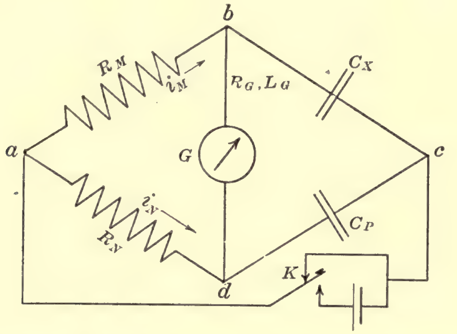


FIG. 224.—Connections for De Sauty method for comparing capacities.

The two bridge arms R_M and R_N are non-inductive resistances. C_X and C_P are the two condensers which are to be compared. The detector, which will be considered as having both inductance and resistance, is at G.

When the key K is against the back stop the condensers are discharged. The arms R_M and R_N are adjusted until on depressing K the detector gives no indication. Then

$$C_X = C_P \frac{R_N}{R_M}.$$

To prove this relation, condition 1 (page 381), will first be applied. In that case, the variable current i_M is that flowing into condenser C_X and the variable current i_N is that flowing into C_P, and at every instant

$$\frac{i_M}{i_N} = \frac{R_N}{R_M}.$$

The potential difference between b and c is

$$V_{bc} = \frac{1}{C_X}\int_0^t i_M dt$$

and between d and c it is

$$V_{dc} = \frac{1}{C_P}\int_0^t i_N dt.$$

V_{bc} must equal V_{dc} since at no time during the establishment of the steady state does any current flow through G. Then

$$\frac{1}{C_X}\int_0^t i_M dt = \frac{1}{C_P}\int i_N dt = \frac{R_M}{R_N}\frac{1}{C_P}\int_0^t i_M dt$$

$$\therefore C_X = C_P\frac{R_N}{R_M}.$$

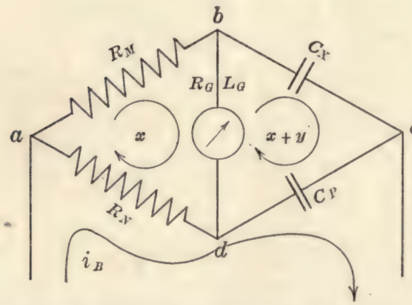


FIG. 225.—Mesh diagram for De Sauty method for comparing capacities

If the condition $\int_0^t i_G dt = Q_G = 0$ be applied, the demonstration is more complicated, since the expression for Q_G must first be deduced and then the condition found which renders it zero. Consider that the bridge is arranged as in Fig. 225.

Assume that during the time of charging the condensers, that is, until the steady state has been established, the meshes are traversed by the variable currents x, $x + y$, and i_B, which finally become zero.

Taking the mesh abd,

$$x(R_M + R_G + R_N) + L_G\frac{dx}{dt} - (x + y)\ R_G - L_G\left(\frac{dx}{dt} + \frac{dy}{dt}\right) - i_B R_N = 0$$

or uniting terms,

$$x(R_M + R_N) - yR_G - L_G\frac{dy}{dt} - i_BR_N = 0. \tag{10}$$

Considering the mesh *bcd*, the potential difference between b and c is

$$V_{bc} = \frac{1}{C_X}\int_0^t (x + y)dt, \quad \text{and similarly for } V_{cd}.$$

Hence

$$\frac{1}{C_X}\int_0^t (x + y)dt + \frac{1}{C_P}\int_0^t (x + y)dt - \frac{1}{C_P}\int_0^t i_Bdt + (x + y)R_G + L_G\left(\frac{dx}{dt} + \frac{dy}{dt}\right) - xR_G - L_G\frac{dx}{dt} = 0$$

or uniting terms,

$$\frac{1}{C_X}\int_0^t (x + y)dt + \frac{1}{C_P}\int_0^t (x + y - i_B)dt + yR_G + L_G\frac{dy}{ds} = 0. \tag{11}$$

(10) and (11) may be integrated from $t = 0$, when the key K is closed, to the time t when the permanent state has been established. Call Q_X the quantity displaced by the current x in that time, Q_G the quantity displaced through the galvanometer by y, the true galvanometer current, and Q_B the quantity displaced by i_B. Integrating (10) and remembering that y is zero at the start and zero at the finish,

$$Q_X(R_M + R_N) - Q_GR_G - Q_BR_N = 0. \tag{10a}$$

Integrating (11) gives

$$(Q_X + Q_G)\frac{1}{C_X} + (Q_X + Q_G - Q_B)\frac{1}{C_P} = 0. \tag{11a}$$

From (10*a*) and (11*a*),

$$Q_X = \frac{Q_GR_G + Q_BR_N}{R_M + R_N} = -Q_G + Q_B\left(\frac{C_X}{C_X + C_P}\right)$$

$$\therefore Q_G = \frac{Q_B\left[(R_M + R_N)\dfrac{C_X}{C_X + C_P} - R_N\right]}{R_M + R_G + R_N}. \tag{12}$$

If $Q_G = 0$,

$$\frac{C_X}{C_X + C_P} = \frac{R_N}{R_M + R_N}$$

or

$$C_X = C_P \frac{R_N}{R_M} \quad \text{as before.} \tag{13}$$

Maxwell Method for Comparing Inductances.—An inductance may be compared with a variable standard by an analogous method due to Maxwell.*

The arrangement of the circuits is shown in Fig. 226. As before, R_M and R_N are adjustable non-inductive resistances. L_X and L_P are the inductances to be compared; they have resistances R_X and R_P respectively. In order to make the adjustment expeditiously, it is necessary to include a variable non-

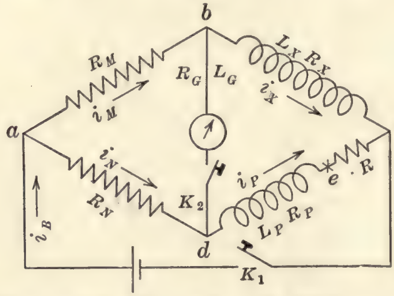


FIG. 226.—Mesh diagram for Maxwell method for comparing inductances.

inductive resistance, R, which can be thrown into the arm L_X if necessary, by changing the battery lead from c to e and to use for L_P a variable standard of inductance having a constant resistance.

The adjustment is made in two steps; a probable value of the ratio $\frac{R_M}{R_N}$ is chosen, and keeping K_1 closed, the resistance R is adjusted until balance is obtained. In this case the arrangement is an ordinary Wheatstone bridge, only the resistances coming into play. When the adjustment is complete,

$$\frac{R_M}{R_N} = \frac{R_X}{R_P + R}. \tag{14}$$

After the balance has been effected, K_1 is released and K_2 kept closed. L_P is now varied until the detector gives no indication when contact is made and broken at K_1. Then

* "Treatise on Electricity and Magnetism," third edition, Art. 757.

$$L_X = L_P \frac{R_M}{R_N}. \tag{15}$$

Several trials with various values of $\frac{R_M}{R_N}$ may be necessary before the proper ratio is found.

In this and other methods where the balance is independent of the frequency, a "buzzer" operating through a telephone induction coil is often a convenient source of supply for the interrupted current, the detector being a telephone.

To prove the relation (15) the first condition stated on page 381 may be imposed. After the adjustment is complete no current passes through the detector at any time, so for all values of t,

$$V_{bc} = V_{dc}$$
$$V_{ab} = V_{ad}$$

$$i_M = i_X$$
$$i_N = i_P$$

$$\frac{i_M}{i_N} = \frac{R_N}{R_M} = \frac{i_X}{i_P}.$$

Therefore

$$i_X R_X + L_X \frac{di_X}{dt} = i_P(R_P + R) + L_P \frac{di_P}{dt}$$

so

$$i_X \left[R_X - \frac{R_M}{R_N}(R_P + R)\right] + \frac{di_X}{dt}\left[L_X - \frac{R_M}{R_N} L_P\right] = 0$$

which must be true for all values of t.

$$\therefore \quad L_X = L_P \frac{R_M}{R_N},$$

and

$$R_X = \frac{R_M}{R_N}(R_P + R).$$

In order to balance the bridge when all four of the arms are inductive, three instead of two conditions must be satisfied, as will be seen from the following.

Suppose that no current flows through the detector at any time. Then referring to Fig. 227

$$V_{ab} = V_{ad}$$
$$V_{bc} = V_{dc}$$
$$i_X = i_M$$
$$i_P = i_B - i_X = i_B - i_M$$
$$i_N = i_B - i_M.$$

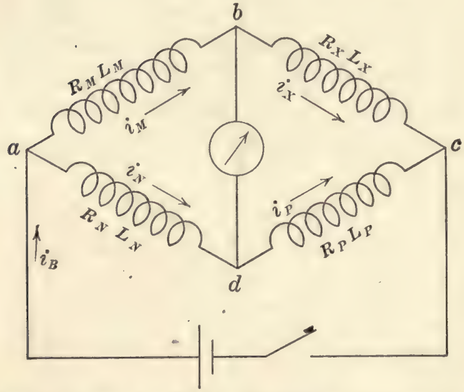


FIG. 227.—Mesh diagram for Maxwell bridge with four inductive arms.

Therefore, making no assumption as to the relation of i_B to t,

$$(R_M + R_N)i_M + (L_M + L_N)\frac{di_M}{dt} = R_N i_B + L_N \frac{di_B}{dt} \qquad (16)$$

and

$$(R_X + R_P)i_M + (L_X + L_P)\frac{di_M}{dt} = R_P i_B + L_P \frac{di_B}{dt}. \qquad (17)$$

Eliminating i_M between (16) and (17) gives

$$\frac{di_M}{dt} = \frac{\frac{di_B}{dt}[-R_PL_N + R_NL_P - R_XL_N + R_ML_P] + i_B[R_MR_P - R_NR_X]}{(R_M + R_N)(L_X + L_P) - (R_X + R_P)(L_M + L_N)}. \qquad (18)$$

Eliminating $\frac{di_M}{dt}$ between (16) and (17) gives

$$i_M = \frac{\frac{di_B}{dt}[L_NL_X - L_ML_P] + i_B[-R_PL_N + R_NL_P + R_NL_X - R_PL_M]}{(R_M + R_N)(L_X + L_P) - (R_X + R_P)(L_M + L_N)}. \qquad (19)$$

The value of $\frac{di_M}{dt}$ derived from (19) when equated to that in (18) gives

$$\frac{d^2 i_B}{dt^2}[L_N L_X - L_M L_P] + \frac{di_B}{dt}[-R_P L_M + R_N L_X + R_X L_N - R_M L_P] + i_B[R_N R_X - R_M R_P] = 0. \quad (20)$$

By supposition (20) must hold for all values of t and, therefore, for the steady state, so

$$R_N R_X - R_M R_P = 0 \quad (21)$$

which is the ordinary condition for the balance of the Wheatstone bridge. In order that (20) may be true for all values of t the coefficients of $\frac{d^2 i_B}{dt^2}$ and $\frac{di_B}{dt}$ must also be zero, so

$$L_N L_X - L_M L_P = 0 \quad (22)$$

$$-R_P L_M + R_N L_X + R_X L_N - R_M L_P = 0. \quad (23)$$

As no assumption has been made concerning the relation of i_B *to* t, equation (20) holds when the bridge is supplied with sinusoidal as well as with variable currents.

The Secohmmeter.[14]—To increase the sensitiveness of the bridge methods for the measurement of self-inductance and capacity which depend upon the use of variable currents, Ayrton and Perry devised the secohmmeter, by which the impulses on the galvanometer needle can be made to follow one another so rapidly that the instrument takes up a steady deflection. The arrangement is essentially a double commutator. One of the commutators reverses the battery connections while the other reverses the galvanometer terminals so that the impulses on the needle of the instrument are always in the same direction. Referring to Fig. 228 the shaded portions of the two commutators are made of an insulating material. The unshaded portions are conducting segments. The brushes aa', bb', and cc', dd', are so placed that the circuits are manipulated in the proper sequence. The secohmmeter is driven at a constant speed by a small motor.

In using this device the speed must not be so high that sufficient time is not allowed for the establishment of the steady

state at each reversal. Improved forms of secohmmeter have been devised by Fleming and Clinton, and at the Bureau of Standards.

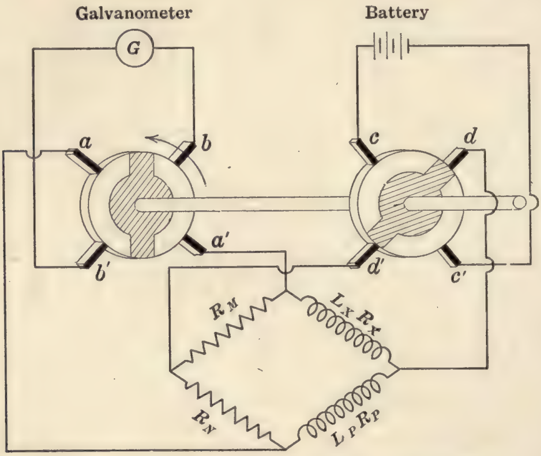


Fig. 228.—Showing connections for secohmmeter.

The Impedance Bridge.—Because of the nearer approach to actual working conditions, capacity and inductance measurements are now made by aid of alternating currents, preferably sinusoidal.

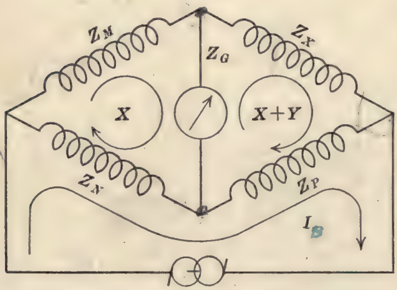


Fig. 229.—Mesh diagram for impedance bridge.

Practically all the recent researches on dielectrics and condensers as well as the precision measurements of inductances have been made by bridge methods, using either the impedance bridge or the Anderson bridge.

In the impedance bridge, which may be applied to the measurement of either inductance or capacity, there are four main conductors arranged as in the Wheatstone bridge. Alternating currents are employed and either two or four of the conductors are reactive.

To deduce the condition for balance the arrangement shown in Fig. 229 may be taken.

It will be assumed that the bridge arms have impedances Z_M, Z_N, Z_X, Z_P, and Z_G and are traversed by sinusoidal currents. All the impedances are expressed in symbolic notation. The mesh currents will be taken as indicated. As cognizance must be taken of their phase relations, these currents must also be expressed symbolically and referred to the same axis, for instance, I_B. Applying Kirchhoff's laws, for the X mesh,

$$X\,(Z_M + Z_G + Z_N) - (X + Y)\,Z_G - I_B Z_N = 0,$$

for the $(X + Y)$ mesh,

$$(X + Y)(Z_X + Z_P + Z_G) - XZ_G - I_B Z_P = 0.$$

Solving for Y, the current through the detector,

$$Y = \frac{I_B(Z_P Z_M - Z_N Z_X)}{Z_G(Z_X + Z_P + Z_M + Z_N) + (Z_M + Z_N)(Z_X + Z_P)}. \tag{24}$$

If the detector and generator be interchanged, the value of the detector current becomes

$$Y = \frac{I_B(Z_N Z_X - Z_P Z_M)}{Z_G\,(Z_X + Z_P + Z_M + Z_N) + (Z_M + Z_X)\,(Z_N + Z_P)}. \tag{25}$$

The condition for no current in the detector is

$$Z_N Z_X = Z_P Z_M. \tag{26}$$

Compare the above with corresponding deduction for the Wheatstone bridge, page 183.

The generator used as a source of power should give a sinusoidal e.m.f. wave.

The ratio arms (M and N) may be non-inductive resistances, highly inductive resistances, or perfect condensers. Bridges with highly inductive ratio arms have been used by Giebe in inductance measurements[20] and by Grover[16] in measurements of the capacity and power factor of condensers.

The detector may be either a telephone or a vibration galvanometer. At low frequencies the latter is to be preferred, for it is a tuned instrument responding freely to currents of only one frequency. With it an accurate balance may be obtained even though the currents are not exactly sinusoidal. Electrostatic disturbances are also avoided.

As the maximum frequency obtainable with the vibration

galvanometer is about 1,800 cycles per second, a limit is set above which the telpehone must be used. The sensitivity of a telephone detector may be greatly increased by having it tuned to the frequency of the supply, especially if that is near the frequency at which the ear is most sensitive (800 to 1,000 cycles per second). Of course with any tuned detector, the periodicity of the current must be kept constant.

Capacity Measurements.—If two perfect condensers are to be compared, they may be placed in the arms P and X (compare with Fig. 225). The arms M and N may be non-inductive resistances. In this case

$$Z_M = R_M$$
$$Z_N = R_N$$
$$Z_X = \frac{1}{j\omega C_X}$$
$$Z_P = \frac{1}{j\omega C_P}.$$

Substituting in (26) gives

$$\frac{R_N}{j\omega C_X} = \frac{R_M}{j\omega C_P}$$

$$\therefore \quad C_X = C_P \frac{R_N}{R_M}.$$

In practice the comparison of ordinary condensers is not so simple, for an energy loss may occur in one or both of them. As the behavior of a condenser with an imperfect dielectric depends on the frequency, it is important that the correct periodicity be employed.

If energy losses be present, the phase of the current in R_M will probably not be the same as that in R_N and no adjustment of these resistances can be found which will cause a zero indication of the detector. If a telephone be used, there will be a considerable range of adjustment over which the sound is faint but never entirely disappears.

If an energy loss be introduced into the arm of the bridge having the smaller power factor, the currents in R_M and R_N may be brought into phase and an exact balance obtained. Wien accomplishes this by the use of a series resistance in the arm containing the better condenser.[15]

The bridge is arranged as shown in Fig. 230. All the resistances are supposed to be non-inductive. The condensers to be compared are at C_X and C_P. One or both may have an imperfect dielectric, and to duplicate their behaviors, it will be assumed that perfect condensers having effective capacities C_X and C_P are in series with resistances r_X and r_P respectively. This is in accordance with the convention mentioned on page 359. r_X and r_P are hypothetical resistances assumed simply as an aid in the demonstration in order to introduce energy losses, their values being such that the behavior of the combination of the perfect condenser C_X and the resistance r_X will be the same as that of the actual condenser at C_X, and the behavior of C_P and r_P the same as that of the actual condenser at C_P. The balance arms R_M and R_N are variable and R_X and R_P are the adjustable resistances used to bring the potential differences V_{cb} and V_{cd} into phase. Two resistances are included because it may not be known at the start which of the condensers has the lower power factor. Only one of the resistances will be used.

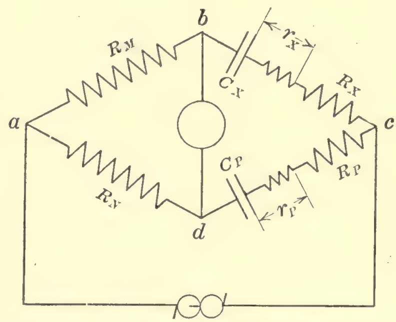


FIG. 230.—Diagram for the Wien impedance bridge.

The balancing is effected as follows: with R_X and R_P both zero, the ratio $\frac{R_M}{R_N}$ is adjusted until the indication of the detector is a minimum. The balance is then improved by adjusting R_X or R_P as the case may require, and still further improved by readjusting $\frac{R_M}{R_N}$ and so on, thus obtaining a perfect balance by successive adjustments. When the balance is effected,

$$C_X = C_P \frac{R_N}{R_M}.$$

This may be proved as follows; in general by (26)

$$Z_P Z_M = Z_N Z_X.$$

In this case

$$Z_M = R_M$$
$$Z_N = R_N$$
$$Z_P = R_P + r_P - \frac{j}{\omega C_P}$$
$$Z_X = R_X + r_P - \frac{j}{\omega C_X}.$$

Substituting in (26) gives

$$R_M(R_P + r_P) - j\frac{R_M}{\omega C_P} = R_N(R_X + r_X) - j\frac{R_N}{\omega C_X}.$$

This one equation being in complex is really equivalent to two, for when all the terms are transposed to the left-hand side the sum of all the horizontal components, "the real terms," must be zero and the sum of all the vertical components, "the imaginary terms," must also be zero. Equating the vertical components,

$$\frac{R_M}{\omega C_P} = \frac{R_N}{\omega C_X}$$

or

$$C_X = C_P\frac{R_N}{R_M}. \tag{27}$$

Equating the horizontal components,

$$\frac{R_M}{R_N} = \frac{R_X + r_X}{R_P + r_P} \tag{27a}$$

Determination of Phase Angle of Condenser.—As previously defined, the phase angle of a condenser, φ, is the deviation of the phase of the current from the ideal lead angle of 90° which would exist in a perfect condenser. To determine the difference of the phase angles of C_X and C_P, $\varphi_X - \varphi_P$, from (27) and (27a),

$$\frac{R_X + r_X}{R_P + r_P} = \frac{R_M}{R_N} = \frac{C_P}{C_X}.$$

When multiplied out and then multiplied through by ω this becomes

$$\omega C_X r_X - \omega C_P r_P = \omega C_P R_P - \omega C_X R_X$$

but

$$\omega C_X r_X = \tan\varphi_X \quad \text{and} \quad \omega C_P r_P = \tan\varphi_P$$
$$\tan\varphi_X - \tan\varphi_P = \omega C_P R_P - \omega C_X R_X. \tag{28}$$

In general, $\tan a - \tan b = (1 + \tan a \tan b)(\tan (a - b))$,

so if the phase angle φ_P of the standard condenser is small, as it usually will be,

$$\tan (\varphi_X - \varphi_P) = \omega C_P R_P - \omega C_X R_X \qquad (29)$$

Either R_X or R_P may be zero as previously noted. If φ_P is known, the power factor of the unknown condenser is readily computed. The values of φ_P and C_P would be determined by a process of stepping up from an air condenser. The curves on page 361 were determined by this method. As it is customary to use a high frequency, 800 cycles per second or greater, residual inductances and capacities in the apparatus must be reduced to a minimum. The sources of error to be considered when refined measurements are to be made are:

1. Inductance or capacity of M and N.
2. Error in the ratio of M and N.
3. Inductance or capacity of R_X and R_P.
4. Electrostatic induction between the bridge and its surroundings.

Determination of Equivalent Capacity and Conductance of Condenser or Short Length of Cable.—The equivalent capacity and the conductance (leakance) of a condenser, or a short length of cable, may be found by means of the Wien bridge. A short length of cable is specified so that the complications arising from distributed capacity, inductance, resistance, and leakance may be avoided, for the frequency employed is likely to be high.

It is desired to find the combination of condenser and resistance in parallel with it which will duplicate the behavior of the actual condenser. No assumption is made as to the nature of the energy loss taking place in the dielectric.

The arrangement of apparatus is that shown in Fig. 230, the specimen being connected in the arm X, R_X being made zero. C_P is a perfect (air) condenser, in series with a non-inductive resistance, R_P, and both are adjustable. To balance the bridge it is necessary to bring the current in the arms N and P into phase with that in the arms M and X. This may be done, as previously indicated, by putting a comparatively small resistance in series with the air condenser, or by shunting that condenser with a

very large resistance; usually the former method is the more convenient.

In general, by equation (26)

$$Z_M Z_P = Z_N Z_X.$$

For this particular case,

$$Z_M = R_M$$
$$Z_N = R_N$$
$$Z_X = \frac{1}{g_X - jb_X}$$
$$Z_P = r_P - \frac{j}{\omega C_P}.$$

Substituting these relations,

$$\frac{R_N}{R_M} = r_P g_X - \frac{b_X}{\omega C_P} - j\left(\frac{g_X}{\omega C_P} + b_X r_P\right).$$

The horizontal component gives

$$\frac{R_N}{R_M} = r_P g_X - \frac{b_X}{\omega C_P}.$$

The vertical component gives

$$b_X = -\frac{g_X}{\omega C_P r_P}.$$

Therefore, the two components of the admittance of the condenser are

$$g_X = \frac{R_N}{R_M}\left(\frac{r_P C_P{}^2 \omega^2}{1 + r_P{}^2 C_P{}^2 \omega^2}\right),$$

and

$$b_X = -\frac{R_N}{R_M}\left(\frac{C_P \omega}{1 + r_P{}^2 C_P{}^2 \omega^2}\right) = -\omega C_X$$

$$C_X = \frac{R_N}{R_M}\left(\frac{C_P}{1 + r_P{}^2 C_P{}^2 \omega^2}\right).$$

The power-factor angle θ_X is given by

$$\tan\theta_X = \frac{b_X}{g_X} = -\frac{1}{\omega C_P r_P}.$$

The conductance g_X is the reciprocal of the *equivalent* insulation resistance of the condenser or cable. This resistance bears

no relation to the dielectric or insulation resistance given by measurements with steady currents. The latter may be many thousand times the equivalent dielectric resistance as determined by alternating currents.

Fleming and Dyke in their researches on the power factor and equivalent conductivity of dielectrics[18] used a bridge in which the arms M and N were formed by two perfect (air) condensers. The arm P contained a perfect (air) condenser in series with a non-inductive resistance. As high frequencies were used (up to 6,000 cycles per second), it was necessary to use a telephone detector.

With this bridge $\frac{Z_M}{Z_N} = \frac{C_N}{C_M}$; therefore, referring to the previous demonstration,

$$g_X = \frac{C_M}{C_N}\left(\frac{r_P C_P{}^2\omega^2}{1 + r_P{}^2 C_P{}^2\omega^2}\right)$$

and

$$C_X = \frac{C_M}{C_N}\left(\frac{C_P}{1 + r_P{}^2 C_P{}^2\omega^2}\right).$$

The tangent of the power-factor angle of the condenser under investigation is given by

$$\tan \theta_X = -\frac{1}{\omega C_P r_P}.$$

When air condensers are used in the bridge arms they must be properly screened or else placed so far apart and so far from the observer that difficulties due to electrostatic induction are avoided. All the connections should be of very fine wire.

Bridge for Measurement of Electrolytic Conductivity.—The impedance bridge, arranged as in Fig. 231, is employed in the measurement of the conductivities of electrolytes.

On account of the capacity action in the electrolytic cell it is necessary to use an adjustable air condenser in parallel with the resistance in the arm P. When the bridge is balanced

$$R_X = \frac{R_M}{R_N} R_P.$$

Wagner Earth Connection.—When a telephone is used as a detector in these bridge methods, difficulties are encountered due to a difference of potential between the observer and the

telephone he is using. This gives rise to a charging current in the instrument which may be sufficient to prevent an exact balance being obtained. The trouble may be eliminated by bringing the telephone and the observer to the same (earth)

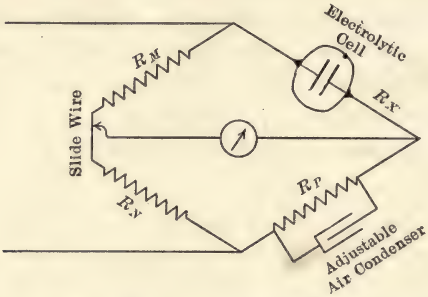


FIG. 231.—Diagram of bridge for determining electrolytic conductivities.

potential by the Wagner Earth Connection[19] which is shown in Fig. 232.

The adjustable auxiliary circuit *efg* is similar in its makeup to the bridge circuit *adc*. It is earthed at *f* and by varying its component parts the impedances of the sections *ef* and *fg* can be adjusted and the potentials of *e* and *g* altered in reference to the earth.

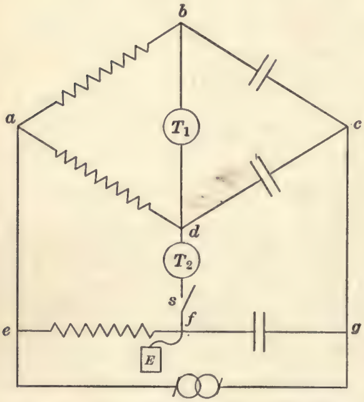


FIG. 232.—Diagram for Wagner earth connection.

With the switch *s* open the bridge is balanced as well as possible, using the telephone T_1. Then *s* is closed and the impedance of the auxiliary circuit adjusted until the sound in T_2 is a minimum. This means that *d* and consequently T_1 are continuously at practically the same potential as *f*, which is earthed. Therefore, there can be no inductive action between the telephone T_1 and the observer. After this adjustment has been made, *s* is opened and the final balance is obtained by adjusting the bridge proper.

Inductance Measurements.—In the comparison of an inductance with a variable standard inductance the connections shown in Fig. 233 may be used.

The ratio arms R_M and R_N are non-inductive resistances. The variable standard of self-inductance is at P (see page 386). L_X is the unknown inductance and R is an adjustable non-inductive resistance which, if necessary, may be placed in series with the unknown inductance by changing the lead from c to e. A and D are sources of alternating and direct current, respectively; T and G are the corresponding detectors (compare Fig. 226).

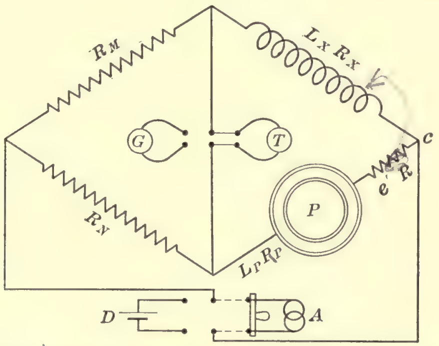


Fig. 233.—Diagram of impedance bridge for comparing inductances.

The impedances of the bridge arms are

$$\begin{aligned} Z_M &= R_M \\ Z_N &= R_N \\ Z_P &= R_P + j\omega L_P \\ Z_X &= R_X + j\omega L_X. \end{aligned}$$

Substituting these values in the general equation (26) and separating the quadrature components, the two conditions which must be fulfilled in order that the bridge may be balanced follow:

From the horizontal component,

$$R_X = \frac{R_M}{R_N} R_P. \tag{30}$$

From the vertical component,

$$L_X = \frac{R_M}{R_N} L_P. \tag{31}$$

Therefore, when measuring an inductive coil a perfect balance implies two things—that the ohmic resistances are balanced as in the ordinary Wheatstone bridge, and that the inductances are in the ratio of the corresponding bridge arms. If there are other than i^2r losses in the arm X they appear in R_X which in this case is an equivalent resistance.

With a bridge properly constructed, its coils being free from inductance and capacity, it is thus possible to make a simultaneous measurement of the inductance and *resistance to alternating currents* of a coil or piece of apparatus.

To assist in carrying out the necessary adjustments in an expeditious manner, it may be noted that non-magnetic conductors of *small cross-section,* used with currents of ordinary frequencies, have practically the same resistance with alternating as with direct current. Therefore, to save time, a preliminary balance may be made with direct current, using the apparatus as an ordinary Wheatstone bridge, thus satisfying the condition

$$R_X = \frac{R_M}{R_N} R_P.$$

If the order of magnitude of the inductance under measurement is entirely unknown, one may begin with $R_M = R_N$ and balance by varying R. It may be necessary to transfer this resistance to the other side of the bridge by changing the lead from c to e. Alternating is now substituted for direct current. The detector will in general give an indication which must be reduced to zero by adjusting the variable standard of inductance. The chances are that on account of the limited range of the standard, L_P cannot be made of such a value as to obtain even a minimum of sound in the telephone. In this case one notices at which end of the scale the indication is the smaller, and then alters the ratio so that the balance point will be thrown toward the middle portion of the scale. The bridge is then rebalanced for direct and for alternating currents. Two or three trials may be necessary in order to obtain a good reading on L_P. If with $R_M = R_N$ the indication is apparently the same at all points of the scale, a large change should be made in the ratio; for instance, to $\frac{R_M}{R_N} = 10$. If this does not give results, $\frac{R_M}{R_N} = \frac{1}{10}$

may be tried and so on. Lastly, a final attempt to obtain a perfect zero indication of T may be made by altering R and L_P *slightly*.

A simple form of impedance bridge adapted to the determination of inductances in terms of capacities and *vice versa* is shown diagrammatically in Fig. 234.

Equal ratio arms are employed; they are advantageous in any bridge arrangement since their equality may be checked at any time by simply reversing them.

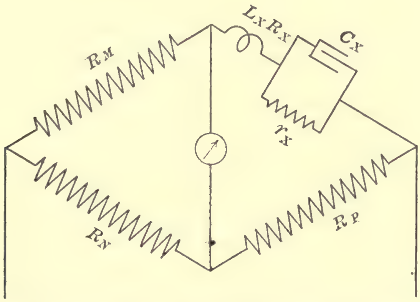


FIG. 234.—Diagram of impedance bridge for comparing an inductance with a capacity.

The arms M and N are equal and they as well as r_X and R_P are non-reactive, so when the bridge is adjusted the arm X is, in effect, non-inductive, the inductance and capacity being balanced. Then

$$R_P = R_X + j\omega L_X + \frac{r_X}{1 + j\omega C_X r_X}. \tag{31a}$$

If L_X is to be determined in terms of a capacity having no appreciable losses, the balance is obtained by adjusting R_P and r_X. From equation (31a),

$$L_X = \frac{C_X r_X^2}{1 + \omega^2 C_X^2 r_X^2} \tag{32}$$

$$R_X = R_P - \frac{r_X}{1 + \omega^2 C_X^2 r_X^2}. \tag{33}$$

If the equivalent capacity and insulation resistance (C_X and r_X) of an imperfect condenser are to be determined, an adjustable

inductance of constant resistance is used at L_X and the balance is obtained by varying L_X and R_P.

From 32 and 33,

$$L_X = C_X r_X \,(R_P - R_X)$$

$$R_P = R_X + r_X - \omega^2 C_X L_X r_X.$$

From these equations,

$$C_X = \frac{L_X}{(R_P - R_X)^2 + \omega^2 L_X^2} \text{ and } r_X = \frac{(R_P - R_X)^2 + \omega^2 L_X^2}{R_P - R_X}. \quad (34)$$

Obviously the frequency must be constant and of known value.

The Impedance Bridge with Four Inductive Arms.[20]—It is desirable to inquire as to the conditions necessary for a balance if all four arms of the impedance bridge contain inductances. This arises from the fact that if very small inductances are to be compared, using currents of high frequency, the residual inductances existing in the ordinary double-wound resistance coils become of great moment. The reactances of the bridge coils may be positive or negative according as the inductance or capacity component preponderates, and at times may be of the same order of magnitude as the reactance under measurement. In a bridge with four inductive arms,

$$\begin{aligned} Z_M &= R_M + j\omega L_M \\ Z_N &= R_N + j\omega L_N \\ Z_P &= R_P + j\omega L_P \\ Z_X &= R_X + j\omega L_X \end{aligned}$$

and at balance,

$$Z_M Z_P = Z_N Z_X.$$

Substituting,

$$R_P R_M + j\omega R_P L_M + j\omega R_M L_P - L_M L_P \omega^2 = R_X R_N + j\omega R_X L_N + j\omega R_N L_X - L_N L_X \omega^2.$$

The horizontal component gives as one condition for balance,

$$[L_N L_X - L_M L_P]\,\omega^2 + R_P R_M - R_X R_N = 0 \quad (35)$$

The vertical component gives as the other necessary condition,

$$R_P L_M - R_N L_X - R_X L_N + R_M L_P = 0 \quad (36)$$

The above results could have been obtained directly from equation (20), page 389. As no assumption was made in the deduction of that equation as to the relation of i_B to t, the bridge current may be assumed as sinusoidal,

$$i_B = I_B \sin \omega t.$$

Substituting in (20) gives

$$[L_N L_X - L_M L_P](-I_B \omega^2 \sin \omega t) + [-R_P L_M + R_N L_X + R_X L_N - R_M L_P](I_B \omega \cos \omega t) + [R_N R_X - R_M R_P](I_B \sin \omega t) = 0$$

which must be true for all values of t.
Consequently the coefficients for both the cosine term and for the collected sine terms must be zero.

$$\therefore -[L_N L_X - L_M L_P]\,\omega^2 + R_N R_X - R_M R_P = 0$$

and

$$R_P L_M - R_N L_X - R_X L_N + R_M L_P = 0$$

The sine terms correspond to the horizontal component in the previous demonstration while the cosine term corresponds to the vertical component.

The Anderson Bridge.—The determination of an inductance in terms of a capacity may conveniently be made by means of the Anderson bridge.[21] This apparatus is a development of the bridge arrangement given by Maxwell.*

The connections are shown in Fig. 235.

All the resistances except R_X are supposed to be non-inductive. The condenser is placed at C and r is an adjustable resistance.

When variable currents are used, as was originally intended, the bridge is first balanced for steady currents, the battery circuit being kept closed. After balance has been attained, the capacity C and the resistance r are adjusted until there is no deflection of the galvanometer when the battery circuit is made and broken. It will be noted that the adjustment of C and r does not disturb the steady current balance but does affect the rate at which the potential of the junction e rises. As the initial values of the potentials of b and e are the same and the final values are the same, there will be no current in the de-

* "Treatise on Electricity and Magnetism," third edition, Art. 778.

tector at any time if the potentials of these two points rise at the same rate.

To determine the condition necessary for a balance, suppose the steady current balance has been attained and that r has been adjusted so that the bridge is also balanced for variable currents.

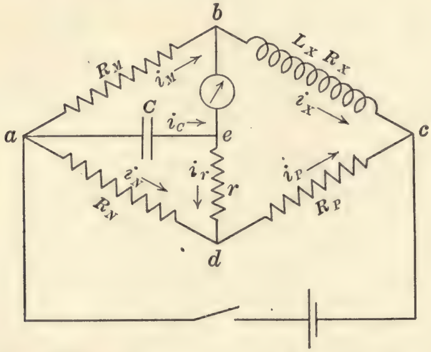


FIG. 235.—Diagram for Anderson bridge.

Referring to Fig. 235, by supposition,

$$R_X = \frac{R_M}{R_N} R_P \tag{A}$$

$$i_G = 0 \text{ continuously}$$
$$i_M = i_X$$
$$i_C = i_r.$$

From the mesh b, c, d, e,

$$i_M R_X + L_X \frac{di_M}{dt} = i_r r + i_P R_P \tag{B}$$

$$\text{P.D.}_{ab} = \text{P.D.}_{ae} = i_M R_M = \frac{1}{C}\int i_r dt$$

$$\therefore \frac{di_M}{dt} = \frac{i_r}{R_M C}. \tag{C}$$

From (A), (B) and (C),

$$L_X = R_M C\left[r + \left(\frac{i_P}{i_r}\right) R_P - \left(\frac{i_M}{i_r}\right)\left(\frac{R_M}{R_N}\right) R_P\right]. \tag{D}$$

From the mesh a, b, e, d,

$$i_M R_M = i_N R_N - i_r r$$

$$\therefore \left(\frac{i_M}{i_r}\right) R_M = \left(\frac{i_N}{i_r}\right) R_N - r$$

but

$$\frac{i_N}{i_r} = \frac{i_P}{i_r} - 1.$$

Substituting these values in (D) gives

$$L_X = R_M C \left[r\left(1 + \frac{R_P}{R_N}\right) + R_P \right]. \tag{37}$$

The Anderson bridge is now used with alternating currents and a vibration galvanometer is employed as the detector. This arrangement has been used at the Bureau of Standards,[21] Washington, in much of the recent and very accurate work on measurements of inductance.

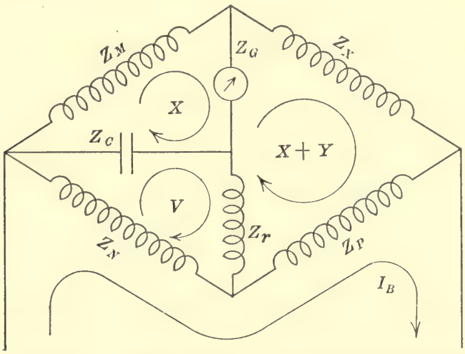


Fig. 236.—Mesh diagram for Anderson bridge.

The necessary conditions for balance are shown below.

The impedances of the various branches are denoted by Z with the proper subscript. The mesh equations are

$$X(Z_M + Z_C) - YZ_G - VZ_C = 0$$

$$V(Z_C + Z_r + Z_N) - X(Z_C + Z_r) - YZ_r - I_B Z_N = 0$$

$$Y(Z_X + Z_P + Z_r + Z_G) + X(Z_X + Z_P + Z_r) - VZ_r - I_B Z_P = 0.$$

Solving for Y, the galvanometer current,

$$Y = \frac{I_B(Z_M Z_C Z_P + Z_M Z_r Z_P + Z_M Z_N Z_P + Z_M Z_N Z_r - Z_C Z_N Z_X)}{\text{denominator, a function of the impedances}}$$

$\therefore$ for balance,

$$Z_M Z_C Z_P + Z_M Z_r Z_P + Z_M Z_N Z_P + Z_M Z_N Z_r - Z_C Z_N Z_X = 0. \quad (38)$$

In the ideal case where the coils of the bridge proper are entirely free from inductance and capacity,

$$\begin{aligned} Z_M &= R_M \\ Z_X &= R_X + j\omega L_X \\ Z_P &= R_P \\ Z_N &= R_N \\ Z_C &= \frac{1}{j\omega C} \\ Z_r &= r. \end{aligned}$$

Substituting in (38),

$$\frac{R_M R_P}{j\omega C} + R_M r R_P + R_M R_N R_P + R_M R_N r = \frac{R_N}{j\omega C}(R_X + j\omega L_X) \quad (39)$$

Separating the quadrature components, the horizontal component gives

$$L_X = R_M C\left[r\left(1 + \frac{R_P}{R_N}\right) + R_P\right]. \quad (40)$$

The vertical component gives

$$R_M R_P = R_N R_X. \quad (41)$$

Thus a perfect balance of the vibration galvanometer implies that both (40) and (41) are satisfied.

To expedite matters, it is usual to make a preliminary balance with direct currents, using an ordinary galvanometer, thus satisfying the condition (41), and then to balance with alternating currents by adjusting r, the vibration galvanometer being employed. This method of procedure assumes that all the resistances involved have the same values for both direct and alternating currents.

Effect of Dissipation of Energy in the Condenser.—In the demonstration a perfect condenser has been assumed. As an energy loss occurs in most condensers, it is important to see how this loss will influence the results. As previously shown, an energy loss is equivalent to an increase of the conductance of the condenser. Suppose that the condenser has been measured

with alternating currents and that its equivalent capacity is C, and its equivalent conductance is $\frac{1}{R_C}$, then

$$Z_C = \frac{R_C}{1 + jR_C C\omega}.$$

Substituting this value in (38),

$$\frac{R_M R_C R_P}{1 + jR_C C\omega} + R_M r R_P + R_N R_M R_P + R_M R_N r = \frac{R_C R_N (R_X + j\omega L_X)}{1 + jR_C C\omega}.$$

The horizontal component gives

$$R_X R_N - R_M R_P = \frac{R_M}{R_C}\Big[r(R_P + R_N) + R_N R_P\Big].$$

The vertical component gives

$$L_X = R_M C\left[r\left(1 + \frac{R_P}{R_N}\right) + R_P\right].$$

That is, the energy loss does not complicate the measurement of the inductance.

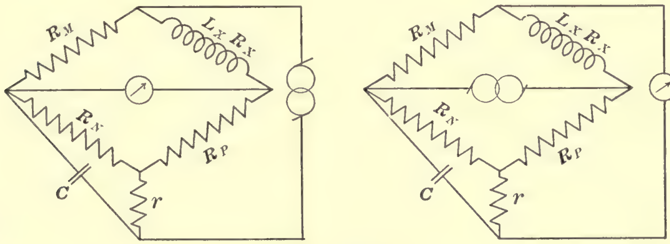


FIG. 237.

Stroude and Oates arrangement of the Anderson bridge. Anderson bridge, Fig. 236 redrawn.

Stroude and Oates[22] modified the Anderson bridge by interchanging the source of current and the detector.

The advantage of the rearrangement is that when the conditions are such that r is high, it is possible to increase the applied voltage and thus maintain the sensitivity of the bridge by keeping the bridge current at a high value.

As the only alteration has been to interchange the source of current and the detector, the formula connecting the self-inductance and the capacity is the same as for the Anderson bridge.

When very small inductances having a magnitude of, for example, 0.001 henry are to be measured, the residual inductances of the bridge coils must be considered. These coils are wound non-inductively, in the usual understanding of the term, but either the inductance or the capacity effect may preponderate. The high resistance coils will give the most trouble.

The Mutual Inductance Bridge.—If variable or alternating currents be used, a Wheatstone bridge which has three non-inductive arms and one inductive arm connot be made to balance, for the potentials at the two ends of the detector circuit can never be in the same time phase. A balance can be obtained, however, by the addition of an adjustable mutual inductance, or air-core transformer of variable ratio, the secondary of which

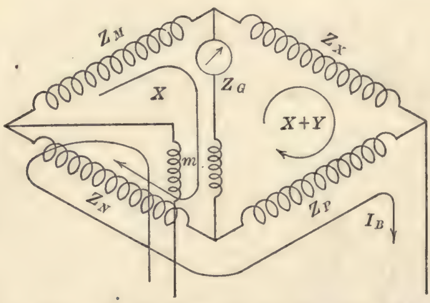


FIG. 238.—Mesh diagram for Hughes bridge.

is connected in series with the detector while the primary is placed in one of the leads from the source of supply to the bridge. The primary, therefore, carries the entire bridge current, and the mutual inductance introduces into the detector circuit a small e.m.f. which is in quadrature with that current.

An apparatus so arranged was used in 1886 by Professor Hughes and the results obtained were given by him in his inaugural address on assuming the presidency of the British Institution of Electrical Engineers. The discussion[23] which followed the presentation of this paper should be read by every student who has any doubts on the question of practice *vs.* theory plus practice. It is sufficient to say here that on account of an inadequate theory of his bridge Professor Hughes misinterpreted the readings which he obtained. H. F. Weber, Rayleigh

and Heaviside showed that the observations obtained by means of the Hughes apparatus are, when correctly reduced, in entire accord with the accepted theory of induction.

The connections for this form of bridge are shown in Fig. 238. They are much like those for the Wheatstone bridge, but in the galvanometer circuit is included the secondary of the air-core transformer of variable ratio, the primary of this transformer being connected in the lead running from the source of current to the bridge. The mutual inductance of the air-core transformer will be represented by m. Assuming sinusoidal currents the mesh equations are:

$$(X + Y)(Z_X + Z_P + Z_G) - XZ_G - I_BZ_P - jm\omega I_B = 0$$

$$X(Z_M + Z_G + Z_N) - (X + Y)Z_G - I_BZ_N + jm\omega I_B = 0.$$

In respect to the sign given to the term involving the mutual inductance, in this and other methods of measurement, it may be either positive or negative depending on the manner in which the device is connected into the circuit. However, the particular connection and the corresponding sign in the equations must be used which will enable a balance to be obtained.

Solving the above equations for Y, the galvanometer current, and substituting the values of the impedances, the arms M, N and P being non-inductive,

$$Y = \frac{I_B[(R_P + jm\omega)(R_M + R_N) - (R_N - jm\omega)(R_X + R_P + jL_X\omega)]}{\text{denominator}}.$$

If only the condition of balance is required, it is not necessary to know the expression for the denominator. For balance, the numerator must be zero or

$$R_P(R_M + R_N) - R_N(R_X + R_P) - m\omega^2L_X + j[(R_M + R_N)m\omega + (R_X + R_P)m\omega - L_XR_N\omega] = 0.$$

Separating the quadrature components, the horizontal component gives

$$R_PR_M - R_NR_X = m\omega^2L_X$$

and the vertical component gives

$$m(R_M + R_N + R_X + R_P) = R_NL_X.$$

In order to obtain a balance both these equations must be satisfied.

Solving for L_X and R_X,

$$L_X = \frac{R_P R_M}{R_N} m \left[\frac{\dfrac{1}{R_N} + \dfrac{R_M + R_N + R_P}{R_P R_M}}{1 + \dfrac{m^2\omega^2}{R^2{}_N}} \right] \tag{42}$$

$$R_X = \frac{R_P R_M}{R_N} \left[\frac{1 - \dfrac{m^2\omega^2(R_M + R_N + R_P)}{R_P R_M R_N}}{1 + \dfrac{m^2\omega^2}{R^2{}_N}} \right] \tag{43}$$

In his paper Professor Hughes treated his observations as if he were dealing with an ordinary bridge, that is, as if $R_X = \dfrac{R_P R_M}{R_N}$.

Heaviside in his examination of the work of Professor Hughes pointed out that balance may be obtained if the secondary of

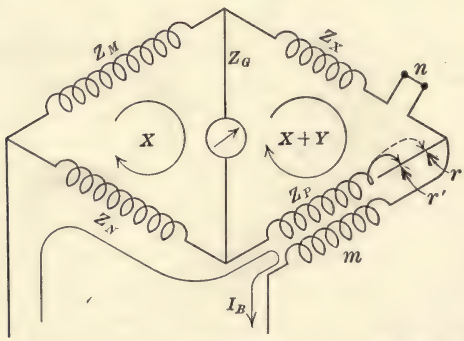


FIG. 239.—Mesh diagram for Heaviside mutual inductance bridge.

the mutual inductance is introduced into one of the main bridge arms as well as if it is used in the galvanometer circuit. Fig. 239 shows a bridge arranged in this manner; as before, Z with the proper subscript denotes the total impedance of the corresponding bridge arm.

The mesh equations are

$$X(Z_M + Z_G + Z_N) - (X + Y)\, Z_G - I_B Z_N = 0$$

$$(X + Y)\,(Z_X + Z_P + Z_G) - XZ_G - I_B Z_P \pm jm\omega\, I_B = 0$$

$$\therefore Y = I_B \left[\frac{Z_P Z_M - Z_N Z_X \pm jm\omega\,(Z_M + Z_N)}{Z_G(Z_M + Z_N + Z_X + Z_P) + (Z_M + Z_N)(Z_X + Z_P)} \right].$$

For a balance,

$$Z_P Z_M - Z_N Z_X \pm jm\omega\,(Z_M + Z_N) = 0.$$

The balance arms of the bridge, M and N, are non-inductive. On substituting the values

$$\begin{aligned} Z_M &= R_M \\ Z_N &= R_N \\ Z_X &= R_X + j\omega L_X \\ Z_P &= R_P + j\omega L_P \end{aligned}$$

and separating the quadrature components, the horizontal component gives

$$R_M R_P = R_N R_X. \tag{44}$$

The vertical component gives

$$R_M\,[L_P + m] = R_N\,[L_X - m]. \tag{45}$$

If $Z_M = Z_N$ (that is, if a bridge with equal ratio arms is used),

$$R_P = R_X$$

and

$$L_X - L_P = 2m.$$

This arrangement is useful in measuring small inductances, for as suggested by A. Campbell,[24] a method of differences can be used thus eliminating the effects of residual inductances in the bridge arms.

Referring to Fig. 239, the inductance to be measured is inserted in the gap n. m is a variable mutual inductance which can be adjusted without changing the inductance or the resistance of the secondary circuit. The inductance of either the arm X or the arm P and the resistance of P must be adjustable.

Let L_X, L_P and R_X, R_P be the values of the inductances and resistances when the bridge is balanced with n short-circuited and the mutual inductance set at zero. Then

$$\begin{aligned} L_X - L_P &= 0 \\ R_X - R_P &= 0. \end{aligned}$$

After this preliminary balance has been obtained, the unknown inductance L'_X, of resistance R'_X, is introduced at n and the balance again obtained by adjusting m and changing the variable resistance from r to r'. Then,

$$L_X + L'_X - L_P = 2m$$

$$R_X + R'_X - [R_P + (r' - r)] = 0$$

$$\therefore R'_X = r' - r$$
$$L'_X = 2m.$$

Obviously inductances from zero up to a maximum of twice the full value of the mutual inductance can be measured.

In obtaining the inductance it is not necessary to know the values of any of the resistances. The variable resistance r may be made of two small and straight wires placed a few millimeters apart and short-circuited by a bridge piece. The inductance of such an arrangement may be calculated, if it is necessary to allow for it.

The apparatus should be so arranged that the equality of the ratio arms may be tested and their adjustment to exact equality facilitated by interchanging them. Any lack of equality affects the value of R_X much more than that of L_X.

Effect of Eddy Currents.—The fundamental assumption on which the theory of any mutual inductance bridge rests is that the current flowing in the primary of the mutual inductance induces an e.m.f. in the secondary which is in quadrature with the primary current. This assumption will be rendered invalid and absurd results will be obtained with the bridge if its construction is such that eddy currents are set up in neighboring masses of metal or in the wire of the coils themselves, if the wire be large. The effect becomes more pronounced as the frequency is increased.

The primary current will induce an electromotive force in the eddy current circuits which will be in quadrature with itself. Assuming that the inductances of these circuits are negligible, the eddy currents will, in turn, induce an electromotive force in the secondary which will be in quadrature with themselves, and therefore in opposition to the primary current. In addition, there is the direct induction from the primary to the secondary,

which induces in the secondary an e.m.f. in quadrature with the primary current. These two components of the electromotive force in the secondary must be added, and obviously the resultant electromotive force will not be in quadrature with the primary current. To illustrate further, the current in the primary, Fig. 240, is I. This is *supposed* to induce an e.m.f. in the secondary which is represented by $-jm\omega I$. Let e represent the path of the eddy current; its mutual inductance with respect to the primary is m_1 and with respect to the secondary is m_2.

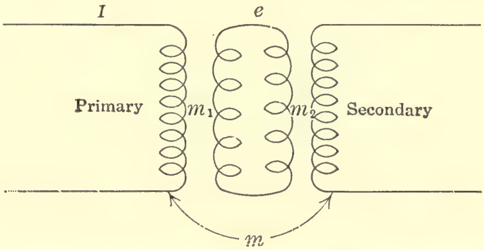


FIG. 240.—Pertaining to eddy current errors in mutual inductance bridge.

The e.m.f. induced in the eddy-current circuit by I will be

$$E_e = -jm_1\omega I.$$

The corresponding current will be

$$I_e = -\frac{jm_1\omega I}{R_e + j\omega L_e}.$$

I_e will induce an e.m.f. in the secondary, given by

$$E'_e = \frac{(-j)(-j)m_1\omega I(m_2\omega)}{R_e + j\omega L_e} = -\frac{m_1m_2\omega^2 I}{R_e + j\omega L_e}$$

$$= -I\left(\frac{R_em_1m_2\omega^2 - jm_1m_2\omega^3L_e}{R_e^2 + \omega^2L^2_e}\right).$$

To obtain the total e.m.f. induced in the secondary, E'_e must be added to $-jm\omega I$. Then

$$E = -I\left(jm\omega + \frac{R_em_1m_2\omega^2 - jm_1m_2\omega^3L_e}{R_e^2 + \omega^2L^2_e}\right).$$

If L_e is negligible, the eddy current induces a component in quadrature with that due to the primary current and of a mag-

nitude depending on the square of the frequency, hence its increasing importance at high periodicities. This shows that all massive metal frames and metal fastenings must be avoided. The coils must be wound with a conductor made up of small strands which are insulated from one another. For economy of space, enameled wire may be used.

Wilson Method for Measuring Inductance.—In this method the reactive component of the potential difference between the terminals of the unknown inductance is measured by a quadrant electrometer.[25] The connections are shown in Fig. 241.

The two sets of quadrants are connected to the terminals of the unknown inductance. One end of the needle circuit is attached to R_X, preferably at the middle. T is an air-core transformer of mutual inductance, m, and V an electrostatic voltmeter

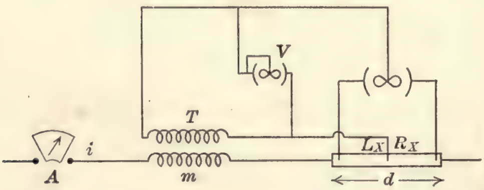


FIG. 241.—Connections for Wilson method for measuring inductance.

for determining the potential of the needle. A is an ammeter for measuring the main current.

When applied to this case, the elementary formula for the deflection of the quadrant electromotor becomes

$$D = \frac{K}{T}\int_0^T \left(2d\left[\pm m\frac{di}{dt} - \frac{d}{2}\right] + d^2\right)dt$$

$$D = \frac{2Km}{T}\int_0^T (d)\left(\frac{di}{dt}\right)dt$$

$$d = R_X i + L_X \frac{di}{dt}.$$

Let the readings of the electrostatic voltmeter and of the ammeter be V and I. Assuming sinusoidal currents,

$$D = \frac{2KV}{I\omega}\left[\frac{1}{T}\int_0^T R_X i\left(\frac{di}{dt}\right)dt + \frac{1}{T}\int_0^T L_X\left(\frac{di}{dt}\right)^2 dt\right].$$

$$\frac{1}{T}\int_0^T i\left(\frac{di}{dt}\right)dt = 0$$

and only the reactive component produces a turning moment. Then

$$L_X\frac{1}{T}\int_0^T \left(\frac{di}{dt}\right)^2 dt = L_X I^2\omega^2$$

$$\therefore D = 2\omega KVIL_X \quad \text{or} \quad L_X = \frac{D}{2\omega KVI} \tag{46}$$

This method may be applied to the measurement of small inductances having a large current-carrying capacity and a small resistance; for example, it has been applied to shunts such as are used for alternating current measurements.

The secondary of the air-core transformer may be the secondary of an ordinary induction coil. The primary may be wound to have a number of turns depending on the current to be dealt with.

The Measurement of Inductances Containing Iron.—The preceding methods are adapted to the measurement of coils with air cores, for in that case the self-inductance is constant, as has been assumed in all the demonstrations. If the coils have iron cores, then owing to the dependence of the permeability on the degree of saturation of the iron, the self-inductance is no longer constant but varies during the cycle.

In such cases the effective inductance may be determined from measurements of the applied voltage, current, frequency and power.

$$L = \frac{1}{\omega I}\sqrt{V^2 - \frac{P^2}{I^2}} \tag{47}$$

The current should be adjusted to the value it has in the ordinary use of the apparatus. With telephonic apparatus it is possible to obtain satisfactory results by bridge methods, for the saturation is so low that the permeability is practically constant.

Measurement of Mutual Inductance.—An obvious method of determining the mutual inductance of two coils is to connect them in series and to measure, by any convenient method, the net self-inductance of the combination; then to reverse one of the

coils and again measure the net inductance. One measurement gives the sum, the other the difference of the mutual and self-inductance effects. If the two net inductances are L' and L'', and the self-inductances of the coils are L_1 and L_2, respectively,

$$L' = L_1 + L_2 + 2m$$
$$L'' = L_1 + L_2 - 2m.$$

Consequently the mutual inductance is given by

$$m = \frac{L' - L''}{4}. \tag{48}$$

Maxwell Method of Comparing Mutual Inductances.—The connections necessary for comparing two mutual inductances by Maxwell's method are shown in Fig. 242 where it is indicated

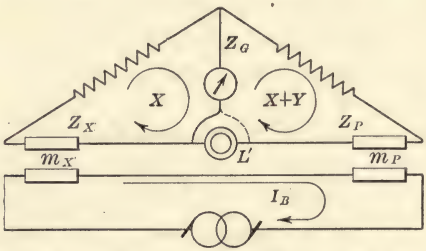


FIG. 242.—Mesh diagram for Maxwell method of comparing mutual inductances.

that alternating currents replace the variable currents assumed in the original demonstration.* The two mutual inductances are m_X and m_P; Z_X and Z_P are the impedances of the respective meshes *omitting* the detector; L' is a variable self-inductance which can be used in series with either m_X or m_P. Its value need not be known.

The mesh equations are

$$XZ_X - YZ_G - j\omega m_X I_B = 0$$

$$XZ_P + Y(Z_P + Z_G) - j\omega m_P I_B = 0$$

from which the detector current is

$$Y = j\omega I_B \left[\frac{Z_X m_P - Z_P m_X}{Z_P Z_G + Z_X(Z_P + Z_G)}\right].$$

* "Treatise on Electricity and Magnetism," third edition, Art. 775.

In order that the detector current may be zero,

$$Z_X m_P = Z_P m_X.$$

When the values of Z_X and Z_P have been substituted and the quadrature components separated, the horizontal component gives

$$\frac{m_X}{m_P} = \frac{r_X}{r_P}$$

or

$$m_X = \frac{r_X}{r_P} m_X \tag{49}$$

and the vertical component gives

$$\frac{m_X}{m_P} = \frac{L_X}{L_P}. \tag{50}$$

As (50) must be satisfied and the self-inductances of the secondaries of the mutual inductances cannot be varied at will, it is necessary to include the variable self-inductance L'.

If the source of current and the detector are interchanged and the secondaries of the mutual inductances are connected in opposition, the arrangement will be balanced if the two secondary e.m.fs. are equal at every instant; that is, when

$$-j\omega \frac{V}{Z_X} m_X = -j\omega \frac{V}{Z_P} m_P;$$

$$\therefore \frac{m_X}{m_P} = \frac{Z_X}{Z_P} \text{ as before.}$$

A disadvantage of the method just given is that r_X and r_P include the resistances of the copper secondaries of the mutual inductances; they must be determined by a separate bridge measurement.

A. Campbell[24] has developed the method so that these resistances are replaced in the formula for m_X by those of carefully calibrated bridge coils. The connections are shown in Fig. 243. Here m_P is a variable standard of mutual inductance; the inductance and the resistance of its primary circuit are fixed.

In carrying out the measurement the switches are first thrown to position 1, the desired values of R_M and R_N unplugged and a

balance obtained by adjusting L'. The arrangement is then a simple impedance bridge, and at balance

$$\frac{Z_M}{Z_X} = \frac{Z_N}{Z_P} \quad \text{or} \quad \frac{Z_M}{Z_N} = \frac{Z_X + Z_M}{Z_N + Z_P}.$$

The opposed secondaries of the mutual inductances are next introduced into the detector circuit by throwing the switches to position 2 and a balance obtained by adjusting m_P.

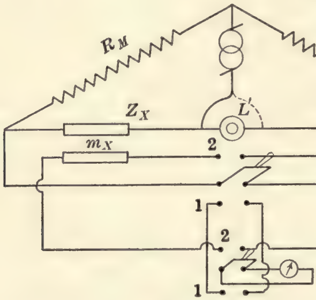


FIG. 243.—Connections for Campbell method of comparing mutual inductances.

At balance the two secondary e.m.fs. must be equal, or

$$-j\frac{\omega m_X V}{Z_X + Z_M} = -j\frac{\omega m_P V}{Z_N + Z_P}$$

$$\therefore \frac{m_X}{m_P} = \frac{Z_X + Z_M}{Z_N + Z_P} = \frac{Z_M}{Z_N} = \frac{R_M}{R_N}$$

and

$$m_X = \frac{R_M}{R_N}\, m_P \tag{51}$$

Measurement of Mutual Inductance in Terms of Capacity.— The connections for the Carey Foster method of comparing a mutual inductance with a capacity, as modified by Heydweiller,[26] are shown in Fig. 244. C is a condenser; r, R_1 and R_2 are non-inductive resistances. r_2 is the resistance and L_2 the self-inductance of the secondary of the mutual inductance.

The impedances are

$$Z_1 = R_1$$
$$Z_2 = (R_2 + r_2) + j\omega L_2$$
$$Z_C = r + \frac{1}{j\omega C}.$$

The mesh equations are

$$X(Z_C + Z_G + Z_1) - (X + Y)\,Z_G - I_B Z_1, = 0$$
$$(X + Y)\,(Z_2 + Z_G) - XZ_C \pm j\omega m_X I_B = 0.$$

Solving for the galvanometer current,

$$Y = \frac{Z_1 Z_2 - j\omega m_X (Z_C + Z_1)}{Z_1 Z_G + (Z_2 + Z_G)\,(Z_C + Z_1)}.$$

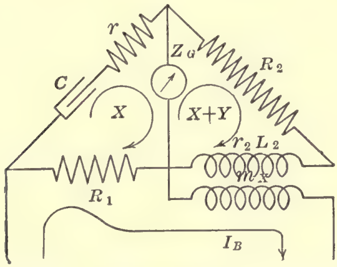


FIG. 244.—Connections for measurement of mutual inductance in terms of capacity.

Therefore the equation for balance is

$$R_1(R_2 + r_2) - \frac{m_X}{C} = j\omega[m_X(R_1 + r) - R_1 L_2].$$

On separating the quadrature components the horizontal component gives

$$m_X = CR_1(R_2 + r_2) \tag{52}$$

the vertical component gives

$$L_2 = \frac{m_X(R_1 + r)}{R_1}. \tag{53}$$

The adjustment of R_2 and C does not disturb the second condition for balance, and an adjustment of r does not disturb the first condition. The two adjustments are thus independent and the arrangement a convenient one. The resistance R_1 should have a large current-carrying capacity. From (53) it is seen that if the resistance r is not present, $m_X = L_2$. This is the lowest usable value of L_2. If the inductance in the branch 2 is below this value it must be increased by adding an inductive coil. In Carey Fos-

ter's original method, the resistance r was absent and variable currents were used.

SOURCES OF CURRENT

In discussing all of these bridge methods sinusoidal currents have been assumed. If a telephone is to be used as a detector and the bridge settings are to be made with precision it is necessary that this assumption be closely fulfilled. As far as the balancing is concerned, when a vibration galvanometer is used the wave form is not so important. There are cases, however, where the balance depends on the frequency, so it is highly desirable that the source of current be one which naturally gives a sinusoidal wave.

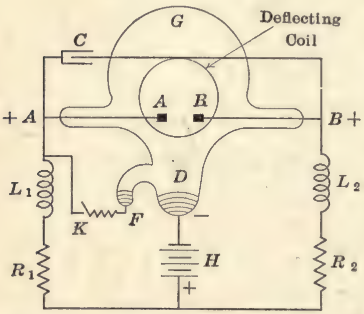


FIG. 245.—Diagram for Vreeland sine wave oscillator.

Vreeland Oscillator.—As the amount of power required is generally small, one is not limited to the use of a dynamo and for bridge work the best source of current is the Vreeland Sine Wave Oscillator.[28]

The wave form obtained from the device is accurately sinusoidal and the frequency, which can be varied from about 160 up to 4,000 cycles per second, is constant for any particular setting of the apparatus.

The device is shown diagrammatically in Fig. 245 and as actually constructed in Fig. 246. The arrangement is such that sinusoidal oscillations of the desired frequency are set up in the primary of an air-core transformer, the secondary of which is connected to the bridge.

The large glass bulb G is the container for a mercury vapor arc in vacuo. The mercury cathode is at D and two carbon anodes are at A and B. H is any convenient source of direct current. R_1 and R_2 are controlling resistances and L_1 and L_2 are two reactors which tend to prevent rapid changes of the direct current which flows from H. In starting the arc an auxiliary electrode F is used and by closing the switch K and tilting the container G, the mercury in F and that in D are brought into contact.

The oscillating circuit *ACB* contains an adjustable condenser, *C*, and the inductance of the two deflecting coils, and can therefore be tuned. The secondary coil from which the current is taken to the bridge is in the magnetic field of the deflecting coils which thus serve the double purpose of causing the arc to oscillate between *D* and *A* and *D* and *B* and of acting as the primary of the air-core transformer.

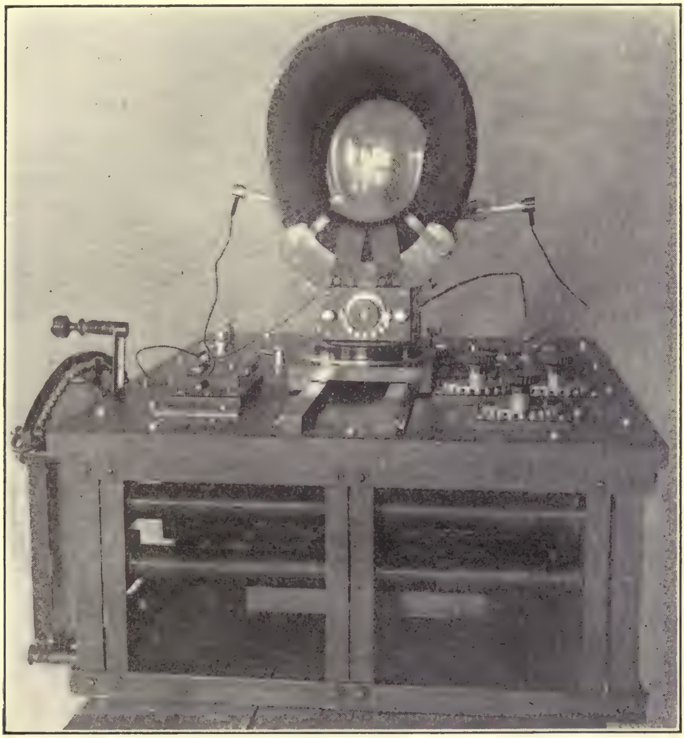


FIG. 246.—Vreeland sine wave oscillator.

If the arrangement were perfectly symmetrical the arc would divide equally between the two anodes, the potentials of *A* and *B* would be equal and no current would flow through the circuit *ACB*. If from any cause the arc to *B* is the stronger, this equality of potentials is disturbed, the potential of *B* is lowered relatively to that of *A*, and a current begins to flow in the oscillating circuit

ACB. The arcs are in the field of the deflecting coils in the circuit *ACB* and these coils are so wound that they still further deflect the arc away from *A* and toward *B*. When the condenser is fully charged, the current in the deflecting coils ceases and the arcs begin to return toward their first condition. The condenser discharges and by means of the deflecting coils the arcs are forced toward *A*.

The frequency with which the condenser is charged and discharged depends upon the inductance and capacity and is given by

$$f = \frac{1}{2\pi\sqrt{L}}\left(\frac{1}{\sqrt{C}}\right).$$

The variations in frequency are obtained by adjusting the capacity *C*. The inductance *L* depends to a certain extent upon the positions of the two deflecting coils. It is, therefore, necessary to determine the frequency after the apparatus has been set up. The calibration is readily made. The current from the secondary is led through a telephone and the note obtained compared by the method of beats with that of a standard tuning fork. After this calibration, the frequency corresponding to any other capacity is readily calculated.

Care must be taken in locating the apparatus, which sets up a considerable stray field. When a telephone is used as the detector the apparatus must be so set up that the note which the oscillator emits does not disturb the observer.

The Microphone Hummer.—If it is not necessary to work at a definite frequency, a simple microphone hummer, as shown in Fig. 247, is a convenient source of current.

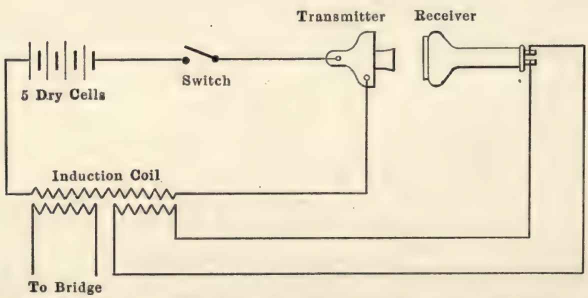


FIG. 247.—Diagram for simple microphone hummer.

This arrangement gives a sharp tone which enables the null point to be accurately determined. The frequency, however, is somewhat uncertain and is controlled by the battery strength and the distance between the transmitter and receiver diaphragms. This simple arrangement has been improved by A. Campbell so that a constant frequency may be obtained; this apparatus is shown in Fig. 248.

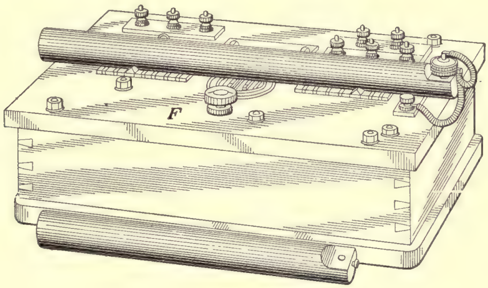


FIG. 248A.

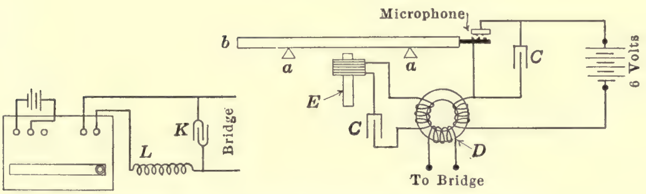


FIG. 248.—Campbell microphone hummer.

The receiver diaphragm is replaced by a bar of mild steel 2.5 cm. in diameter and 23.7 cm. long (for a frequency of 2,000). The bar is supported at its nodal points on two adjustable bridge pieces. The frequency may be altered by changing the bar and at the same time altering the capacity of the condenser so that 800, 1,000 or 2,000 periods per second may be attained. The hummer is started by lightly striking the bar and then adjusting, by the milled head *F*, the height of the magnet *E*, which maintains the vibration. To prevent electrostatic disturbances, the winding *D* is covered with an electrostatic shield which may be connected to earth.

When using the hummer for measuring small capacities or

high inductances, it is advantageous to connect it to the bridge through an auxiliary tuning circuit as shown in the diagram, where K is a subdivided condenser and L an adjustable inductance. The values of these are adjusted to produce resonance with the fundamental frequency of the hummer and by this method it is quite possible to obtain pressures up to 50 or 100 volts at the bridge terminals if only a small amount of current is desired.

Electrical Resonators.—The most obvious method for obtaining the current is by the use of small dynamos of high frequency, but difficulties present themselves because of harmonics in the wave forms. For high periodicities it is necessary to use a telephone as a detector, and to obtain a good null point a pure sine wave is necessary. This may be attained by eliminating, by means of electrical resonators or barriers, all but the harmonic of the desired frequency. It is to be kept in mind that resonating arrangements *partially* eliminate all but a selected harmonic.

If an efficient selective device be used, a complex wave form may become advantageous, for the harmonic of the desired frequency may be selected, for instance, the fundamental, the third, the fifth, and so on; and one machine run at a constant speed will serve for measurements at a number of different frequencies.

Fleming and Dyke Resonator.—Fleming and Dyke in their researches on the power factor and conductivity of dielectrics[18] used a small dynamo having a normal frequency of about 900 cycles per second. The wave form of this machine was far from sinusoidal, the third and fifth harmonics being very pronounced. By the use of a resonator, they were able to select and use in their bridge either the fundamental of 920 periods per second, the third (2,760) or the fifth harmonic (4,600), the machine being operated at a constant speed.

Referring to Fig. 249, C_1 is an adjustable paraffined paper condenser of 20 microfarads capacity. The adjustable inductance L_1 is made by winding single layer coils of No. 14 wire on two pasteboard tubes, one of which can be slipped within the other. The two coils are connected in series by a flexible wire and the inductance is adjusted by inserting the inner tube to a greater or less extent. Three of these inductances may be necessary to cover the range of the fundamental, the third and the

fifth harmonics. As in all electrical resonators, the ohmic resistance of these coils must be kept low if the resonant point is to be sharply defined.

The relation between the inductance and capacity necessary to resonate a harmonic of frequency f is given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

or if C be in microfarads and L in millihenrys,

$$f = \frac{5{,}033}{\sqrt{LC}}$$

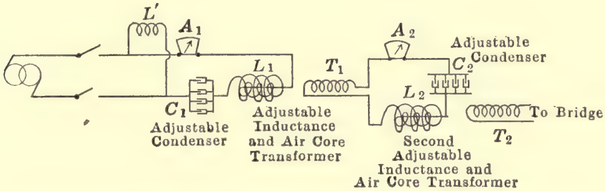


FIG. 249.—Diagram for Fleming and Dyke resonator.

The capacities and inductances used by Fleming and Dyke were as follows:

Frequency	L_1 in millihenrys	C_1 in microfarads
920	1.5	20
2,760	0.81	4
4,600	0.60	2

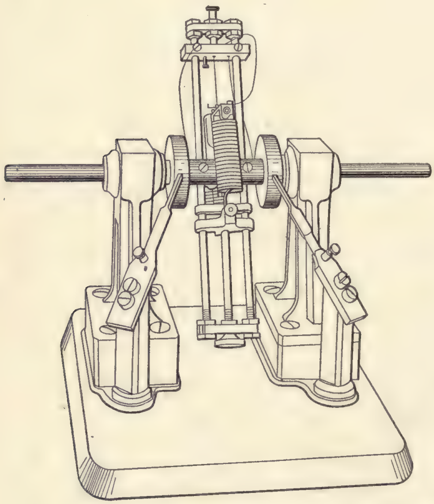
To obtain a pure sine wave it was found necessary to use a second resonator electromagnetically coupled with the first by an air-core transformer formed by thrusting a secondary coil wound on a paper tube, T_1, into the first adjustable inductance, L_1. The capacities used in this second resonator circuit could be varied from 0.25 to 8.25 microfarads. As indicated, the bridge current is derived from a second air-core transformer, its secondary, T_2, being inserted in L_2.

DEVICES FOR MAINTAINING CONSTANT SPEED

In many methods of measurement, particularly when dealing with inductances and capacities, it is necessary accurately to

control the speed of the electric motor which drives the contact-making device or the alternator which furnishes the current. It is presupposed that the voltage of the supply is kept constant. Hand regulation may be employed if some very sensitive form of detector be used which will at once make evident a change of speed, but in careful work this necessitates an additional observer whose only duty is to note the speed indicator and keep the control rheostat adjusted. Therefore, some arrangement which will be automatic in its action is required.

The Giebe Speed Regulator.[29]—A shunt-wound direct-current motor may be controlled by a form of centrifugal governor devised by Giebe.



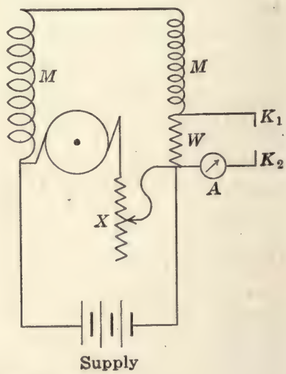


FIG. 250.—Giebe speed regulator.

Fig. 250 shows the diagram of the circuits and a general view of the governor. The resistance X is for the rough adjustment of the speed.

When the speed rises too high, the governor short-circuits a resistance in the field circuit and thus decreases the speed.

Referring to Fig. 251, the frame D carries the mechanism and is rigidly attached to the shaft R which is directly connected to the motor. The electrical connections to the contacts K_1 and K_2 are made through the slip rings V. The weight P slides on a

guide wire W and must move without appreciable friction; it is drawn inward by the spiral springs, F, the tension of which may be regulated. When the motor is started the centrifugal force causes the weight to move out in opposition to the control exercised by the springs. If the speed be high enough, contact between K_1 and K_2 will be made and the resistance short-circuited. The motor then slows down a little and the contact is broken; the speed then rises and the cycle is repeated. Thus the speed is kept constant, subject to very slight oscillations about its mean value.

The position of the piece Q to which the rear ends of the springs are attached may be adjusted by the micrometer screw T (pitch 1 mm.) and when properly adjusted may be clamped in position on the rods SS. The platinum contact K_1, which is carried by a flat spring of moderate strength, is insulated and mounted on the weight P and is connected to one of the slip rings by a flexible wire. The contact K_2 is a flat plate of platinum and the bar H which carries it may be set at any desired distance from the axis. As the shaft runs through the device two springs are used as indicated in the smaller figure.

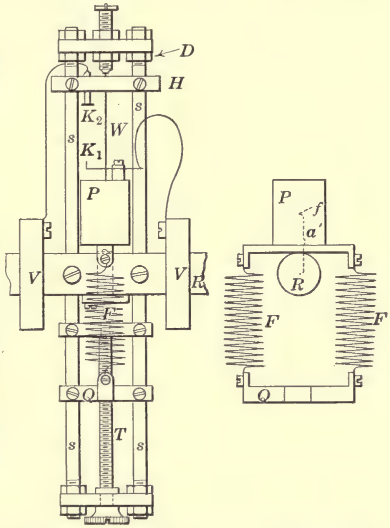


FIG. 251.—Details of Giebe speed regulator.

It is obvious that on account of the shaft the center of gravity of the weight can never be brought to the axis of rotation. Let a' be its minimum possible distance from the center of the shaft, M be the mass of the weight and C the constant of the springs, that is, the force in dynes necessary to extend the springs 1 cm. Let r be the distance of the center of gravity of the weight from the center of the shaft when the latter is rotating with an angular velocity ω. It will be assumed that the springs have been given an initial tension T_1, that is, when the device is at rest in a horizontal position and the weight is in contact with the shaft, a

tension, T_1, has been applied to the springs by means of the micrometer screw.

When the angular velocity is ω and the center of gravity is r cm. from the axis, the centrifugal force will be

$$K_C = M\omega^2 r$$

and the tension on the springs will be

$$K_S = C(r - a') + T_1$$

At equilibrium $K_C = K_S$ and

$$M\omega^2 r = C(r - a') + T_1.$$

The equilibrium may be stable, neutral, or unstable, depending on the initial tension T_1. For, suppose that at some instant when the center of gravity of the weight is distant r from the axis, K_C happens to be equal to K_S. If the weight is given a slight displacement outward, δr,

$$\frac{\delta K_C}{K_C} = \frac{\delta r}{r} \text{ and } \frac{\delta K_S}{K_S} = \frac{C\,\delta r}{C(r - a') + T_1} = \frac{\delta r}{r - \left(a' - \frac{T_1}{C}\right)}.$$

$\frac{T_1}{C}$ is the initial extension of the spring. It will be denoted by e'; then

$$\frac{\delta K_S}{K_S} = \frac{\delta r}{r - (a' - e')}.$$

If a' is greater than e' the denominator of the expression for $\frac{\delta K_S}{K_S}$ is less than r, $\frac{\delta K_S}{K_S} > \frac{\delta K_C}{K_C}$ and the weight will return toward its original position, that is, the equilibrium is stable. If $a' = e'$, then $\frac{\delta K_S}{K_S} = \frac{\delta K_C}{K_C}$ and the equilibrium is neutral. If a' is less than e' the denominator is greater than r and $\frac{\delta K_S}{K_S} < \frac{\delta K_C}{K_C}$. In this case, the weight will suddenly fly outward to the extent of its travel, the spring being insufficient to make it return to its original position. This is the case of unstable equilibrium.

If the springs be given increasing tension the equilibrium remains stable until $T_1 = Ca'$ or $e' = a'$, that is, until the tension is that which would bring the center of gravity of the weight to

the center of the shaft if the motion of the weight in that direction were not limited. This may be called the critical value of the tension, since if it is exceeded, the weight will suddenly fly outward.

While the device may be operated with any one of the three adjustments implied above, it is most sensitive and regulates best when the tension is near its critical value. Referring to the equation of equilibrium,

$$M\omega^2 r = C(r - a') + T_1$$

or

$$\frac{M}{C}\omega^2 r = r - \left(a' - e'\right).$$

If $Ca' = T_1$ or, what is equivalent, if $a' = e'$,

$$\omega = \sqrt{\frac{C}{M}}.$$

This may be called the critical speed, since for any higher speed the equilibrium is unstable.

Why the apparatus regulates most satisfactorily in the neighborhood of the critical speed becomes evident if values of ω and r be plotted; this has been done in Fig. 252, two values of e' being used.

It has been assumed in Fig. 252 that the construction of the regulator is such that the center of gravity of the weight can never be nearer the axis than 3 cm. or more distant than 6 cm. It is seen that as the critical speed is approached the change in the position of the weight for a given increase in the angular velocity becomes vastly increased. This means a corresponding increase in the sensitiveness of the apparatus.

If $a' > e'$ the weight arrives at its ultimate position slowly and the contact of K_1 and K_2 may be uncertain. If $a' < e'$ the motion is sudden and the pressure between the contacts considerable. For the best results e' should be slightly larger than a', that is, the initial tension on the springs should be such that the weight is in unstable equilibrium at the speed which it is desired to maintain.

Any given regulator has only one speed at which it works with entire satisfaction, that is, at $\omega = \sqrt{\frac{C}{M}}$.

If it is desired to regulate the motor at some other speed either C or M must be altered.

To adjust the tension of the springs so that the center of gravity is in the proper initial position, that is, to give $a' - e'$ its correct value, it is necessary to experiment with the completed appara-

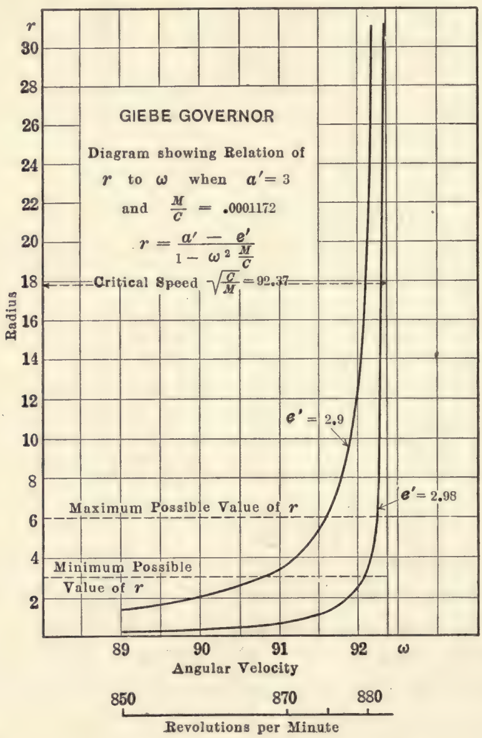


FIG. 252.—Pertaining to Giebe speed regulator.

tus. Tests show that the regulator will keep the speed constant to about $1/10{,}000$ part of its mean value.

The Leeds and Northrup Automatic Speed Controller.—The constant speed necessary for driving secohmeter devices, as well as sufficient energy (about 150 watts at 70 volts) for most of the measurements of inductance and capacity made in an ordinary laboratory, may be conveniently obtained by use of a device

of the Leeds & Northrup Co., in which a small rotary converter or motor-generator set is controlled by an electrically operated tuning fork.

The connections of the apparatus are shown in Fig. 253. The circuits necessary for driving the fork are shown by dotted lines and the synchronizing circuits by solid lines.

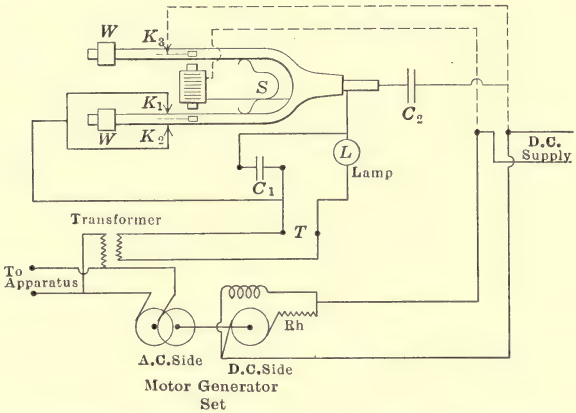
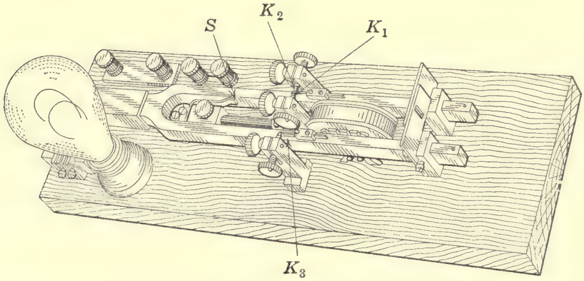


Diagram for Leeds and Northrup speed controller.



Electrically driven tuning fork.

FIG. 253.

The speed of the direct-current motor is first regulated by a rheostat in the armature circuit so that the motor tends to drive the alternator at a speed about 10 per cent. above synchronism.

If it is desired to maintain a frequency of 60 cycles, for instance, the fork is regulated by means of the movable weights W and the movable spring S, which gives a fine adjustment, until its rate is

60 complete vibrations a second. When the alternator gives exactly 60 cycles, the contacts K_1 and K_2 will be made at a definite point in the voltage wave and no effect will be produced. If, however, the direct-current motor tends to speed up and drive the alternator a little above synchronism, the contacts K_1 and K_2 are made when the electromotive force has a higher value and a larger current will flow from the secondary of the transformer through the lamp. This throws an additional load on the motor, the magnitude of which is dependent upon the departure from synchronism.

As there is a resistance in the armature circuit of the motor, this load slows the machine down and brings it back to synchronism. Thus by a series of small and rapidly applied loads (120 a second) the motor is held in check. If the motor tends to slow down, the contact is made when the voltage is smaller, the load on the motor is decreased and the speed maintained constant. Motors up to 1 kw. may be controlled in this manner. As with all adjustable tuning forks, the weights W should be set exactly opposite each other. The adjustment of the contacts is important. When the fork is at rest, the contact springs should be equally distant from K_1 and K_2, about $\frac{1}{32}$ in. To reduce the sparking, the condensers C_1 and C_2 are shunted around the breaks at K_1, K_2, and K_3. The binding posts at C_1 allow the insertion of additional capacity around the breaks K_1 and K_2, if this be found necessary. The transformer is used in order to separate entirely the load from the synchronizing current, this being necessary in order to avoid leakage, etc. Usually a 1:1 ratio is employed, but when the larger sized motors are used the sparking at the contacts may be reduced by transforming to a higher voltage, 1:2, and using a high-voltage lamp for the load.

The Wenner Speed Controller.—This device enables motors of 5 kw. and under to be controlled by an electrically driven tuning fork of the desired frequency. The connections are indicated in Fig. 254.

When the extra field resistance a is in circuit, the field current is cut down and the motor increases in speed, while if a is short-circuited, the field is increased and the speed decreased. The fundamental idea is to arrange a special rotating switch and a contact controlled by the tuning fork, so that when the motor

speed rises, the resistance *a* will be short-circuited for a greater percentage of the time, thus increasing the average strength of the field and, in consequence, bringing the speed back to its original value.

The contact is controlled by the electrically driven fork and is so arranged that it is closed for half a complete period, that is, for half the time. Contact 2 is a slip ring, which is electrically connected to the half-ring 3; thus its circuit is also made for half the time. As contacts 1 and 3 are in series, the resistance *a* is short-circuited only when *both* contacts 1 and 3 are made.

In the normal running position, switch 3 lags 90° behind switch 1 in time phase. Thus the resistance *a* is short-circuited one-fourth of the time and is in circuit three-fourths of the time.

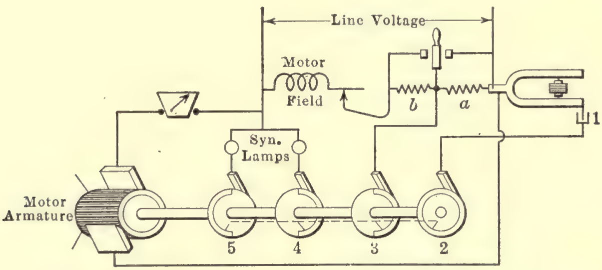


FIG. 254.—Diagram for Wenner speed controller.

If the motor tends to run too fast, it gains on the fork in time phase and the short-circuit around *a* is closed for a longer time, thus increasing the average strength of the motor field. This slows down the machine. On the other hand, if the motor tends to run too slow, the resistance *a* is short-circuited for a shorter time, the average strength of the motor field is decreased and the speed comes back to its original value. The resistance *b* is used in synchronizing, and should be about three-fourths of the resistance *a*. The machine is started with resistance *b* in circuit. When synchronism is indicated by the lamps, *b* is replaced by *a*. The rotating switches 4 and 5, which are in electrical connection with the slip ring 2, are so set that when the motor and the fork are in synchronism the lamps are of equal brilliancy. If the motor tends to hunt, it will be shown by the ammeter.

THE VIBRATION GALVANOMETER

In 1891 Max Wien suggested that it was possible to greatly increase the sensitivity of the detectors used in alternating-current measurements, where zero methods are employed, by taking advantage of the principle of resonance. To do this, the moving member of the detector is mechanically tuned so that its natural period is the same as that of the alternating electro-magnetic forces which cause its deflection. The idea was realized by

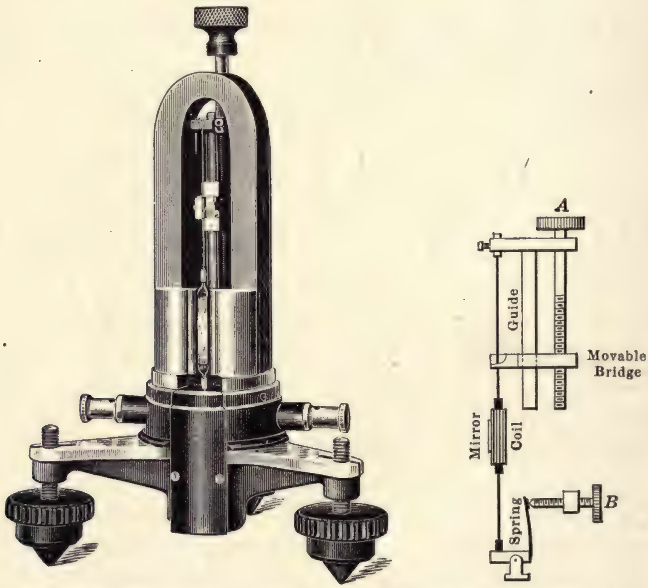


FIG. 255.—Vibration galvanometer, Leeds and Northrup Co.

Wien in his "optical telephone." The later development of this instrument into the vibration galvanometer has been due more especially to Wien, Rubens, A. Campbell, Duddell and Drysdale, who have utilized both the moving needle and the moving coil principles.[30]

The instrument is read by the mirror and scale method and the optical arrangement should be such that when no current is passing, a *sharply* defined image may be seen on the screen. When the bridge, or other apparatus to which the galvanometer

is connected, is out of balance, the galvanometer coil will be set in vibration and this image will become extended into a band of light.

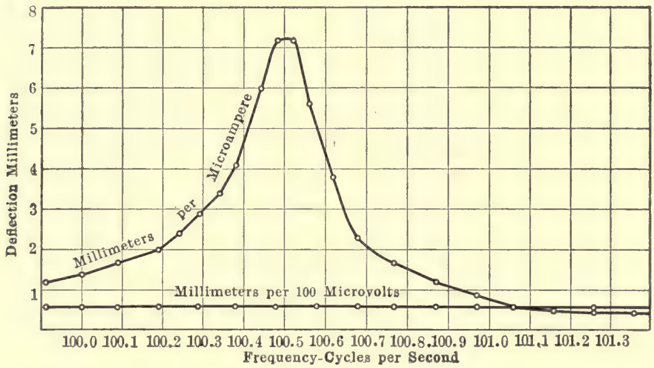


Fig. 255*A*.—Showing effect of tuning, on current and voltage sensitivities of a vibration galvanometer.

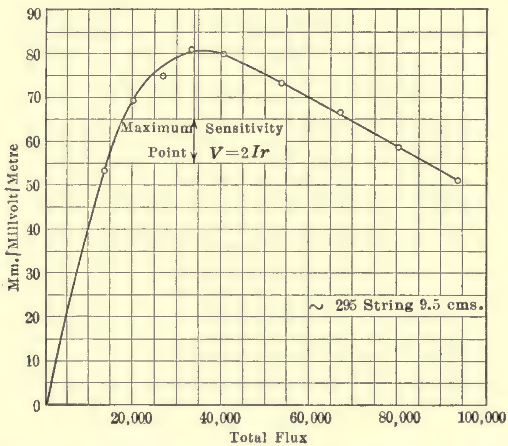


Fig. 255*B*.—Showing effect on the voltage sensitivity of a vibration galvanometer when the flux cut by the coil is varied.

The instrument shown in Fig. 255 is a D'Arsonval galvanometer so designed that the coil may be given a high rate of vibration. The small sketch at the side shows how the rate may be varied, that is, how the instrument may be tuned to the frequency of the circuit. A flat strip suspension is used, the

effective length of which may be adjusted by turning the milled head, A, at the upper end of the vertical screw, thus securing a coarse adjustment. The fine adjustment is made by turning the milled head, B, which, by means of the spring, S, controls the tension on the suspension.

On account of the large restoring moment which must be employed to obtain a high rate of vibration, the sensitivity of a vibration galvanometer for direct currents is very small. It is only when the period of the galvanometer and that of the current coincide that the current sensitivity rises to a high value. This is well illustrated by Fig. 255A. From the figure it is clear that if the sensitivity is to be maintained, the frequency of the current must be constant; in the case shown a change of 0.2 per cent. in the frequency reduces the current sensitivity by about 70 per cent.

The characteristic of responding freely to only one frequency permits many measurements to be made with non-sinusoidal currents, provided the harmonics are not so pronounced that they "force" the vibration of the movable system. In one very good commercial form of vibration galvanometer the sensitivity for the third harmonic is only $\frac{1}{4,000}$, and for the fifth harmonic only $\frac{1}{12,000}$ of that for the fundamental. This selective sensitivity is one reason why, within the range where they are both effective, the vibration galvanometer is superior to the telephone as a detector, unless the telephone is tuned to the frequency of the current.

As current of constant frequency is essential, it is not always possible to use commercial electric circuits as sources of power in those alternating-current measurements where the vibration galvanometer is employed.

Current Sensitivity.[31]—The relation between current and deflection for the vibration galvanometer is obtained by solving equation 9, page 25, in which $i = I_N \sin (N\omega)t$. $N\omega$ is $N2\pi$ times the fundamental frequency of the current; for the fundamental $N = 1$, for the third harmonic $N = 3$, etc.

The equation according to which the vibration takes place is

$$P\frac{d^2\theta}{dt^2} + k\frac{d\theta}{dt} + \tau\theta = CI_N \sin (N\omega)t \tag{54}$$

An idea of the magnitudes of the constants in (54) will be given by data applying to an instrument investigated by Wenner.

$$
\begin{aligned}
C &= 1.4 \times 10^5 \text{ c.g.s.} \\
\tau &= 5{,}700 \quad \text{"} \\
k &= 0.018 \quad \text{"} \\
P &= 0.015 \quad \text{"} \\
\text{Resistance} &= 30 \text{ ohms} \\
\text{Resonating frequency} &= 100 \text{ cycles per second.}
\end{aligned}
$$

The vector solution of (54) is

$$\theta_N = \frac{CI_N}{[\tau - (N\omega)^2 P] + jkN\omega} \tag{55}$$

If ω_0 is 2π times the frequency of the vibrating coil of the galvanometer,

$$\theta_N = \frac{CI_N}{P[\omega_0^2 - (N\omega)^2] + jkN\omega} \tag{56}$$

The maximum sensitivity will be obtained when the instrument is exactly tuned to the frequency of the circuit by changing τ or when $\omega_0 = \omega$, $N = 1$. When tuned, the galvanometer is but little affected by the harmonics in the current wave. For example, the deflection due to the fundamental will be

$$\theta_1 = \frac{CI_1}{k\omega} \tag{57}$$

and that due to a current of N times the fundamental frequency will be, as $kN\omega$ is then negligibly small,

$$\theta_N = \frac{CI_N}{P\omega^2[1 - N^2]}.$$

That is, for the same value of the current,

$$\frac{\theta_1}{\theta_N} = \frac{P\omega[1 - N^2]}{k} \tag{58}$$

Using the data given above for a particular instrument,

$$\frac{\theta_1}{\theta_3} = 4{,}200 \text{ approximately;}$$

or the sensitivity to the third harmonic is less than $1/4{,}000$ that for the fundamental.

The resonance range of a vibration galvanometer is an arbitrary measure of the exactness of tuning required if a high sensi-

tivity is to be maintained, and is defined as the fractional change in the frequency of the current which will reduce the sensitivity of the instrument to one-half its maximum value. It is highly desirable if the galvanometer is to be used for general laboratory purposes, that the resonance point be not too sharply defined. That is, the resonance range of the instrument should be large, as great as 0.2 of 1 per cent.

To express the resonance range in terms of the constants of the galvanometer:
when the instrument is perfectly tuned,

$$\theta_1 = \frac{CI_1}{k\omega}.$$

If the frequency of the supply is slightly raised, that is, if N is made a little greater than 1, the deflection becomes

$$\theta_N = \frac{CI_N}{\omega\sqrt{P^2\omega^2[1 - N^2]^2 + k^2N^2}}.$$

In determining the resonance range the change in N is supposed to be such that $\frac{\theta_N}{I_N} = \frac{\theta_1}{2I_1}$ so

$$k^2(4 - N^2) = P^2\omega^2[1 - N^2]^2$$

and as N is very nearly 1,

$$1 - N^2 = \frac{k\sqrt{3}}{P\omega}, \text{ approximately.}$$

$$N = \frac{\text{Frequency of current which halves the maximum amplitude}}{\text{Resonating frequency}} = 1 + R_i$$

where R_i is the resonance range for current.

$$\therefore\ R_i = \frac{k\sqrt{3}}{2\,P\omega}, \text{ approximately.} \qquad (59)$$

Voltage Sensitivity.[31]—When a vibration galvanometer is so used that the voltage sensitivity is important, it should be noted that as the coil vibrates it cuts the flux in the air gap and thus sets up a back e.m.f. which is in time quadrature with the deflection and has a component in opposition to and a component in quadrature with the current.

The e.m.f. which is effective in forcing the current through the

circuit is the vector difference of the applied and the back e.m.fs.

The back e.m.f. is given by

$$E_B = -C\frac{d\theta}{dt} = -j\omega C\theta = -j\frac{\omega C^2 I}{P[\omega_0^2 - \omega^2] + jk\omega}$$

$$= \frac{-I\omega^2 C^2 k - jI\omega C^2 P[\omega_0^2 - \omega^2]}{P^2[\omega_0^2 - \omega^2]^2 + k^2\omega^2} \tag{60}$$

If r and L are the resistance and inductance of the reactive circuit in which the galvanometer is inserted and V is the applied voltage,

$$I(r + j\omega L) = V + E_B$$

and

$$\frac{\theta[P(\omega_0^2 - \omega^2) + jk\omega](r + j\omega L)}{C} = V - j\omega C\theta$$

$$\therefore \theta = \frac{CV}{rP[\omega_0^2 - \omega^2] - k\omega^2 L + j\omega[kr + LP(\omega_0^2 - \omega^2) + C^2]} \tag{61}$$

In this case, where the circuit is reactive, the sensitivity is not a maximum when $\omega_0 = \omega$ but when τ is so adjusted that

$$P[\omega_0^2 - \omega^2] = -\frac{C^2\omega^2 L}{r^2 + \omega^2 L^2}. \tag{62}$$

The corresponding value for the magnitude of θ is

$$\theta = \frac{CV\sqrt{r^2 + \omega^2 L^2}}{\omega[rC^2 + k(r^2 + \omega^2 L^2)]} \tag{62a}$$

From (62) and (62a) it is seen that if C is large it may be possible to increase the deflection by placing an inductance in series with the galvanometer and slightly raising the frequency of the supply, for the fractional increase in the numerator of (62a) may be greater than that in the denominator.

To obtain the greatest possible deflection when the instrument is used in a circuit of fixed inductance, both the torsional constant of the suspension, τ, and the coil constant, C, must be adjusted. C may be varied by changing the strength of the flux threading the coil. When both τ and C are adjusted, the magnitude of θ is a maximum when

$$\left.\begin{aligned} &P(\omega_0^2 - \omega^2) = -\frac{C^2\omega^2 L}{r^2 + \omega^2 L^2} \\ &\text{and} \\ &C^4 = \frac{r^2 + \omega^2 L^2}{\omega^2}[P^2(\omega_0^2 - \omega^2)^2 + k^2\omega^2] \end{aligned}\right\} \tag{63}$$

The corresponding maximum value of θ, obtained by 63 and 61, is

$$\theta = \frac{V}{2\omega\sqrt{kr}}.$$

In general the equivalent impedance of the circuit including the galvanometer is

$$Z = \left[r + \frac{C^2\omega^2 k}{P^2[\omega_0{}^2 - \omega^2]^2 + k^2\omega^2}\right] + j\omega\left[L + \frac{C^2P[\omega_0{}^2 - \omega^2]}{P^2[\omega_0{}^2 - \omega^2]^2 + k^2\omega^2}\right] \quad (64)$$

when conditions (63) are imposed, this reduces to

$$Z = 2r + jo.$$

Therefore, when the deflection has been made a maximum by adjusting both r and C the current is in phase with the applied voltage and

$$I = \frac{V}{2r} \quad (65)$$

In this case half the energy supplied to the instrument is expended mechanically and half in electrical heating.

The sensitivity to a voltage whose frequency is N times the resonating frequency may be seen from (61). If the voltage, V_N, is applied at the galvanometer terminals,

$$\theta_N = \frac{CV_N}{Pr\omega^2[1 - N^2]} \text{ approximately.} \quad (66)$$

By a process similar to that used on page 438 it may be shown that the resonance range for voltage, R_v, is

$$R_v = \frac{\sqrt{3}(kr + C^2)}{2Pr\omega} \text{ approximately.} \quad (67)$$

It may be possible to increase the voltage sensitivity of the galvanometer by applying the potential difference through a step-up transformer of the proper ratio. The step-up ratio, A, required for maximum sensitivity is given by

$$A^2 = \frac{kr + C^2}{r_1 k} \quad (68)$$

where r_1 is the resistance of the primary and r that of the galvanometer circuit. By the use of a transformer of the above ratio the voltage sensitivity is increased in the ratio $\frac{A}{2}$.

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CHAPTER VIII

INDUCTION INSTRUMENTS

The Induction Principle.—In 1826 Arago discovered that if a copper disc be rotated about a vertical axis and immediately below a magnetic needle which is pivoted coaxially with the disc, the needle is deflected in the direction of rotation of the disc. If the speed be sufficiently high the force acting on the needle will overcome the earth's directive force and the needle will be set in rotation. The inverse experiment may be performed; if the magnet be rotated on its pivot and the disc be free to move, the disc will follow the magnet. The correct explanation of the phenomena was given by Faraday, who showed the motion to be due to the reaction between the magnet and the currents induced in the disc.

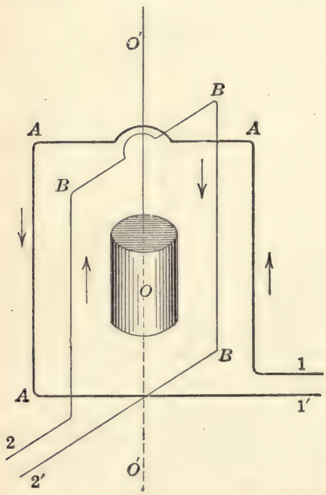

Fig. 256.—Illustrating Ferraris' method of producing a rotating magnetic field.

In this inverse experiment, the phenomena are those associated with a rotating magnetic field; that is, a field of constant intensity whose angular position in space is continually changing. In 1885 Ferraris showed that a rotating magnetic field could be obtained by the action of currents in suitably placed coils.

Let the coils be arranged as in Fig. 256, and consider the magnetic field at the point O. At any instant the field perpendicular to the plane of the coil A at this point is given by $h_A = k_A i_A$ where k_A is a constant depending upon the geometry of the coil, and i_A is the instantaneous value of the current. If sinu-

soidal currents are employed, $h_A = k_AI_A \sin \omega t$. The field due to the second coil, B, will be $h_B = k_BI_B \sin (\omega t - \beta)$ where β is the time-phase difference of the two currents. If the coils are of equal magnetic strengths ($k_AI_A = k_BI_B$) and are placed with their planes perpendicular, and if the two currents are in time quadrature ($\beta = 90°$), the fields at any instant are as shown in Fig. 257.

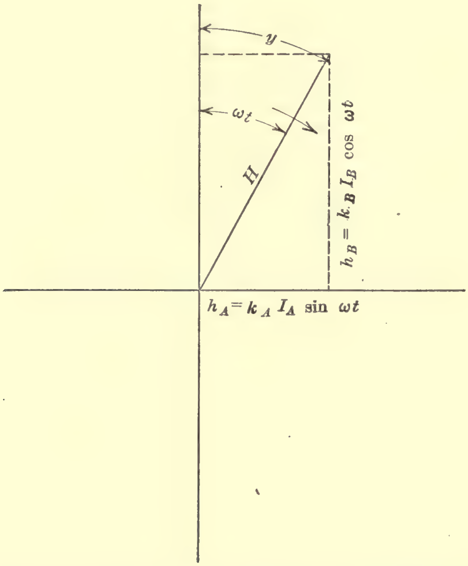

FIG. 257.—Showing components of rotating field.

The resultant field is

$$H = \sqrt{k^2{}_AI^2{}_A \sin^2 \omega t + k^2{}_AI^2{}_A \cos^2 \omega t} = k_AI_A, \text{ a constant.}$$

That is, the field at O is of constant intensity. Its direction at any instant is given by the angle y;

$$\tan y = \frac{\sin \omega t}{\cos \omega t} = \tan \omega t$$

$$\therefore y = \omega t.$$

The resultant field, therefore, rotates with a constant angular velocity, ω, making one complete rotation in the time required for one complete cycle of the current.

If a copper cylinder be suspended by a thread so that its axis

is vertical and includes the point O, the rotating field will cut the cylinder, induced currents will be set up and the cylinder will rotate just as does the disc in the inverse of Arago's experiment. This was one of Ferraris' classic experiments. Ferraris, however, did not appreciate the importance his discoveries would assume when developed along engineering lines, for he states that "These calculations and experimental results confirm the evident *a priori* conclusion that an apparatus founded on this principle cannot be of any commercial importance as a motor, and while we may study the dimensions in order to increase notably its power and output, it would be useless here to enter upon any consideration of this problem. Still, the experiments described may be of some interest."[1]

The possible application of the principle to measuring instruments was, however, mentioned by him.

Application to Measuring Instruments.—The strength of the field and therefore the action on the cylinder will be greatly

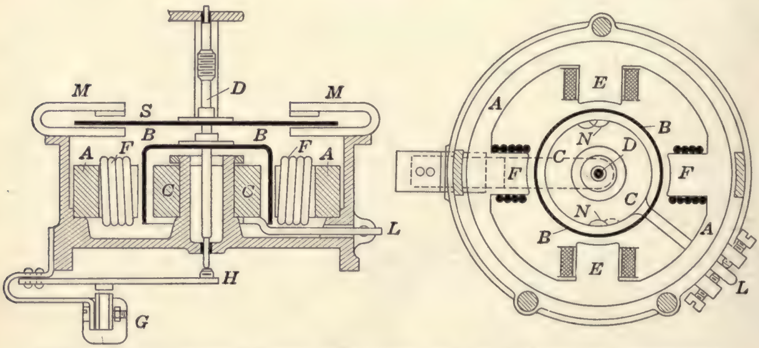

FIG. 258.—Magnetic circuit and rotor of an induction instrument.

increased if the coils be provided with laminated iron cores. Fig. 258 shows the magnetic circuit of an induction-type, watt-hour meter made at one time by Siemens and Halske.

In the figure, AA is a laminated ring with four poles FF, EE projecting toward the center and C is a circular laminated iron core. It is evident that the poles F and F produce a field whose general direction in the right-hand diagram is horizontal, while the field due to E and E is vertical. The strength of field in the narrow air gap will be considerable and in this field is placed the

hollow aluminum drum *B* which forms the movable element. The similarity of this arrangement to that of Ferraris is evident.

The general explanation of the action of induction ammeters, voltmeters, watt-meters and watt-hour meters may be based on a consideration of the properties of rotating magnetic fields.* In this text, however, the motion of the rotor, or movable member, will be considered to be due to the reciprocal action of one pair of poles on the currents induced by the other pair of poles.

Referring to Fig. 258, the fields due to coils *FF* induce currents in the cylindrical aluminum rotor. These induced currents will tend to flow in closed paths perpendicular to the axis of the coil.

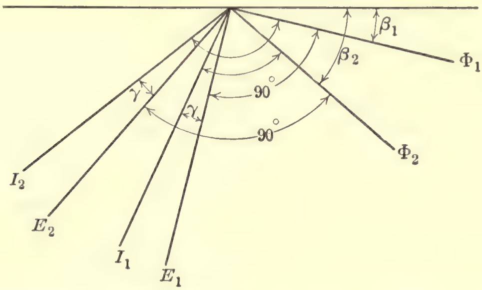

Fig. 259.—Time-phase diagram for induction instrument.

The various current filaments will lag behind the induced e.m.fs. by angles dependent on the resistances and inductances of the current paths in the cylinder. Call γ their equivalent phase displacement. These currents are in a position to be deflected by the fields set up by coils *EE*. Similarly the currents induced by the flux from *EE* will be acted upon by the flux from *FF*. Assuming sinusoidal currents, the time-phase relations of the various quantities are shown in Fig. 259. Let the fluxes set up by *FF* and *EE* be Φ_1 and Φ_2 respectively.

The angles β_1 and β_2 are measured from an arbitrary zero line. The time-phase difference between the two fluxes, Φ_1 and Φ^2, is $\beta_2 - \beta_1$. E_1 is the e.m.f. induced in the cylindrical or disc rotor by the flux Φ_1 and E_2 is that due to Φ_2. E_1 sets up induced

* Such an explanation is given in Solomon, "Electricity Meters," p. 111.

currents in the rotor, which on account of the inductances of the eddy-current paths lag behind E_1. The equivalent value of these currents is I_1, which lags behind E_1 by the angle γ. Likewise, the equivalent current I_2 lags behind E_2. The turning moment is due to the reaction between Φ_1 and I_2 less that between Φ_2 and I_1. The fluxes and currents are *assumed* to be sine functions of the time. The instantaneous torque will be of the form

$$\tau = Ki_1\phi_2 - Ki_2\phi_1$$

and the average torque will be

$$\mathrm{T} = \frac{1}{T}\int_0^T (Ki_1\phi_2 - Ki_2\phi_1)\, dt$$

$$= K'\Phi_2 I_1 \cos(\gamma + 90° + \beta_1 - \beta_2) - K'\Phi_1 I_2 \cos(\gamma + 90° + \beta_2 - \beta_1)$$

$$= -K'\Phi_2 I_1 \sin(\gamma - (\beta_2 - \beta_1)) + K'\Phi_1 I_2 \sin(\gamma + (\beta_2 - \beta_1)).$$

At any given frequency the induced currents are

$$I_1 = \frac{K_1\Phi_1 f}{Z}$$

and

$$I_2 = \frac{K_1\Phi_2 f}{Z}$$

where Z is the impedance of the eddy-current paths. Therefore,

$$\mathrm{T} = \frac{K''f\Phi_1\Phi_2}{Z}\{-\sin(\gamma - (\beta_2 - \beta_1)) + \sin(\gamma + (\beta_2 - \beta_1)\}$$

$$= \frac{K'''f\Phi_1\Phi_2}{Z}\cos\gamma\sin(\beta_2 - \beta_1). \qquad (1)$$

The accelerating torque is proportional to the product of the two fluxes, to the sine of the time-phase angle between them, and to the frequency.

If the armature is in motion, there will be a retarding effect due to its movement through the two alternating fields. This retarding effect will be proportional to the mean square values of the fields and to the angular velocity of the armature ω', or to

$$\left[\frac{\Phi_1{}^2}{2} + \frac{\Phi_2{}^2}{2}\right]\omega'.$$

By proper design this term is kept as small as possible in watt-hour meters, and the armature is seldom allowed to make more than one revolution per second.

Induction Ammeters and Voltmeters.—As an illustration of the application of these principles to measuring instruments, the Westinghouse induction ammeters and voltmeters may be taken.[2,3] Fig. 260*A* shows the electrical and magnetic circuits of the ammeter. The line current enters by the terminals TT and flows through the coil P, giving rise to the flux, Φ_P, which crosses the air gap in the horizontal direction. The magnetic circuit carries a second winding, S, which with the coils AA' forms a closed circuit.

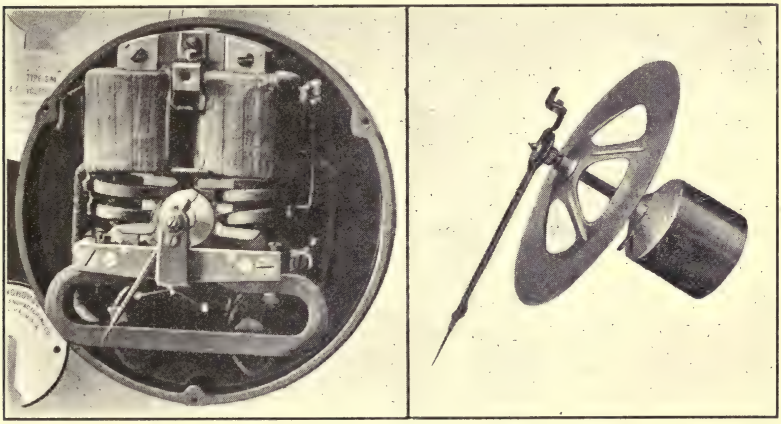

Fig. 260.—Westinghouse induction ammeter.

The current induced in this secondary circuit sets up a flux, Φ_S, which crosses the air gap in the vertical direction. The two fluxes are in the proper space relation to produce a torque on the thin aluminum cup which forms the movable element. They must also have the proper time-phase relation. This is attained by the transformer action of the two windings, P and S.

Referring to the vector diagram accompanying Fig. 260*A*, the flux Φ_P links both P and S. It is therefore the mutual flux and induces in the winding S an e.m.f., E_S, which lags 90°. The flux, Φ_S, threads the secondary winding, S, but not the primary winding, P, consequently it is the secondary leakage flux; it will

be in time phase with I_S, the current which flows in S. The induced voltage, E_S, must overcome the reactance drop due to Φ_S and the ohmic drop in S. The fluxes Φ_P and Φ_S differ in time phase, being somewhat more than 90° apart, and will produce a torque on the movable element.

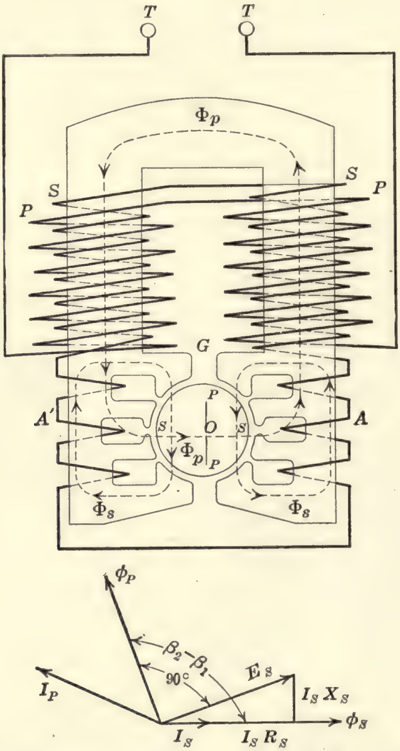

FIG. 260A.—Electric and magnetic circuits of Westinghouse induction ammeter.

I is the line current and as the arrangement is a sort of current transformer,

$$\Phi_S = K_2 I$$

$$\Phi_P = \frac{K_3 I}{f}$$

These values inserted in (1) give for the torque,

$$T = K_4 I^2 \frac{\cos\gamma}{Z}[\sin(\beta_2 - \beta_1)] \tag{2}$$

$$\beta_2 - \beta_1 = 90° \text{ approx.}$$

The torque and therefore the deflection are proportional to the mean square of the line current.

Frequency and Temperature Effects in Westinghouse Ammeter.—In order that an instrument may be of the highest utility, its indications must be free from the effects of change of frequency and of temperature. Practically, these effects must be reduced to amounts which are consistent with good engineering practice, since they cannot be made zero. As Z, γ, and $(\beta_2 - \beta_1)$ vary with the frequency, the factor $\frac{\cos\gamma \sin(\beta_2 - \beta_1)}{Z}$ can be made only approximately constant. By careful design, the maximum difference in the readings corresponding to a definite current, for any two frequencies between 25 and 60 cycles, may be reduced to about 0.5 per cent.

In order that the temperature effect be *nil*, for any definite current, $\frac{\Phi_S \Phi_P}{Z}\cos\gamma$ must be constant with respect to temperature. This relation is readily attained as the arrangement is electrically equivalent to a current transformer. It is to be remembered that in a current transformer if the primary current be kept constant, the secondary current will be practically unaffected by considerable changes in the resistance of the secondary circuit. This means that the mutual flux, Φ_P, increases in proportion to the secondary resistance. The flux Φ_S is due to the secondary current and is, therefore, fixed in value; consequently, if by the use of the proper materials in the secondary, Φ_P be given a temperature coefficient equal to that of $\frac{Z}{\cos\gamma}$, the required adjustment is attained. Therefore, the secondary circuit is made partly of copper and partly of resistance wire of low temperature coefficient, the proper proportion of the two being determined experimentally, so that allowance can also be made for the temperature coefficient of the iron and of the controlling spring.

In the voltmeter the primary coil is wound with fine wire, and an external non-inductive resistance wound with wire of zero temperature coefficient is used. The proportion of ohmic re-

sistance to total impedance is made very high, so that the current in the primary coil is practically independent of the frequency.

The Induction Wattmeter.—The induction principle is used in switchboard wattmeters.

Fig. 261 shows, in diagram, the magnetic and electric circuits of a switchboard instrument made by the Westinghouse Company.

The core is built up of thin stampings. The voltage coil, in series with a reactor, is connected across the circuit; it magnetizes the core as indicated by the arrows. This potential coil flux crosses the air gap in the horizontal direction. The current coils give rise to the flux in the vertical direction. These two fluxes are, therefore, in the appropriate space relation for producing a torque on the thin aluminum cylinder which is pivoted in the air gap.

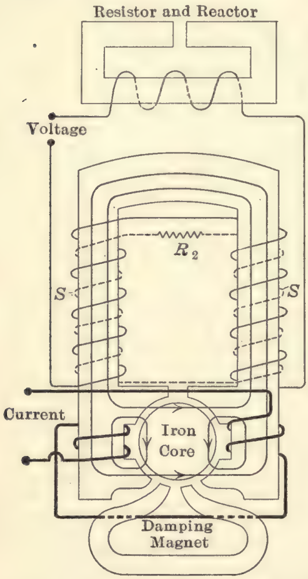

FIG. 261.—Electric and magnetic circuits of Westinghouse induction wattmeter.

As the air gaps are large, the fluxes will be proportional to the currents. Consequently at a fixed frequency,

$$T = \frac{K_5 VI \cos \gamma}{Z} \sin (\beta_2 - \beta_1).$$

As before, $\beta_2 - \beta_1$ is the time-phase difference of the fluxes.

The time-phase diagram is given in Fig. 262.

The potential circuit of an induction wattmeter is purposely made highly inductive so that Φ_1 naturally lags 75° or 80° behind the applied voltage; it will not lag 90° on account of the energy losses in the iron cores and in the windings.

The angle α is the amount by which Φ_1 falls short of being in quadrature with V.

$$\alpha + (\beta_2 - \beta_1) + \theta = 90°,$$

so the torque is given by

$$T = K_5 VI \frac{\cos \gamma}{Z} \sin (90 - \alpha - \theta) = K_5 VI \frac{\cos \gamma}{Z} \cos (\alpha + \theta).$$

The power is $VI \cos \theta$. If $\alpha = 0$, that is, if the useful potential-coil flux is adjusted so that it is exactly 90° out of time phase with the applied voltage, the torque will be proportional to the power in the circuit. This, of course, is the proper adjustment and must be attained by the addition of special phase shifting or lagging devices by which the *useful potential-coil flux* in the air gap is brought into quadrature with the line voltage.

If the useful flux lags behind the applied voltage by less than 90°, that is, if α is +, the wattmeter is said to be under-lagged; if $\alpha = 0$, it is exactly lagged and if α is −, that is, if the flux has been caused to lag more than 90°, the instrument is over-lagged.

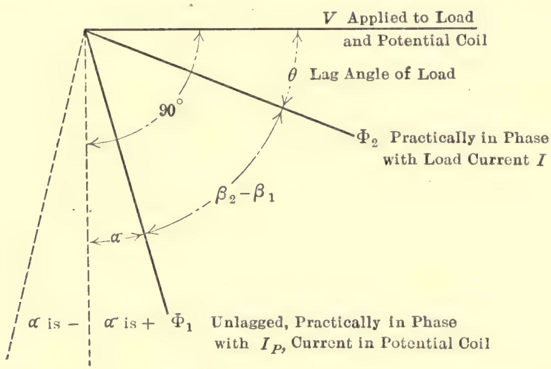

FIG. 262.—Time-phase diagram for induction wattmeter.

The construction of the magnetic circuit of the potential coil should be such that the useful potential-coil flux is naturally nearly 90° out of phase with the applied voltage; the required amount of additional lagging is thus reduced. The less the reliance placed on the lagging device the better will the instrument behave when it is subjected to wide variation of frequency and to distorted wave form.

In the instrument shown in Fig. 261 the desired quadrature relation of the applied voltage and the useful potential-coil flux is obtained by use of a second winding, S, the circuit of which may be closed through the resistance R_2.

It will be seen that this lagging arrangement is a sort of transformer with a large primary leakage flux, much of which is in the series reactor. There is considerable resistance in the

potential-coil circuit so that when the circuit of R_2 is open the instrument is under lagged, that is, the angle between the applied voltage and the useful potential-coil flux is less than 90°.

Referring to the diagram, Fig. 263, (in which a 1/1 ratio is assumed), the flux which threads the voltage winding and the secondary winding, S, (shown in Fig. 261) is the mutual flux Φ_M. This flux is also the useful potential-coil flux; it crosses the air gap in the horizontal direction and cuts the hollow aluminum cylinder which is pivoted in the gap. This cylinder forms the rotor or movable member of the wattmeter. Φ_M induces an e.m.f., E, in both the voltage and the secondary windings. The applied voltage, V, must overcome the voltage, E, induced by Φ_M, the ohmic drop in the primary circuit I_1R_1, and the reactive drop in the primary circuit, I_1X_1, due to the

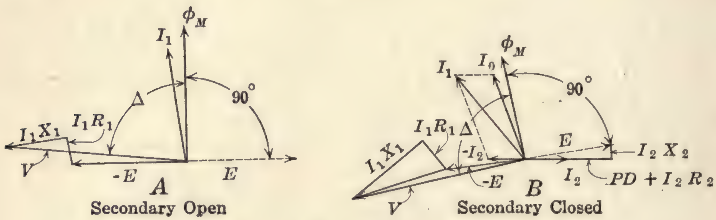

Fig. 263.

primary leakage flux. As indicated in Fig. 263A, the angle Δ between the applied voltage, V, und the useful potential-coil flux, Φ_M, is less than 90°.

When the secondary circuit is closed through the resistance, R_2, (see Fig. 263B) a current, I_2, flows in it. The total primary current, I_1, is the vector sum of $-I_2$ and the no load current I_0. It is seen that closing the secondary circuit rotates I_1 counter-clockwise so that when I_1X_1 and I_1R_1 are added to $-E$ the vector, V, which represents the applied voltage, is also rotated counter-clockwise and the angle Δ between the applied voltage, V, and the useful flux, Φ_M, is increased.

In order to adjust the lagging so that Δ is 90° it is necessary to calibrate the instrument with a load of unity power factor, and then to calibrate it with a load of low power factor, about 0.5. If the two results do not agree, the resistance R_2 (Fig. 261)

must be altered and another trial made, and so on until the results do agree.

As the correct action of an induction wattmeter depends on having the angle Δ exactly 90°, it is obvious that a change of frequency, with its accompanying change of reactance of the potential coil, will introduce errors. An induction wattmeter which is adjusted for use on a 60-cycle circuit will be in error if used at 25 cycles. As the correctness of the instrument is dependent on the frequency, it will be affected by changes of wave form. See the discussion of the induction watt-hour meter, page 473.

In another lagging arrangement, not commonly used in America, the potential coil is shunted by a non-inductive resistance and this divided circuit placed in series with a reactance. This whole combination forms the potential circuit of the instrument.

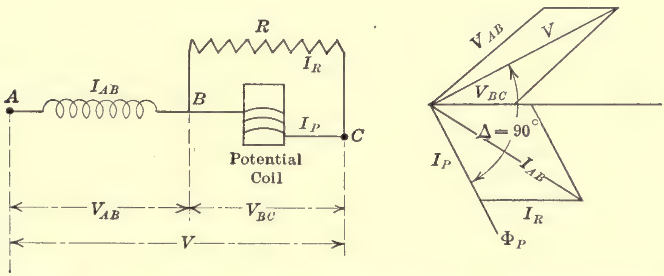

FIG. 264.—Diagram for lagging arrangement for induction wattmeter.

This arrangement is indicated in Fig. 264.

The equations are:

$$\dot{I}_{AB} = \dot{I}_R + \dot{I}_P$$
$$\dot{V} = \dot{V}_{AB} + \dot{V}_{BC},$$

By altering the non-inductive resistance, R, the component, I_R, may be varied and the phase of I_P, or more correctly, the phase of Φ_P, which will differ a little from that of I_P, may be changed until $\Delta = 90°$.

In the induction wattmeter, temperature changes affect the resistance of the rotor or movable element; the induced currents are thus cut down about 0.4 per cent. per degree of temperature

rise. Consequently, unless there is some special compensation for temperature effects, this form of wattmeter has a large temperature coefficient. In the Westinghouse instrument shown on page 452 the compensation is made, as in the ammeter, by adjusting the temperature coefficient of the circuit containing R_2.

For the discussion of the induction watt-hour meter, see the chapter on "Electricity Meters," page 457.

The advantages claimed for the induction type of instrument for switchboard work, are:

1. Extremely long scales, due to the fact that the movable element can turn through nearly 360°.
2. Compactness; this reduces the size of the switchboard panel.
3. Errors due to external fields of the fundamental frequency are small.
4. Robust construction, which facilitates repairs.
5. The ratio of torque to weight of moving element is large.

There are also certain disadvantages, some of which may be overcome by careful design.

1. The indications are affected by changes of frequency and wave form; these errors are very important in induction watt-hour meters, especially at low power factors.
2. There are temperature errors caused by changes in the resistance of the armature due both to changes of room temperature and to self-heating in the instrument.
3. The torque always increases with the load on the instrument, so in cases of sudden overload the pointer may be thrown violently against the stop and either bent or displaced.
4. Alternating currents must be used in checking the instruments, this is sometimes inconvenient when dealing with portable instruments.

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1. "Electrodynamic Rotation Produced by Alternate Currents," G. Ferraris, *The Electrician*, vol. 36, 1895–96, p. 281. (Paper was read March 18, 1888.)

2. "A New Form of Induction Ammeter or Voltmeter," Paul MacGahan, *Electric Journal*, vol. 4, 1907, p. 113.

3. "Induction Type Indicating Instruments," P. MacGahan, *Trans American Institute Electrical Engineers*, vol. 31, part II, 1912, p. 1565.

4. "Electricity Meters," H. G. Solomon, 1906, Charles Griffin & Co., London.

CHAPTER IX

ELECTRICITY METERS

Watt-hour Meter.—In connection with the supply of electrical energy for lighting and power, it is necessary to have some form of integrating meter; that is, a meter which will give, not the rate at which energy is supplied to the circuit, but the total amount of energy supplied during a given time, as for instance, a month. Such a meter must evaluate $\int_{t_1}^{t_2} vidt$; v and i are the instantaneous values of the voltage and current; the time during which the energy is supplied is $t_2 - t_1$. It is customary to express this time in hours. The energy is then stated in watt-hours, or, more frequently, in kilowatt-hours.

The necessity for accurately measuring electrical energy is apparent from the fact that in the United States, alone, the charges for the electrical energy furnished for light and power were, for the year 1916, approximately $450,000,000.

The essentials of the watt-hour meter will be better appreciated if one approved form of the instrument be described. The Thomson watt-hour meter for direct currents will be selected, for this was the first successful commutating meter. It was placed on the market in the latter part of 1889, has passed through the usual processes of development, and is still regarded as one of the best of its class.

The instrument consists of a small motor which is provided with a magnetic brake. The motor drives a counter whose indications on a system of dials are proportional to the total number of revolutions which have been executed by the armature.

No iron is used in either the field or the armature of the motor, therefore all magnetic effects are directly proportional to the currents.

The fields of the motor are placed in the main circuit in series with the load. The armature, in series with a suitable resistance, is connected across the supply mains. The driving torque of the

motor is proportional to VI and the retarding torque due to the brake is proportional to the angular velocity of the disc. Broadly speaking, therefore, the energy supplied to the circuit during a given time will be proportional to the number of revolutions of the armature during that time; in other words, to the reading on the dials of the counter.

Referring to Fig. 265, the main current passes through two similar field coils of low resistance F. In a two-wire meter these coils are connected in series. The potential circuit, which contains the armature A, is connected on the line side of the main

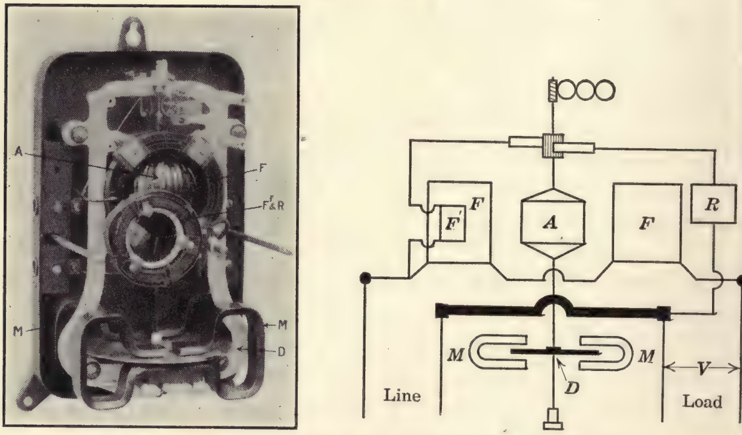


FIG. 265.—Thomson watt-hour meter for direct currents.

coils. The armature (now made in a spherical form to reduce the weight) is carried by an upright spindle which at its upper end gears by means of a worm into the very light train of wheels which moves the pointers of the counter over the dials. The current is carried to the armature through silver-tipped brushes, which rest with a very slight pressure on a silver commutator of small diameter. The brushes are adjusted before the meter leaves the factory and should be very carefully handled.

In the early designs of this meter, the series resistance, R, was wound non-inductively on cards and carried in an envelope at the back of the instrument. It is now combined with the light-load coil, F', mentioned below. In a 110-volt instrument, the total resistance of the potential circuit is about 2,500 ohms.

The lower end of the shaft is provided with a removable steel pivot which rests in a sapphire or diamond jewel carried by a spring support in the end of the jewel screw (see Fig. 267). This screw can be turned back so that the disc D is clamped against the magnets M; the pivot is thus relieved of all strain during transportation.

The brake disc, D, now made of aluminum, moves through the fields of the permanent magnets, M. To change the retarding torque and therefore the speed of the meter, the distance of the poles of the magnets from the axis of the disc may be altered.

In order to compensate at light loads for the effects of mechanical friction, a field coil of fine wire, F', is connected in series with the armature. A small permanent driving torque is thus obtained. By adjusting the position of the coil with respect to the armature this torque may be made such that the registration at light loads is commercially correct; at the same time the load current necessary to start the meter is much reduced. The effect of the light-load coil at the higher loads is insignificant.

To show that the number of revolutions during a given time is practically proportional to the energy supplied to the load *via* the meter,

Let V = line voltage.
I = line current.
I_a = current in potential circuit or armature.
R = resistance of potential circuit.
ω = angular velocity of armature.
h = field due to drag magnets.
r = resistance to eddy currents in brake disc.
K_S = driving torque due to light-load coil.
K_M = initial friction torque.

K and k, with various subscripts, are constants or proportionality factors.

The flux through the armature due to the main coils, F, will be proportional to I and that due to the starting or light-load coil, F', will be proportional to I_a, so

$$\text{total flux through armature} = k_F I + k_{F'} I_a.$$

The back e.m.f. in the armature circuit will be proportional to the product of the flux and the angular velocity.

$$\text{Back e.m.f.} = k(k_F I + k_{F'} I_a)\omega$$

The armature current will be

$$I_a = \frac{V - kk_F I\omega}{R + kk_{F'}\omega} = \frac{V - kk_F I\omega}{R} \text{ approximately.}$$

The driving torque due to the main coils may be represented by $K_F I I_a$ and that due to the light-load coils by $K_{F'} I_a^2$. As the meter is operated at a constant voltage and the torque due to the light-load coil is small, it is allowable to consider this quantity as constant; it will be denoted by K_S. The total accelerating torque is

$$\left(\frac{K_F}{R}\right) VI - \left(\frac{K_F k k_F}{R}\right) I^2\omega + K_S \tag{1}$$

The total retarding torque is that due to the mechanical friction of the meter (including windage) plus that due to the magnetic brake. The friction torque has a certain initial value, denoted by K_M, and increases more rapidly than the speed; its value may then be represented by $K_M + K_{M'}\omega^2$. The brake torque is proportional to the angular velocity of the disc and if the temperature of the disc and the magnets is constant, may be expressed by $\dfrac{k_D h^2 \omega}{r} = K_D\omega$. (2)

Consequently the total retarding torque is

$$K_M + K_D\omega + K_{M'}\omega^2 \tag{3}$$

For steady motion the accelerating and retarding torques must be equal. Equating (1) and (3) gives for the angular velocity of the disc,

$$\omega = K_1 VI - K_2 I^2\omega + K'_S - K'_M - K'_{M}{}'\omega^2 \tag{4}$$

The terms on the right-hand side of the equation which involve ω are small corrections due to the back e.m.f. and the change of friction torque with the speed.

It is seen that if the light-load coil is adjusted so that at no-load the meter is just on the point of starting (K'_M very slightly greater than K'_S), the angular velocity of the armature will be practically proportional to the power supplied to the load; and it at once follows that the total number of revolutions, N, executed by the armature in a given time is proportional to the energy supplied to the load during that time.

For meters as actually constructed

$$\text{watt-hours registered on dials} = \dot{K}_h N. \quad (5)$$

This is fundamental and applies to all watt-hour meters. The factor K_h is called the watt-hour constant, and is the number of watt-hours of energy necessary to cause one revolution of the movable system.

General Discussion of Essential Characteristics.—A consideration of the uses to which the watt-hour meter is put will show that the instrument should possess certain characteristics.

In order that the first cost and the expense of maintenance may not be too great, electricity meters must be simple in design and must contain no parts which are subject to rapid deterioration.

It is desirable that the reading in kilowatt-hours be given directly by the dials, especially in small meters; this avoids the necessity for multiplying the dial readings by a constant. A possible source of misunderstanding between the consumer and the supply company is thus avoided.

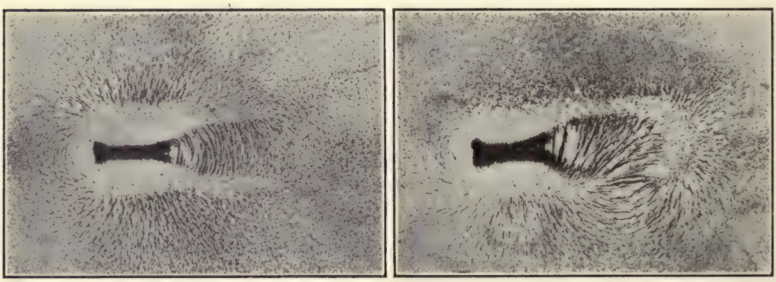
The meter should be protected by a case which can be sealed and which is dust, water and insect proof; the arrangement should be such that there is little likelihood that a short-circuit can occur during the removal and the replacement of the meter cover. It is desirable that the electrical connections to the meter be so made that it is not possible to tamper with the instrument; in certain cases special devices are used for covering all the connections from the service wires to the meter so that it is impossible for the customer to draw current which does not pass through the meter.

Permanency of calibration is a prime requisite. On a large system it is not practicable to test and adjust the majority of the meters oftener than once a year. Consequently, they must maintain their accuracy for at least this period. In the case of large consumers where the amount of money involved is considerable the inspections are more frequent.

To attain permanence of calibration the friction at the pivots and commutator and the retarding torque of the magnetic brake must remain constant. It is essential that the commutator and brushes be capable of operating continuously for long periods without undue increase of friction and without attention. To

secure this result Elihu Thomson, after experimenting with many materials, was led by a knowledge of the mechanical and electrical properties of silver, and experience with contacts made of it, to adopt the pure metal for both the commutator and the brushes. This solved the greatest problem in the design of commutating meters.

The magnets used in connection with the retarding disc or brake must retain their strength. This involves the choice of a proper magnet steel, a correct design (a nearly closed magnetic circuit) and the artificial ageing of the magnets by partial demagnetization and by the proper heat treatment at moderate temperatures. From equation (2) in the demonstration already



Normal distribution. Distribution after a short circuit.

FIG. 266.—Showing effect of a short-circuit on the distribution of magnetism in the drag magnets of a direct-current watt-hour meter.

given (page 460), it will be seen that a 1 per cent. change in the strength of the retarding magnets affects the accuracy of the meter by 2 per cent.

The magnets should be so placed that their strength is not likely to be altered by the field due to the current coil.

Besides the natural deterioration of the magnets there is the chance of an accidental change due to short-circuits, and the magnets should be so arranged that this effect will be minimized. The enormous and sudden rush of current through the current coils during a short-circuit sets up a magnetic field which may be strong enough to change entirely the distribution of magnetism in the drag magnets and cause the meter to over-register. Such a change in the distribution of magnetism is illustrated in Fig. 266.

The field coils must be firmly held and kept apart by spacing

blocks so that they cannot alter their positions or draw together and crush the armature if a short-circuit occurs.

As friction losses in the instrument are unavoidable, they must be reduced to a minimum and must remain practically constant over long periods of time. Therefore the moving parts should be light, the jewels and pivots of the best, and kept in good order. The jewels should be carried by spring supports as this construction increases the life of the jewel by the elimination of "hammering" and consequently assists in maintaining accuracy at light loads.

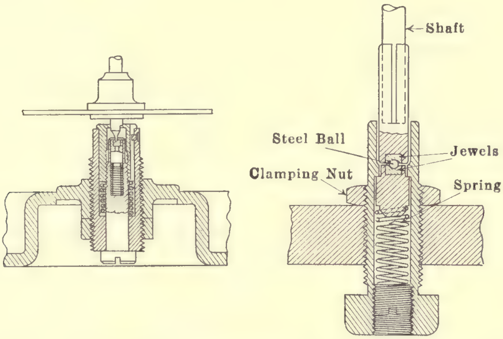


FIG. 267.—Jewel supports for watt-hour meters.

Cupped diamond jewels are now used in direct-current meters of large capacity; this contributes materially to the maintenance of accuracy at light loads.

In order to reduce the wear on the moving parts of a meter, the full-load speed is limited to about 50 revolutions per minute.

The commutator should be of small diameter and smooth and must be kept free from oil and dust of all kinds; the brushes must be smooth and the brush pressure properly adjusted.

The counter should have the minimum possible friction and the worm and wheel connection to the armature spindle should be properly adjusted. Steady pins should be used to insure permanence of the adjustment.

The torque of the meter should be high so that the unavoidable irregularities in friction may not cause inaccuracies, for in time the pivot and jewel as well as the commutator become rough, especially if the meter be subject to vibration and sudden jars.

The commutator is very likely to be troublesome, especially if any sparking occurs due to the presence of dust or oil.

The fact that a meter has a high torque is advantageous only when it is associated with a light moving element. The ratio

$$\frac{\text{Full-load torque}}{\text{Full-load speed} \times \text{friction}}$$

should be large if the accuracy of the meter is to be only slightly affected by the wear of the jewel, pivot and commutator.

Of late years the weight of the movable element has been much decreased by the use of a spherical self-supporting armature and an aluminum brake disc.

The effect of changes of room temperature on the accuracy of the meter should be reduced to a minimum. It is evident that the net effect of temperature on the resistance of the disc and on the drag magnets should be balanced by the change in the resistance of the armature circuit. The average temperature coefficient of the meter between 20° and 40°C. should not be more than 0.2 per cent. per degree at either 10 per cent or 100 per cent of full-load current. Tests show[8] that the temperature coefficients of representative direct-current watt-hour meters of American manufacture vary between +0.26 and +0.07 per cent per degree C., most of them being about +0.1 per cent.

As the armature circuit carries considerable current and is in part made of copper, its resistance (R in formula 1) will rise and decrease the driving torque when the meter is first connected in circuit; consequently, tests should not be made until the permanent state of temperature has been reached, which may require about 20 minutes.

When a load is thrown on the meter, the heat liberated in the current coils also raises the temperature of the copper-wound armature and increases the resistance of the potential circuit, thus decreasing the registration.[8] This effect increases with the load current and, up to the time of attainment of temperature equilibrium, with the length of time the current is left on.

From equation (2) it will be seen that the effect of the self-heating of the meter on the magnetic brake is to decrease the retarding torque and cause the meter to speed up. However, as

the brake is at a considerable distance from both the current and potential coils, the error is practically that due to the change in temperature of the potential circuit.

The net result of the heating due to the current coils is that if the meter is adjusted at full-load by changing the position of the drag magnets and then at light load by means of the light-load coil, the registration will be correct at light and at full-load, will be a little too great at intermediate loads and a little too small at overloads, the general form of the percent-registration curve being that shown at *AHF* in Fig. 268.

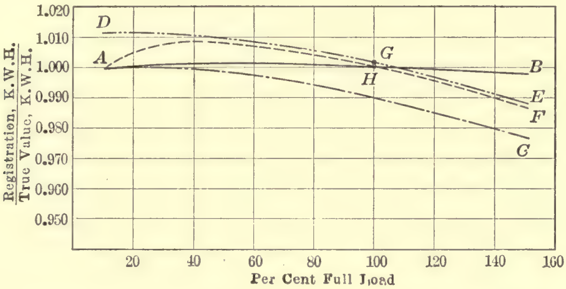


FIG. 268.—Pertaining to self-heating error due to current coils of direct-current watt-hour meter.

In detail, if there were no errors due to the heat from the current coils the per cent-registration curve, if the meter were adjusted to register correctly at full-load and at light load, would be that derived from equation (4), all the coefficients being constant. If the adjustments were made at 10 per cent load and full-load, the curve would be *AHB*.

The effect of the heating is to increase *R*. This causes the upper parts of the curve to droop, the result being the curve *AC*. By moving the drag magnets the curve *AC* may be shifted bodily so that it takes the position *DGE* and by means of the light-load coil the registration at some low load (10 per cent load) may be made correct. The percent-registration curve is then *AHF*. The registration is correct at 10 per cent load and at full-load. These self-heating errors are important in portable rotating standard watt-hour meters such as are referred to on page 495.

The meter should be unaffected by local magnetic fields.

Trouble may be experienced from improper wiring, leads carrying large currents being placed too near the meter. This would be likely to occur in heavy direct-current switchboard work, for in this case the meter coils consist of only a few turns and the busbars at the back of the switchboard may be very near the meter. To obviate this trouble a special astatic wattmeter has been designed. It is shown diagrammatically in Fig. 268*A*.

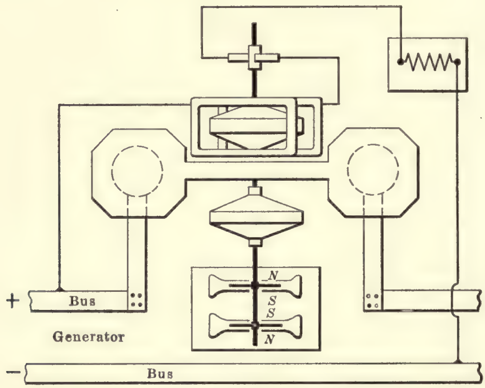


FIG. 268*A*.—Diagram for astatic watt-hour meter.

The spindle carries two equal armatures, one operating in the field above and the other in the field below a straight conductor. In consequence of this arrangement, variations of the local field, which affect its strength equally at the upper and lower armatures, have no effect on the registration. The drag magnets are so placed that if the strength of one is increased by the extraneous field, that of the other is diminished. The whole retarding device is enclosed in an iron shield.

Any watt-hour meter should maintain its accuracy under varying conditions of voltage and load. In general, in the neighborhood of the station, the voltage on a system of electrical supply will remain nearly constant, especially if the system be large; but at a distance, owing to insufficient copper in the conductors, the voltage variations may be considerable and the accuracy of the meter should not be affected by them. Because of heating, the resistance of the potential circuit of the meter is dependent on the line voltage. Consequently, a change in line

voltage does not produce a proportionate change in the speed of the armature. At moderate and full loads an increase of voltage will tend to make the meter register too little. At light loads, where the light-load coil F' furnishes a considerable part of the driving torque, an increase of voltage tends to make the meter register too high, for the torque of the light-load coil, which increases as the square of the current in the armature circuit, may more than counterbalance the tendency of the meter to register too low due to the increase in the resistance of the potential circuit. Practically, the effect of voltage variations will depend on the load on the meter, for the position of the drag magnets

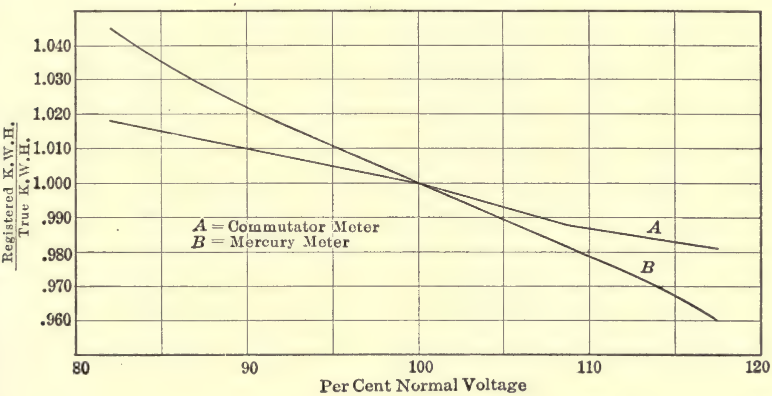


FIG. 269.—Showing effect of voltage variation on the registration of direct-current watt-hour meters at full-load current.

and of the light-load coil is adjusted at some standard voltage. For direct-current meters, at full-load current, a variation of from 10 per cent above to 10 per cent below the normal voltage should not affect the accuracy by more than 3 per cent and at 10 per cent of the rated full-load current, the effect should not be more than 5 per cent.

Accuracy at light loads is of great importance, for it is seldom that the meter in an installation is worked at full capacity. Indeed, for a great portion of the time the load on the meter may be but a small fraction of its rated capacity, and under these conditions it is essential that the energy be measured as accurately as possible.

The meter should rotate continuously with 2 per cent of

rated full-load current, and at 10 per cent of rated full-load current should register correctly to within 3 per cent. The light-load adjustment must be such that the meter does not "creep," that is, rotate continuously when the consumer is not using energy. This should be true even when the supply voltage is 10 per cent higher than that at which the meter was adjusted. Electrical supply companies which give careful attention to their meters, instruct testers to leave them so adjusted that they register correctly to within 1 per cent at from 5 to 10 per cent of full-load and to within 1 per cent at full-load.

The periodic service tests on a large distribution system where careful attention is given to the upkeep of the meters may be expected to show results comparable with the following:

Commutating meters	Light load, 5 to 10 per cent. of full-load	Full-load
Per cent of total number of meters which register between 98 and 102 per cent of the correct value	60.0	90.5
Per cent of total number of meters which register between 95 and 105 per cent of the correct value	91.9	98.3
Per cent of total number of meters which register between 90 and 110 per cent of the correct value	98.3	98.9
Induction meters		
Per cent of total number of meters which register between 98 and 102 per cent of the correct value	84.0	93.8
Per cent of total number of meters which register between 95 and 105 per cent of the correct value	97.7	98.6
Per cent of total number of meters which register between 90 and 110 per cent of the correct value	98.6	98.8

The tests were made at intervals of from six months to a year.

The aim of careful supply companies is to so maintain their meters that a full-load accuracy of 98 per cent or better is obtained.

The energy losses in the meter must be small, for the potential coil of the instrument is in circuit continuously, even though the consumer is using no energy. The expense of energizing the potential coils falls on the supply company; in the aggregate it may be a considerable item.

When the consumer uses energy the voltage at the load is diminished by the *IR* drop in the current coils of the meter. This of course varies with the load. For a small direct-current meter (5 amp.) the drop in the field coils may be about 1 per cent of the line voltage when the rated full-load current is used. The expense of energizing the field coils falls on the consumer.

Use of Watt-hour Meters on Three-wire Circuits.—For metering on low-tension, three-wire, direct-current circuits such as are used in congested districts in cities, two ordinary two-wire meters may be used, one with the current coils in the positive lead, the potential circuit being connected between this lead and the neutral wire, while the other is similarly connected on the negative

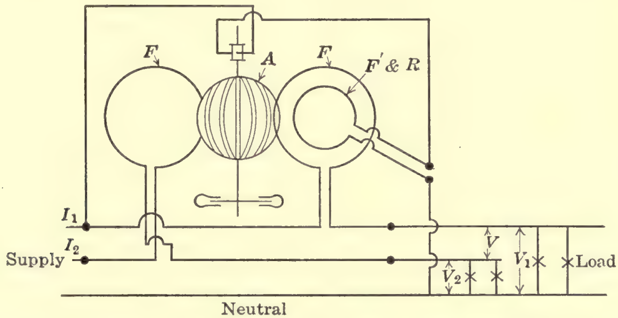


FIG. 270.—Diagram for three-wire direct-current watt-hour meter.

side of the circuit. Such an arrangement is free from errors due to the unbalancing of the voltage of the circuit and to unequal currents in the positive and negative leads; it is, therefore, a desirable arrangement when the circumstances are such that these effects may become very large.

Ordinarily, a single three-wire meter is employed. In this instrument, one of the current coils is in the positive, while the other is in the negative lead.

When traversed by equal currents, the two current coils should have equal effects on the armature. The potential circuit may be connected between the positive or the negative lead and the neutral or between the positive and negative leads. Fig. 270 shows diagrammatically a three-wire meter with the first connection.

Theoretically, the three-wire meter is subject to certain errors; for instance, with the connection shown in Fig. 270 the potential lead being on main No. 1, an error will occur if the voltages are unbalanced, for the power given to the circuit is

$$P = V_1I_1 + V_2I_2.$$

The angular velocity of the armature is supposed to be proportional to this quantity, while in reality it is proportional to

$$V_1(I_1 + I_2).$$

Therefore, with a steady load, the correction which must be added to the reading, reduced to watts, to obtain the true power, is

$$C = I_2(V_2 - V_1).$$

This correction will be positive or negative depending on whether V_2 or V_1 is the larger. With the potential lead connected to main No. 2,

$$C = I_1(V_1 - V_2).$$

If the potential circuit is connected between the positive and negative mains, the angular velocity of the disc will be proportional to

$$\tfrac{1}{2}V(I_1 + I_2)$$

where V is the potential difference between the positive and negative mains. The correction in watts which must be added to the reading to obtain the true power, will be the difference between P and this quantity, or

$$C = \frac{(I_2 - I_1)(V_2 - V_1)}{2}.$$

The error in the registration will be zero if the currents in the current coils are equal, even though the voltages are unbalanced. It will also be zero if the voltages are balanced, even though the currents flowing in the two current coils are unequal. If the leads to the meter and load be of considerable resistance and the voltages are balanced before any current is drawn, the meter will always read too high when unequal currents are taken by the two sides of the load; for if I_1 is greater than I_2 the quantity $(V_2 - V_1)$ is positive and the correction negative. If I_1 is less than I_2, $(V_2 - V_1)$ is negative and again the correction is negative.

If alternating currents are used and the loads are reactive, these relations are still further complicated by the phase displacements of the currents. The corresponding corrections are readily deduced.

Practically it is impossible to make allowance for these errors, for the loads on the two sides of the installation are continually shifting; the best that can be done is to see that the load distribution is such that the two sides are well balanced.

The Use of Commutating Watt-hour Meters on Alternating-current Circuits.—Before the introduction of induction watt-hour meters, it was customary to employ commutating meters on reactive circuits. In this case the reactance of the potential-coil circuit introduces an error, for the potential-coil current in it is not in time phase with the voltage applied to the load and the mean product of the currents in the fixed and movable elements of the watt-hour meter will not be proportional to the power delivered to the circuit. The error so arising will be insignificant when the instrument is used on a non-inductive load, but when the power factor is low, the error may become of importance. A general discussion of this phase-angle error will be found in the section on the "Electrodynamometer Wattmeter," page 309.

Lag Coil.—It is necessary to adjust the phase difference between the current in the armature and the current in the field coil so that this phase difference is the same as that between the voltage applied to and the current in the load, and to do this without greatly altering the current in the field coils. This is accomplished by the use of the "lag coil," which is a non-inductive shunt placed around the field coils of the instrument.

Its action may be made clear by the following:

Let R_P = resistance of potential-coil circuit.
L_P = inductance of potential-coil circuit.
R_C = resistance of current coils.
L_C = inductance of current coils.
R_S = resistance of "lag coil" or shunt around current coils.
I = load current.
I_S = current in lag coil.
I_C = current in current coil.
I_P = current in potential coil.
V = line voltage.

$P.D._S$ = potential difference at terminals of current and lag coils.
θ_P = phase displacement of potential-coil current with respect to V.
θ_C = phase displacement of current in current coils with respect to line current.
ω = 2π times frequency.

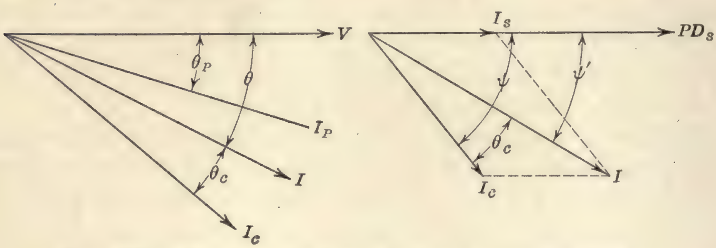


FIG. 271.—Diagrams for lag coil of commutating meter.

On account of the reactance of the field coils, the current I_C in them will lag behind $(PD)_S$ as indicated. The vector sum of I_S and I_C is the total or load current, I. By adjusting the resistance R_S the current in the field coil I_C may be made to lag behind the main current I by an angle equal to the lag of the current in the potential coil behind the applied voltage; in other words, θ_C may be made equal to θ_P. The instrument then becomes practically correct.

Analytically, for the potential circuit,

$$I_P = V\frac{R_P - j\omega L_P}{R_P{}^2 + (\omega L_P)^2}.$$

The lag of this current behind the potential will be

$$\theta_P = \tan^{-1}\frac{\omega L_P}{R_P}.$$

For the shunted current coils

$$I_S = \frac{(PD)_S}{R_S}$$

$$I_C = PD_S\frac{R_C - j\omega L_C}{R_C{}^2 + (\omega L_C)^2}$$

I_C lags behind PD_S by the angle $\Psi = \tan^{-1}\frac{\omega L_C}{R_C}$.

The main current is

$$I = I_S + I_C = PD_S\left[\frac{R_C}{R_C{}^2 + (\omega L_C)^2} + \frac{1}{R_S} - j\frac{\omega L_C}{R_C{}^2 + (\omega L_C)^2}\right]$$

and lags behind $(PD)_S$ by the angle Ψ'

$$\Psi' = \tan^{-1}\frac{R_S L_C \omega}{R_S R_C + R_C^2 + (\omega L_C)^2}$$

$$\tan\theta_C = \tan(\Psi - \Psi') = \frac{L_C\omega}{R_S + R_C}$$

To make $\theta_C = \theta_P$,

$$\frac{L_P}{R_P} = \frac{L_C}{R_S + R_C}.$$

The proper value of R_S is therefore

$$R_S = \left(\frac{L_C}{L_P}\right) R_P - R_C,$$

which is independent of the frequency.

The Induction Watt-hour Meter.—It has been shown above that commutating watt-hour meters may be made to register correctly on circuits of all power factors. Formerly, lagged meters of this class were in common use on alternating-current circuits; they have now been superseded by induction watt-hour meters for the following reasons. The moving element of the induction meter may be made very light and at the same time the torque may be kept high. This reduces the wear on the lower pivot and jewel and lessens the chance of errors due to pivot friction. There is no commutator to become rough through wear and sparking, thus increasing the friction, and there are no brushes to keep in order. The net result is a great decrease in the first cost of the meters and in the cost of maintaining them, a decrease in the current necessary to start the meters and an increase in the accuracy of the registration at light loads (see page 468). This last point is of the utmost importance. The losses in the potential coils are less in the induction than in the commutating meter and as the loss goes on continuously 24 hours a day, this fact is of importance.

Fig. 272 shows in a schematic manner the essential parts of an induction watt-hour meter. The explanation of the creation of an accelerating torque in this instrument is the same as that given for the induction wattmeter, page 448. The accelerating torque is balanced by the retarding torque of a magnetic brake as in direct-current meters.

The terminals T_3 and T_4 are connected to the supply while the load is connected between T_1 and T_2.

PC is the coil of the highly inductive potential circuit; it is connected across the line. Most of the flux through this coil passes down the central core and returns *via BA* and *CE*. Some of it, however, goes to the potential coil lug *PL* and thus magnetizes it. The flux which proceeds outward from *PL* cuts the pivoted disc *D* which forms the movable element of the meter.

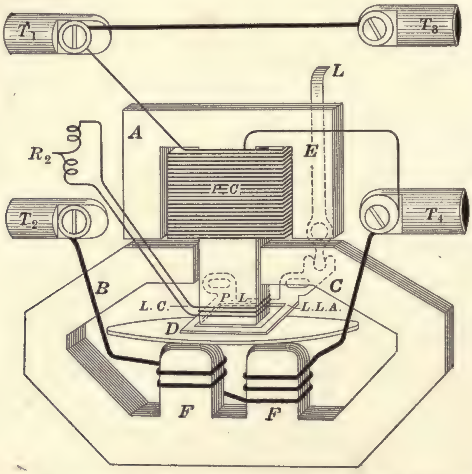


FIG. 272.—Showing electric and magnetic circuits of induction watt-hour meter.

The line current flows through the oppositely wound series coils *FF*. On the passage of currents in all the coils, the flux from *PL* induces currents in the disc which are acted upon by the flux due to *F*, and the flux from *FF* induces currents in the disc which are acted on by the flux due to *PL*. With sinusoidal currents a driving torque is thus generated whose value is

$$\mathrm{T} = K'VI \sin(\beta_2 - \beta_1) - k_1 \left[\frac{\Phi_1{}^2}{2} + \frac{\Phi_2{}^2}{2}\right]\omega'. \quad \text{See page 448.}$$

The retarding torque is due to the movement of the disc through the air gaps of the drag magnets, which in this meter are placed diametrically opposite *PL*.

For steady motion the driving torque must equal the retarding

torque of the brake, or $K_D\omega'$, where ω' is the angular velocity of the disc. The term

$$k_1\left[\frac{\Phi_1^2}{2} + \frac{\Phi_2^2}{2}\right]\omega'$$

represents the drag due to the motion of the disc through the alternating fields in the air gaps. It will vary with the load on the meter, and be a source of error, but by correct design this error may be made negligibly small, and the angular velocity of the movable element becomes

$$\omega' = KVI \sin (\beta_2 - \beta_1).$$

But the potential circuit is highly inductive, so (see Fig. 262)

$$\omega' = KVI \cos (\theta + \alpha) \tag{6}$$

where θ is the power factor angle of the load and α is the departure from exact time quadrature of the useful potential-coil flux and the potential applied to the load (see page 453).

If the meter is properly lagged, that is, if $\alpha = 0$, the angular velocity of the disc is proportional to the power and the total number of revolutions executed in any time interval is proportional to the kilowatt-hours of energy supplied during that time.

The Lag Adjustment.—Exact time quadrature of the useful potential-coil flux and the voltage applied to the load is obtained by the use of a lag coil, LC, which is wound about the potential-pole tip. The circuit of this coil is completed by the resistance R_2 which is adjusted until the desired phase relation ($\Delta = 90°$) is established. A simplified diagram of the potential circuit is shown in Fig. 272*A*. It will be seen that the arrangement is equivalent to a transformer with large leakage reactances. For simplicity a 1 : 1 ratio is assumed. The disc, which serves as the rotor, projects into the air gap, a_2, and is cut by the total flux in the gap, Φ_2. The mutual flux which cuts the primary and secondary is Φ_M and the primary and secondary leakage fluxes are Φ_{L1} and Φ_{L2}, respectively, Φ_{L2} being the leakage flux which cuts through the disc. The total flux through the primary is Φ_1. When the secondary is open the angle Δ is less than 90° due to the IR drop in the potential coil. When the secondary

is closed a magnetomotive force proportional to and in phase with I_2 is introduced. The leakage flux Φ_{L2} will be in time phase with and substantially proportional to I_2; likewise the leakage flux Φ_{L1} will be in time phase and substantially proportional to I_1. The mutual flux Φ_M is proportional to and in phase with I_o, I_o being the vector sum of I_1 and I_2 as in any transformer. Φ_1 is the vector sum of Φ_M and Φ_{L1}, and Φ_2 is the vector sum of Φ_M and Φ_{L2}. $-E_1$, the induced voltage in the primary, will obviously be 90° ahead of Φ_1. By the proper adjustment of I_2, Φ_{L2} may be

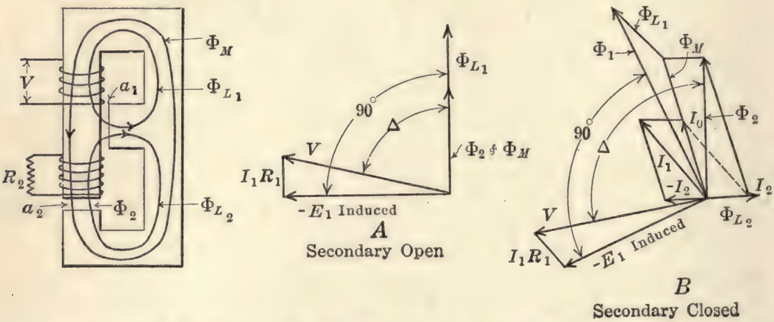


FIG. 272A.—Diagram for lag adjustment of induction watt-hour meter.

made of such magnitude that Φ_2, the total flux cutting the disc, is swung clockwise so that angle Δ is made 90°. In order to make the adjustment, one must have at command a source of sinusoidal current which has the voltage and frequency for which the meter was designed and from which loads can be taken at unity power factor and some lower power factor, 0.5, or thereabouts. One must also know whether the power factor is due to a lagging or a leading current.

The meter is set up and adjusted by moving the drag magnets so that it registers correctly at unity power factor. If the registration is correct the constant K, calculated by the formula

$$K = \frac{Pt}{N3{,}600},$$

will agree with that stated by the makers. (P is the power, t is the time in seconds required for N revolutions.) The light-load adjustment is now made.

Keeping these adjustments the same, the meter is then tested at the lower power factor. If it is correctly lagged, the values of K from the two tests will be the same. Suppose, however, that the constant given by the second test is greater than that obtained at unity power factor. This shows that the meter runs too slow at low power factors. Therefore, if the current lags, see equation (6),

$$\cos(\theta + \alpha) < \cos\theta$$
$$\theta + \alpha > \theta$$

therefore α is a positive angle (see Fig. 262) and the meter is underlagged. This means that the resistance R_2 in Fig. 272 must be decreased. After the change in R_2 has been made the test is repeated, and so on until the two constants agree.

If the current had been leading, θ negative, the result would have been

$$\cos(-\theta + \alpha) < \cos(-\theta)$$
$$-\theta + \alpha > -\theta.$$

Here α must be a negative angle and the meter is overlagged.

In this connection, attention may be called to the fact that the statement that a power factor is 0.5, for example, may give little indication of the conditions under which the meter is operating, for both the P.D. and current waves may be irregular. With a distorted P.D. wave, one may obtain various current waves, depending upon the method of regulation which is used, the power factor always being 0.5. For instance, if inductances be used, the upper harmonics in the current wave will be suppressed to a certain extent. If the change from unity to a low power factor (0.5) is made by using a three-phase circuit, as shown on page 503, the fundamental will be lagged 60°, but the harmonics will not appear in their proper phase relations.

The most exacting test for the lagging is when $\theta = 90°$, for in that case, the meter will register unless $\alpha = 0$. It is difficult to adjust θ to exactly 90°. A natural method is to take the voltage and current from the two phases of a two-phase circuit, but the two e.m.f.'s may not be exactly 90° apart and the regulating devices together with the current coils of the instruments may shift the phase of the current slightly.

Correct lagging is especially important when induction meters

are used for measuring energy supplied for industrial purposes. Induction motors which are commonly used may be only partially loaded and therefore operating at low power factors. This is the condition at which it is most necessary to keep the potential and current fluxes of the meter in the proper phase relation.

Light-load Adjustment.—The principle underlying the devices used for the light-load adjustment is that of the shaded pole motor. In this type of motor the flux from the stator is split into two portions which are displaced in time phase. Consequently, the forces acting upon the movable element are unbalanced and a tendency towards rotation results. Referring to Fig. 272, which shows the electric and magnetic circuits of one type of watt-hour meter made by the General Electric Co., *PL* is the potential lug. Immediately below it is a stamping *LLA* which forms a short-circuited coil of a single turn; it is made of sheet metal of the appropriate resistivity, and so mounted that it can be displaced in its own plane either to the right or to the left by moving the lever *L*.

Suppose that the potential coil is energized, that there is no load on the meter and that *LLA* is placed symmetrically with respect to the potential pole. Currents will be induced in *LLA* which will cause a back magnetomotive force, but as *LLA* is symmetrically placed with respect to the pole tip, the flux cutting the disc will be symmetrical with respect to the pole and all in the same time phase. Consequently there will be no tendency for the disc to turn. Now suppose the loop to be displaced toward the left—the part of the pole covered by it will be "shaded," that is, owing to the induced currents in the loop, the flux from that portion of the pole will be decreased and displaced in phase when compared with that from the unshaded portion at the right of the loop. Thus the disc is acted on by two sets of fluxes which differ in time phase and there is a travelling field and a tendency to rotation. By giving the loop the proper displacement the friction may be compensated so that the disc will begin to move as soon as a very small load is put on the circuit.

Sources of Error in Induction Watt-hour Meters.—The readings of the induction watt-hour meter are subject to a number of

errors inherent in the construction of the instrument, which are not found in instruments based on the electrodynamometer principle. In the main, these errors are due to incorrect phase relations of the various fluxes and to saturation effects in the iron. They are most troublesome at low power factors and with badly distorted wave forms. The following curves apply to a single meter, in which the errors were much exaggerated. Unless otherwise specified the wave forms contained no irregularities or peaks.

Temperature Errors.—The temperature of the instrument may be altered either through self-heating or change of room tempera-

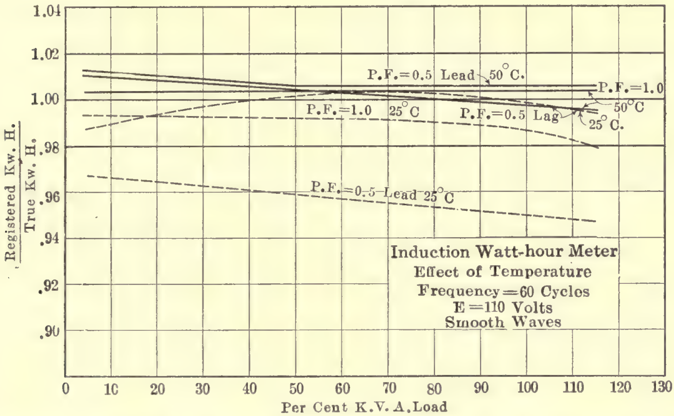


FIG. 273.—Showing effect of temperature on induction watt-hour meter.

ture. If the temperature rises, the resistance of the disc increases so that while the driving torque is decreased, the retarding torque is lessened in about the same proportion, the two effects thus tending toward compensation. There are certain other effects; for instance, the drag magnets decrease in strength and as their effect depends on the square of their strength, a 1 per cent change will change the retarding torque 2 per cent. The resistances of the potential and lag coils change and disturb the lag adjustment; the permeability of the iron and the iron losses are also changed. The net effect is very different in different meters.

The effect may be of importance in the use of portable rotating standard watt-hour meters. With some types of such meters

it may be necessary to insert a thermometer in the instrument in such a manner as to give the mean temperature, and to provide a calibration card which will give the necessary corrections for the ordinary range of atmospheric temperature.

Fig. 273 shows the effect of change of temperature on an induction watt-hour meter of accepted design.

Frequency Errors.—The effect of a departure from the normal frequency may be shown qualitatively as follows.

Suppose that the frequency is doubled, the voltage, current, and power factor of the load remaining fixed. Assuming that the resistance of the potential coil is small, the potential-coil flux will be halved. However, the currents induced in the disc by this

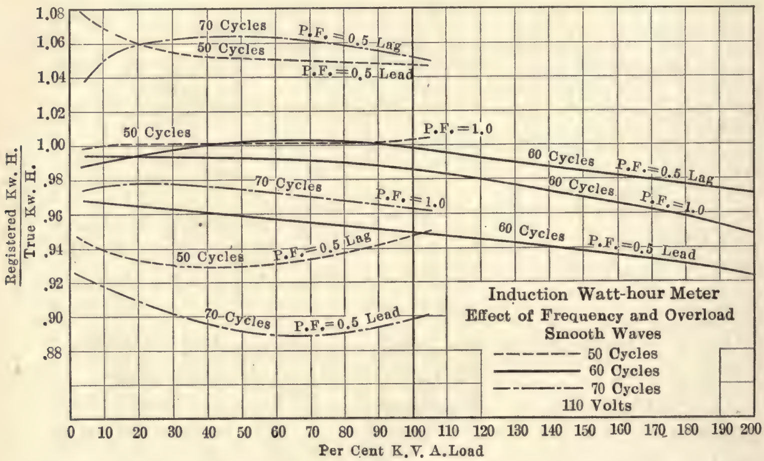


FIG. 274.—Showing effect of frequency on induction watt-hour meter.

flux will remain as before, for, while the flux is only one half as great, it is varying at twice the frequency. The induced currents react with the current-coil flux, which is fixed. The net result is that the alteration in this portion of the accelerating torque is that due to the changed time-phase relation of the fluxes. The currents induced in the disc by the current-coil flux are doubled, for though the value of the flux is not changed it is varying at twice the normal frequency. These doubled currents react with the halved potential-coil flux, so again the effect is that due to the changed time-phase relation of the fluxes.

An increase in frequency will also change the distribution and the lag of the currents in the movable member and increase the impedance of the disc.

In the practical case a change of frequency upsets the lagging, that is, the time-phase relation of the current and potential coil fluxes, which has been adjusted at some standard frequency. This being so, one would expect that the effects of a change of frequency would not be very marked with loads of unity power factor but might be considerable if the power factor were low. Such is found to be the case; see Fig. 274 which also shows the effect of an overload on the registration at normal frequency.

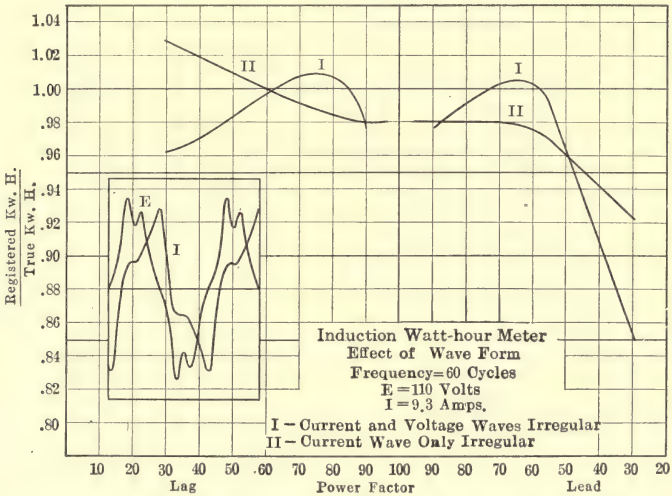


FIG. 275A.—Showing effect of poor wave form on the registration of an induction meter.

Effect of Wave Form.—As the registration is affected by variations in frequency, one naturally expects that changes of wave form will affect the accuracy of the meter, especially at low power factors. This is shown by the curve, Fig. 275A.

The theory of the induction wattmeter given on page 452 rests on the assumption of sinusoidal waves of current, voltage and flux. As the flux wave is the time integral of the voltage wave, the form of the flux wave due to the potential coil will not be the

same as that of the voltage applied at the potential terminals unless the last be sinusoidal. The currents induced in the disc by the current coils depend on the time rate of change of the flux due to the current coils and will differ from the current wave in form unless this form be sinusoidal.

When the circuit conditions are such that the wave form is greatly distorted, this source of error may give rise to inaccuracies in metering, the reasons for which become apparent only when the wave form has been determined by an oscillograph or other wave-tracing device.

In a certain case of this kind where induction meters persistently refused to operate correctly, in spite of the fact that factory tests showed them to be commercially correct, it was found that the e.m.f. wave form was as shown in Fig. 275*B*.

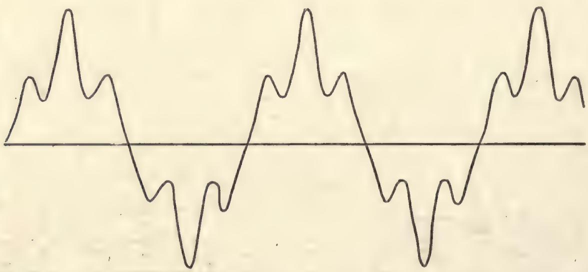


FIG. 275*B*.—PD. wave. When the generator was connected to a transmission line the wave form was so badly distorted that induction meters could not be relied upon.

Effect of Voltage Variation.—Departure of the voltage from its normal value may influence the accuracy of the meter because of change in the resistance of the potential coil winding and of saturation effects in the iron core of the potential coil. For the ordinary range of voltages on a circuit which is nominally operated at a constant potential, these effects will not be large. This is illustrated by Fig. 276.

Polyphase Watt-hour Meters.—For metering in polyphase circuits where the two-wattmeter method is applicable, a special form of induction watt-hour meter has been developed. An example of this is shown in Fig. 277.

The instrument consists of two complete induction watt-hour meters mounted in the same case and with the two discs rigidly

fastened to the same shaft. The driving torque is therefore the sum of the torques due to the two members; that is, it is proportional at any instant to the power in the circuit. The retarding

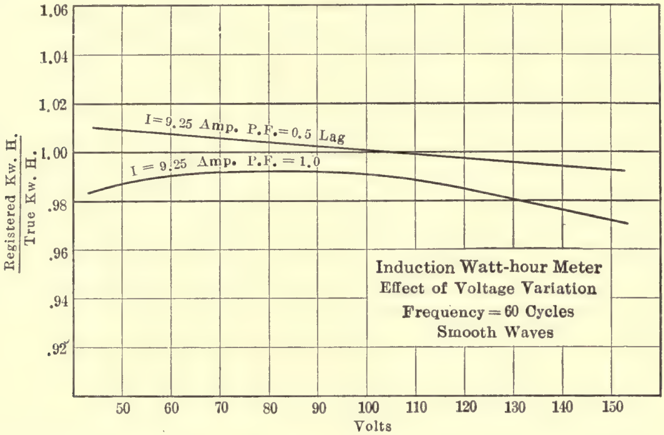


FIG. 276.—Showing effect of voltage variation on induction watt-hour meter.

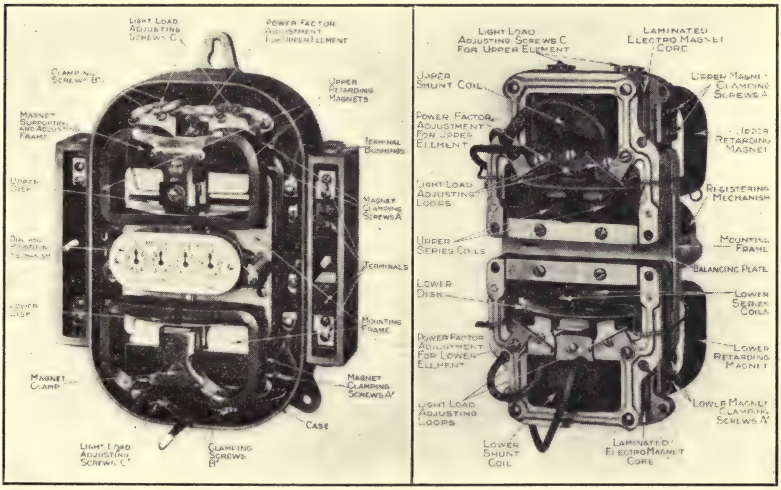


FIG. 277.—Polyphase watt-hour meter, Westinghouse Co.

torque is furnished by two sets of drag magnets, one applied to each disc. Of course, there must be no interference between the two elements.

Each element is complete in itself and must be adjusted so that it registers correctly at both high and low loads, as well as at both unity and low power factors. It is essential, when carrying out these adjustments, that both potential circuits be kept energized, otherwise the moving element will experience an abnormal retarding torque.

Suppose the upper element to be under adjustment; both sets of drag magnets are placed in what seems to be a reasonable position and the adjustment is made as in a single-phase meter. When it is completed, attention is given to the lower element, the constant of which must be varied and made equal to that of the upper element without changing the constant of the latter.

It is not permissible to change the position of the drag magnets; the change in the constant must be effected by altering the driving torque of the lower element. This may be done by varying the fluxes and for this purpose taps are sometimes brought out from the potential coil by which the number of active turns may be altered. In the induction meters now made by the General Electric Co., this adjustment is effected by changing the position of the lower current coils by means of a screw.

A very good check on the equality of the two elements may be obtained by operating the potential circuits in parallel and the current circuits in series and opposed; under these conditions the disc should not rotate.

When induction meters are used on loads which have a rectifying effect, such as three-phase arc furnaces, they must be inserted in the *primary* of the transformer which supplies the load.

MERCURY MOTOR METERS

The principle utilized in the mercury motor meters is that illustrated by the familiar Barlow's wheel, in which a current flows radially in a pivoted copper disc so placed between the poles of a magnet that it is cut by the flux. On the passage of the current the disc is set in rotation.

The advantages of the mercury motor meter for direct current are the elimination of the commutator, as well as the wire-wound armature and the brushes, and the decrease of the wear on the lower pivot and jewel. These things tend to decrease the expense of maintenance.

A practical difficulty has been that, in time, the mercury is very likely to become contaminated and cause an increase in the friction of the meter.

The Mercury Ampere-hour Meter.—Aside from special uses, some of which will be referred to later, ampere-hour meters are intended for use on constant-potential circuits, for if the potential be *kept constant* their readings form as just a basis for that part of the charges which are dependent on the amount of energy furnished as do those of the watt-hour meter. When the ampere-hour meter is so used, the register is graduated to read in kilowatt-hours, the voltage of the circuit having some definite value.

In America the watt-hour meter is now used almost exclusively in lighting installations, but in Great Britain the ampere-hour meter is extensively employed.

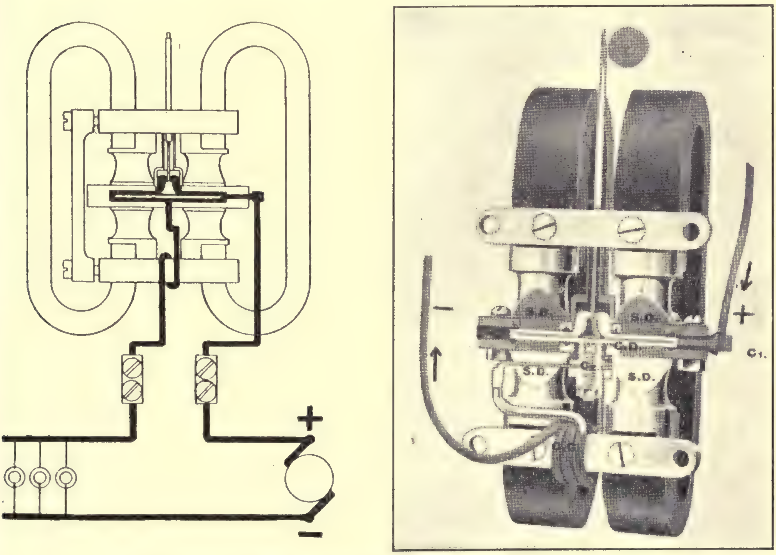


FIG. 278.—Working parts of Ferranti ampere-hour meter.

Ferranti Ampere-hour Meter.—The Ferranti ampere-hour meter was one of the earliest forms of electricity meters, its development having been begun as early as 1883.

Fig. 278 shows the essential parts of this meter as at present constructed. The motor is of the Faraday disc type.

The current enters at the + terminal C_1, flows through the mercury to the amalgamated edge of the copper disc CD, then through the disc to its central portion, which is amalgamated, and out by the terminal C_2. Thus the current in the disc is in the field of the permanent magnets SD and a driving torque is imparted to the disc armature. To protect the copper from the action of the mercury the top and bottom surfaces of the disc are platinum-plated and enameled, except directly above C_2. As the armature moves through the fields of the two magnets SB and SD, there will be the usual braking action due to eddy currents. The fluid friction of the mercury also contributes a retarding action and as this increases with the speed, that is, with the customer's load, the meter is compounded by a coil of a few turns, CC, on the lower iron crossbar. When the current is increased, the strength of the field SD is also increased and hence the driving torque becomes larger. However, the action of the magnetic brake remains the same, for the poles at SD are so arranged that when the field at SD is increased that at SB is diminished.

The buoyancy of the armature is adjusted by a weight on the spindle until the disc just sinks. Friction between the pivot and jewel is thus reduced to a minimum. A sealing device is used so that the mercury will not be spilled during transportation.

Sangamo Meter.—In America the mercury motor meter has been developed by the Sangamo Electric Co. which began the work in 1904.

The main body of the mercury chamber is made of a moulded insulating compound (see Fig. 279). The two current terminals, E_1, E_2, are diametrically opposite each other, and above the lower part of the chamber which contains the copper disc armature is a spirally laminated ring of soft iron (return plate). On the spindle above the disc is a hardwood float; this takes the pressure from the lower bearing, which becomes merely a guide; in fact, a slight thrust is exerted against the bearing plate of the upper ring jewel.

In all Sangamo meters the copper armature discs are now slit radially. The current is thus caused to flow directly from terminal E_1 to E_2 without spreading over the disc. By this means the torque is increased about 40 per cent.

The cover of the mercury chamber is made with a central tube

projecting downward. The clearance of the spindle in the tube is about 0.006 in. and the form of the chamber is such that the mercury cannot be spilled even though the instrument be inverted.

As the current flows diametrically across the disc, the flux must be directed upward on one side of the spindle and downward on the other side. The driving field is furnished by either a permanent or an electro-magnet, according to circumstances; the

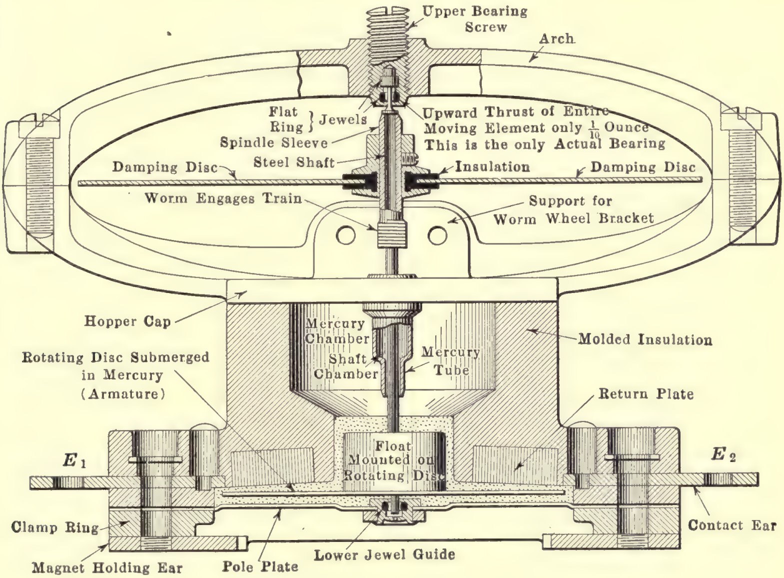


FIG. 279.—Section of working parts of Sangamo meter.

poles are immediately beneath the chamber and contiguous to the current lugs. The magnetic circuit is completed by the spirally laminated soft iron return plate. The necessary "braking" action is due to induced currents in the armature disc and in the usual aluminum damping disc provided for that purpose.

The Sangamo Ampere-hour Meter.—Aside from the ordinary lighting and power installations, there are certain operations, such as electroplating and the charging and discharging of storage batteries, where it is desirable to register the total quantity of electricity rather than the energy. For this purpose, the

Sangamo ampere-hour meter has been developed. In this instrument the driving field is produced by a large permanent magnet.

In electric automobile and truck work, an ampere-hour meter will give an indication of the state of charge of the battery.

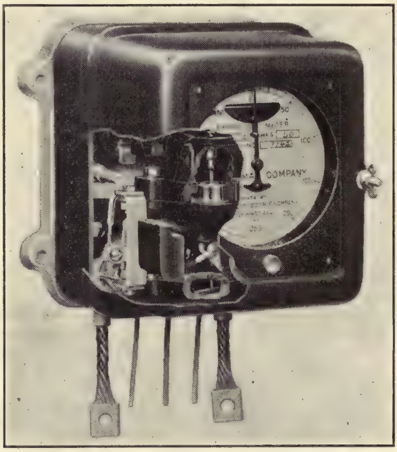


FIG. 280.—Sangamo ampere-hour meter.

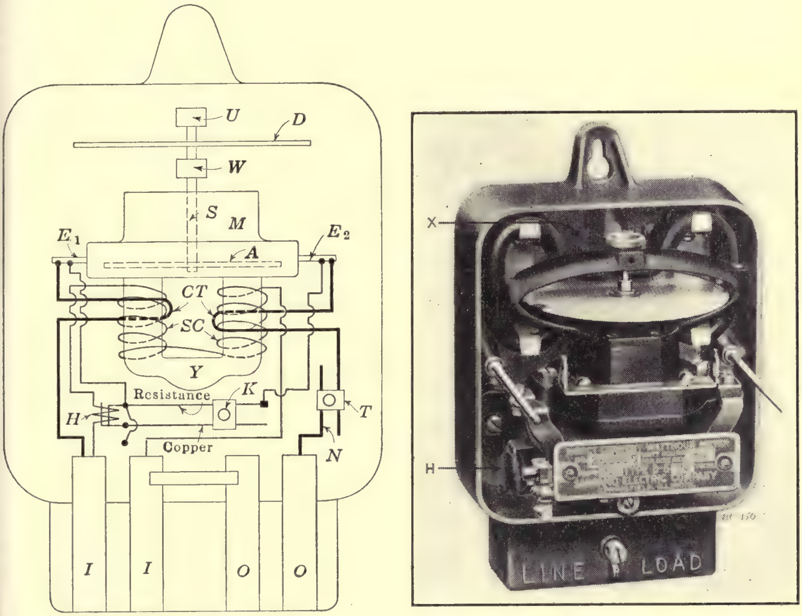
The Mercury Watt-hour Meter.—The electrical connections for the Sangamo direct-current watt-hour meter are shown in Fig. 281*A*. The main-line current passes across the copper armature disc in the direction E_1E_2. The U-shaped electromagnet Y, which furnishes the necessary field, is connected across the line.

The light-load adjustment is obtained by the use of a thermocouple H which is inserted in a shunt circuit between E_1 and E_2, and heated by a resistance coil which forms a part of the potential circuit of the meter. The couple sends a small current through the disc in the same direction as the load current. The effect of the thermo-couple is controlled by altering the position of the connecting link K. The couples are now made reversible, so that the same meter may be used in either the positive or negative side of the line.

As the fluid friction naturally becomes unduly large with increase of armature speed, the instrument is compounded by taking the main circuit around the U-magnet at CT. This improves the action of the instrument at heavy load and at overload.

For high capacities, meters of 10 amp. are used with shunts provided with heavy connecting cables. For obtaining the final adjustment of the multiplying power of the shunt the high-resistance wire *N* and sliding terminal *T* are provided.

The full-load drop through the armature of a 10-ampere meter without a shunt is about 30 millivolts; in the 20 to 80-ampere meters with internal shunts it is about 60 millivolts; for the



A B

FIG. 281.—Sangamo direct-current watt-hour meter.

external shunts the drop is about 75 millivolts. This higher drop is necessitated by the resistance of the connecting cables. The loss in the potential circuit of a 110-volt meter is about 4.5 watts; with 220-volt and 550-volt meters it is about 9 and 22 watts, respectively. In the two latter, the larger part of the loss is in the added wire resistance. The full-load torque is about 6 cm.-gm.

In this meter the drag magnets are fixed in position, and a 25 per cent variation in the braking action may be obtained by the use of a magnetic shunt on the drag magnets. The shunt

is a disc of soft iron mounted on a fine-pitched screw, as shown at X, Fig. 281B. The drag magnets are shielded by the cast-iron frame of the instrument.

The ability of the Sangamo meter to withstand severe mechanical shocks and jars and its freedom from the influence of stray fields, which if they do cut the armature are directed either upward or downward on both sides of the spindle, render it applicable to car tests in street-railway work. For this purpose a special form of register, with a resetting device for registering the consumption of energy during a single trip, has been developed. The register has also the ordinary totalizing dials.

METER TESTING

To maintain the accuracy of the meters in any distribution system, it is necessary that they be tested periodically. On account of the risk of altering the constant of any form of motor-meter during transportation all tests must of course be made on the meters as installed.

In many States, laws have been enacted which permit a customer, in case he is dissatisfied with his bill, to request the services of the appropriate public service commission in order that a test may be made by a disinterested party.

Referring to the fundamental formula for the watt-hour meter, (page 461), for meters as actually constructed, the watt-hours registered = K_h times (number of revolutions of disc). K_h is the watt-hour constant of the meter; its value is usually marked on the meter disc. In some types of meters the constant is expressed in watt-seconds on the dials for each revolution of the disc. In any case the constant K is a fixed ratio depending on the arrangement and ratio of worm, wormwheel, gear train, and the dial units of the watt-hour meter. In any meter tests which are made by timing the disc as it rotates one must be certain that the register used on the meter has the proper constant.

In case of a dispute between the customer and a supply company the meter must be tested as found, that is before any adjustments are attempted. The records of these tests are necessary in order that the customer and the company may arrive at an understanding.

To test a watt-hour meter, it is necessary merely to determine

the rate of revolution of the disc, then multiply this value by the test constant of the meter, and compare the result with the number of watts indicated by standard instruments which are so connected in the circuit as to measure the same amount of power as the watt-hour meter under test. The energy is supposed to be supplied at a constant rate;

$$\text{watts by watt-hour meter} = P' = \frac{K_h N 3{,}600}{t}.$$

For direct-current meters:

$$\text{Correct watts} = P = VI;$$

N = number of revolutions of the disc.

t = time in seconds for N revolutions.

V = corrected average voltage measured at the potential terminals of the meter.

I = corrected average current flowing through the series coils of the watt-hour meter.

If the voltage and current fluctuate badly, VI should be the *average watts* during the test.

If a three-wire meter, with the potential circuit connected between one side of the main circuit and the neutral wire, is calibrated with both current coils connected in series, then the value of K_h to be used in the above formula for P' is one half that marked on the disc.

The different manufacturers of meters use various modifications of the fundamental formula, and one should be sure that the test constant given by the maker is used in the proper manner.

To illustrate, for all meters made by the General Electric Co., watt-hour constant = test constant.

$$K_h = K_t \qquad\qquad P' = \frac{K_t N 3{,}600}{t}.$$

For Fort Wayne meters, type K, watt-hour constant = $\dfrac{\text{test constant}}{36}$

$$K_h = \frac{K_t}{36} \qquad\qquad P' = \frac{K_t N 100}{t}.$$

For the Fort Wayne meters, types K_1, K_2, K_3, K_4, watt-hour constant = test constant

$$K_h = K_t \qquad\qquad P' = \frac{K_t N 3{,}600}{t}.$$

For Sangamo meters, watt-hour constant $= \dfrac{\text{test constant}}{3{,}600}$

$$K_h = \frac{K_t}{3{,}600} \qquad P' = \frac{K_t N}{t}$$

For Duncan meters, watt-hour constant = test constant

$$K_h = K_t \qquad P' = \frac{K_t N 3{,}600}{t}.$$

For Westinghouse meters, watt-hour constant $= \dfrac{\text{test constant}}{3{,}600}$

$$K_h = \frac{K_t}{3{,}600} \qquad P' = \frac{K_t N}{t}$$

except for type $CW-6$, for which watt-hour constant = test constant

$$K_h = K_t \qquad P' = \frac{K_t N 3{,}600}{t}.$$

The watt-second constant, K_s, is the number of watt-seconds of energy necessary to cause one revolution of the movable element. It is equal to the watt-hour constant multipled by 3,600, the number of seconds in an hour.

The register constant, K_r, is the factor by which the reading of the register must be multiplied in order to ascertain the total amount of electrical energy which has been supplied to the load *via* the meter. For meters of small size, such as are used in the majority of cases, the modern practice is to make this factor unity, for it is likely that the small consumer will fail to understand why the supply company, in making out his bill, should multiply his meter reading by a factor of 2 or 4 for instance. In meters of large size, it is necessary to use register constants of 10, 100 and so on, for otherwise the value in kilowatt-hours of one dial division becomes too large.

The register ratio, R_r, is the number of revolutions of the wheel meshing with the worm or pinion on the shaft of the movable element, which is necessary to cause the first or most rapidly moving dial hand to make one revolution.

The gear ratio, R_g, is the number of revolutions of the movable element required to cause the first dial hand to make one revolution.

Common Sources of Inaccuracy.—If the meter is very slow, or cannot be brought up to speed, the trouble may be due to:

1. Commutator and brushes pitted, oily and dirty.
2. Commutator segments short circuited.
3. Lint or magnetic particles between drag magnet and disc.
4. Disc may not run true, or may be out of position.
5. Pivot worn.
6. Jewel rough or cracked.
7. Dirt in jewel.
8. Undue friction in the worm and the registering train.
9. Upper guide bearing pressed down on shoulder of spindle.

The meter may register too much, due to weakening of the magnets, through ageing or by a short circuit on the customer's premises.

Methods of Testing.—There are several methods of making tests to determine whether the meter is registering correctly or not. They differ in the arrangement employed for ascertaining the true amount of power or energy delivered to the load *via* the meter. The arrangements commonly used for this purpose are:

1. Indicating instruments.
2. A calibrated load box together with a voltmeter.
3. A rotary standard watt-hour meter.

When indicating instruments or a calibrated resistance are employed, the time in seconds required for a whole number of revolutions of the moving element of the watt-hour meter is determined by means of a stop watch. In direct-current work, calibrated ammeters and voltmeters of the moving-coil type are used to determine the true watts. In alternating-current work a calibrated indicating wattmeter is used. If small meters are tested, one must be sure that the results are not complicated by the loss occurring in the voltmeter or in the potential coil of the indicating wattmeter. The watts given by the meter are calculated by the appropriate test formula and compared with the results given by the indicating instruments. "The percentage of accuracy" is given by $\frac{\text{meter watts}}{\text{true watts}} \times 100$. The "rate" of the meter is given by $\frac{\text{meter watts}}{\text{true watts}}$.

In carrying out the test the meter should be timed for as much as 60 sec. if accurate results are desired. This tends to reduce the errors due to the personal equation in timing and counting as well as the errors due to the stop watch.

Great care must be exercised in the purchase and in the maintenance of the stop watches, for they are the weakest element in this method of testing. A watch may keep good time but be inaccurate as a stop watch. It is important that the indicating hand start and stop promptly without jumping and reset to exactly zero. The starting and stopping errors are of great importance. In order that one may be sure that the watch is in good condition, it should be tested at several points before beginning work.

The index hand of a stop watch moves forward by a succession of jumps separated by intervals during which the hand is at rest, so that though the watch beats $\frac{1}{5}$ sec., the hand is in motion only about $\frac{1}{100}$ sec. at each beat, that is, while the escapement is in action. There is thus a possibility of an error of nearly $\frac{1}{5}$ sec. due to the peculiar mechanism of the watch. In timing for 30 sec., this might give rise to an error of about two-thirds of 1 per cent., in addition to all the other errors due to the imperfect mechanical action of the mechanism and the personal equation of the observer.

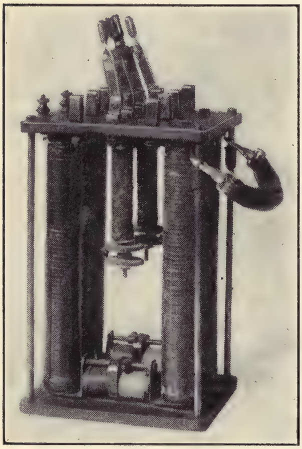


Fig. 283.—Load box for meter testing.

Load Boxes.—A calibrated load box is frequently used in testing meters of small size. Such an arrangement is shown in Fig. 283.

The coils should be wound non-inductively, so that the box is applicable to both direct and alternating-current circuits. The resistance material should have a very low temperature coefficient. The switches should be of such a construction that contact resistances are reduced to a minimum. The box is tested in the laboratory with different combinations of switches and under a series of applied voltages

differing by 0.5 volt. Therefore, when it is used on a test, it is necessary only to observe the applied voltage in order to determine the load on the meter. It is to be noted that a 1 per cent. error in the voltage reading will cause a 2 per cent. error in the watts.

The resistor in the box, shown in Fig. 283, consists of four units, two having a resistance of approximately 220 ohms each, and two having a resistance of about 22 ohms each. The following loads may be obtained:

At 110 volts	At 220 volts
25 watts approximately	100 watts approximately
50 " "	1,000 " "
100 " "	
250 " "	
500 " "	
1,000 " "	

The connecting cables are included in the measurement when the box is calibrated. They should be composed of a large number of fine wires so that they may be very flexible and in order that the effect of a break in any individual wire may be small.

When such a load box is used for routine tests, it is accompanied by a calibration card which gives the watts corresponding to various applied voltages. The advantage of this method of testing in direct-current work is that it is necessary to provide and to read only one instrument.

Portable Standard Watt-hour Meters.—Routine tests are frequently much facilitated by using a standard watt-hour meter instead of an indicating wattmeter and a stop watch. The standard is a portable watt-hour meter with a special register, readable to 0.01 of a revolution, which allows the number of revolutions of the movable element to be read with precision. This register must be so arranged that it may be promptly started and stopped by the use of a push button.

In case such an instrument is used, after having connected its current coils in series and its potential coils in parallel with those of the meter under test, one has only to compare the number of revolutions made by the standard during a certain time with the number made by the meter under test during an equal time, for example, that required for a definite number of revolutions

of the meter under test, and then allow for the meter constants. For example, denote by x the meter under test, and by s, the standard meter. The average powers given by the two meters are P_x and P_s; then

$$P_x = \frac{(K_h)_x N_x 3{,}600}{t} \qquad P_s = \frac{(K_h)_s N_s 3{,}600}{t}.$$

The "percentage of accuracy" is $\frac{P_x}{P_s} 100 = \frac{(K_h)_x N_x 100}{(K_h)_s N_s}$

It would be an obvious convenience if the meters had equal watt-hour constants.

The advantages of this method are the elimination of the use of the stop watch by the tester, independence of load and voltage fluctuations, and the reduction of the working force, for only one man is necessary. Independence of load and voltage variations is a most decided advantage, for at times, especially if high-capacity meters are being tested, it is necessary to use the consumer's load, and this may be fluctuating.

Rotary standards are now made for both alternating and direct currents.

The alternating-current standard is started and stopped by making and breaking the potential circuit. In the direct-current instrument the potential circuit is kept closed so that the armature and disc rotate continuously; the register is thrown into and out of gear by an electrically operated clutch.

It is essential that the construction of rotary standards be such that their accuracy will not be affected by the necessary handling during transportation. This means that the geometry of the coil system and of the brake must not alter, and that the friction must remain constant. To insure this last it is necessary to provide means for raising and clamping the movable system so that the pivots and jewels may not be injured.

To reduce the irregularities due to unavoidable friction, the commutators used in direct-current standards should be of small diameter and the brush pressure constant. A high ratio of torque to weight of the moving element is most desirable.

When the instrument is connected into the circuit, care should be taken that the field and the armature coils are at practically the same potential, especially if the voltage is so high that a multiplier is used.

For the greatest utility, the current range of the rotary standard should be large, so that it may be used for testing meters of a number of different capacities. At the same time, the ampere-turns due to the fixed coils must be large even when small meters are tested. This insures that the standard will never be operated on what is the equivalent of a light load. Therefore, the current coils must be wound in sections so arranged that they can conveniently be connected in various series-parallel combinations by some reliable means. The full-load ampere-turns of all the sections should be the same.

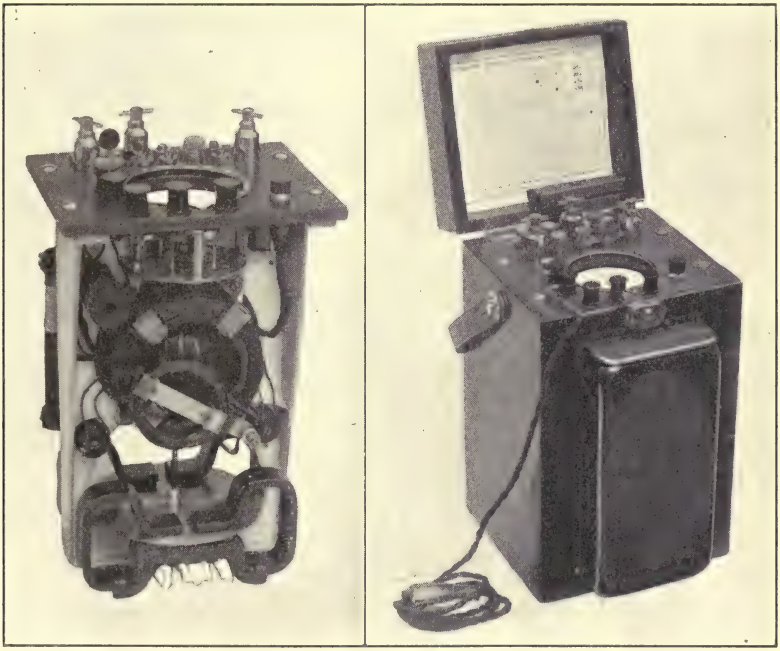


FIG. 284.—Rotating standard watt-hour meter for direct currents. General Electric Co.

Large electrical companies now use rotary standards, which are accurately maintained by their laboratory departments, as secondary standards when checking and adjusting service meters before they are sent out for installation on the consumer's premises.

While the rotary standard is very convenient and in some

cases necessary, one must not forget that great care must be taken if accurate results are to be obtained.

The direct-current instrument is heavier and less convenient than that for alternating current, and is not so commonly used. When the instrument is calibrated, and when it is used, it is

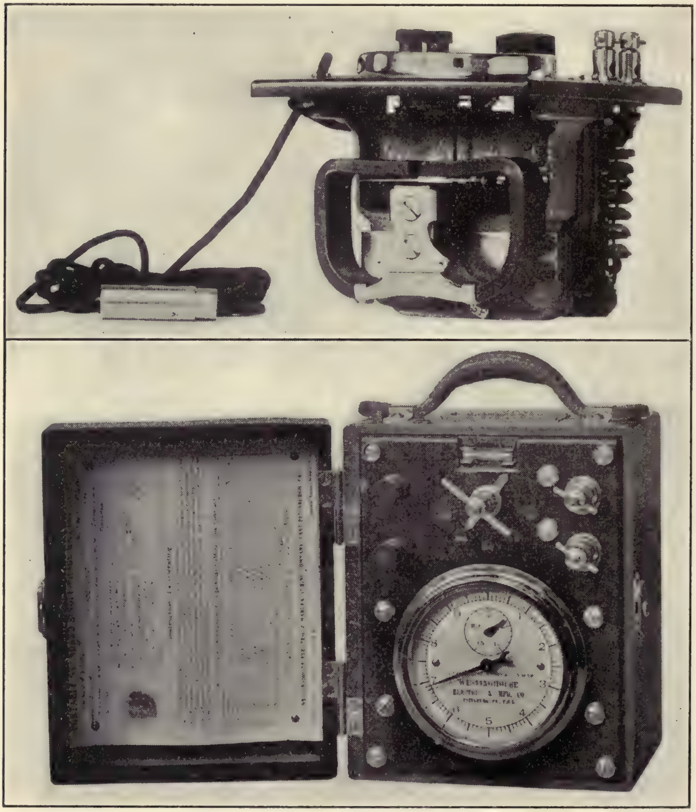


FIG. 285.—Rotating standard watt-hour meter for alternating currents, Westinghouse Co.

necessary to keep the potential-coil circuit of a direct-current rotary standard energized for a considerable time (about 30 min.) before any readings are taken—long enough for the entire armature circuit, the disc, and the drag magnets to attain their permanent states of temperature, since the resistance of the armature circuit, the resistance of the disc to eddy currents and

the strength of the drag magnets are all dependent upon temperature. The heat liberated in the current coils also influences the accuracy of the meter for it, too, affects the temperature of the potential coil, the disc and the drag magnets. This self-heating error may be of importance in careful tests if the meter is so used that it must carry a large current for a long time. The alternating-current standard is subject to the errors found in meters of the induction type (see pages 478, 505). Therefore, in case of a serious dispute between the consumer and the supply company, it is preferable to use indicating instruments if possible.

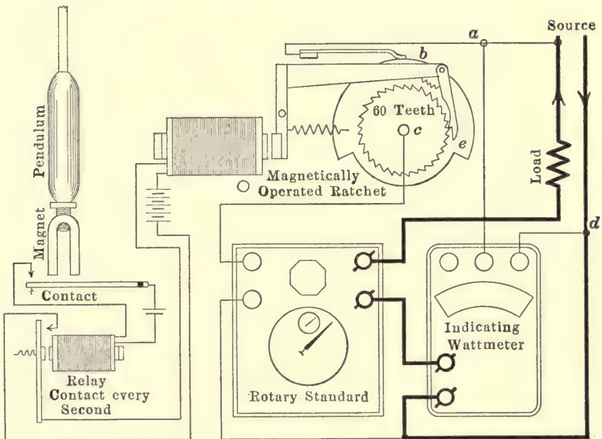


FIG. 286.—Timing device for calibrating watt-hour meters.

The use of rotary standards takes the determination of the time element from the tester, who must of necessity use a stop watch, and hands it over to the laboratory department, where much more accurate timing devices may be maintained.

A timing device designed for use in calibrating rotary standards is shown diagrammatically in Fig. 286.

The master clock which operates the device has a pendulum which beats seconds (½ period). At each beat of the pendulum the relay contact is made and the ratchet wheel is advanced one tooth, carrying with it the contact sector *e*. The duration of the contact of the spring *b* corresponds to 36 teeth on the ratchet wheel, in other words, to 36 sec. The potential circuit,

a to *d*, operates the clutch, if direct-current meters are being tested. With alternating-current meters, as shown in the figure, the contact sector *e* is included in the potential circuit. In either case, the power by the meter is

$$P = 100K_hN.$$

Fictitious Loads and Arrangements for Phase Shifting.—In the laboratory it is often convenient, and sometimes necessary, especially when meters of high capacity are tested, to avoid the consumption of energy which would result from loading the meter in the ordinary way. Also, in service tests after the meter has been installed it is often necessary to test at definite loads

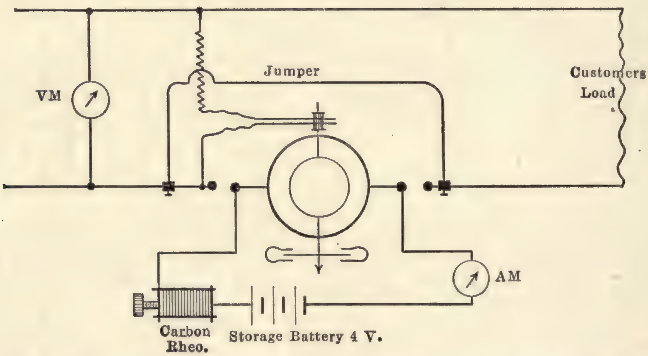


FIG. 287.—Connections for testing a watt-hour meter by use of a fictitious load.

and under constant conditions. This is frequently impossible if the customer's load is relied upon. It is not feasible to use large rheostat loading boxes on account of their expense and inconvenience. In such cases, the potential and current circuits may be separately excited from two sources; the potential circuit from the line as usual and the current circuit from a low-voltage source.

For direct-current work, up to 500 amp., two Edison storage cells and a compact carbon rheostat, as indicated in Fig. 287, are very convenient. By turning back the handle of the rheostat, the circuit is broken when the readings are not being taken. The weight of the cells for testing 500-amp. meters is about 180 lb.

It will be noticed that the customer's load is carried by the jumper, which is put on before the meter is taken out of service, thus avoiding any interruption of the circuit. *In this and other cases where jumpers are used, it is essential that they be so applied that the normal field at the armature of the meter is not disturbed.*

For tests of alternating-current meters after installation, it is possible by the use of a special step-down transformer connected across the mains, to obtain large fictitious loads. This implies that the controlling devices may be made simple and compact and the whole apparatus portable. Such devices are on the market and are sold under the name of phantom load boxes. It is to be remembered that the percentage accuracy of an alternating-current meter depends on the power factor of its load, so it is necessary to be sure that the transformer arrangement does not introduce complications due to phase displacements.

Phase-shifting Devices.—In testing and adjusting alternating-current meters in the laboratory, one must be able to vary the

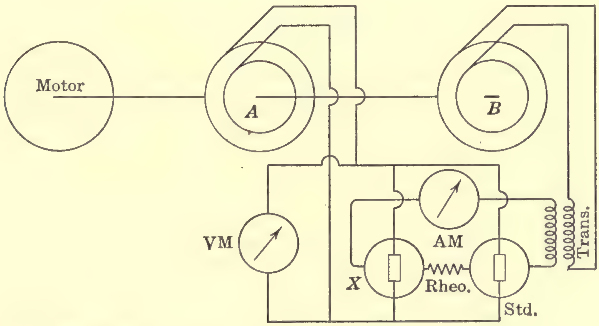


FIG. 288.—Diagram for phase-shifting motor-generator set.

effective power factor of the load, preferably without any attendant alteration in either the current, voltage or wave form. An arrangement for this purpose is shown diagrammatically in Fig. 288.

It consists of two motor-driven machines, the armatures of which are rigidly coupled; one field is stationary while the other is so mounted that it can be displaced about the axis of the shaft by a wormwheel and sector and its angular position read on a graduated arc. The displacement may be effected by a remote-

control arrangement. Machine A energizes the potential coils of the meters while machine B supplies the current coils; B is either of low voltage and large current capacity or else works through a step-down transformer. Both machines should give sinusoidal waves, for the operation of induction meters, especially

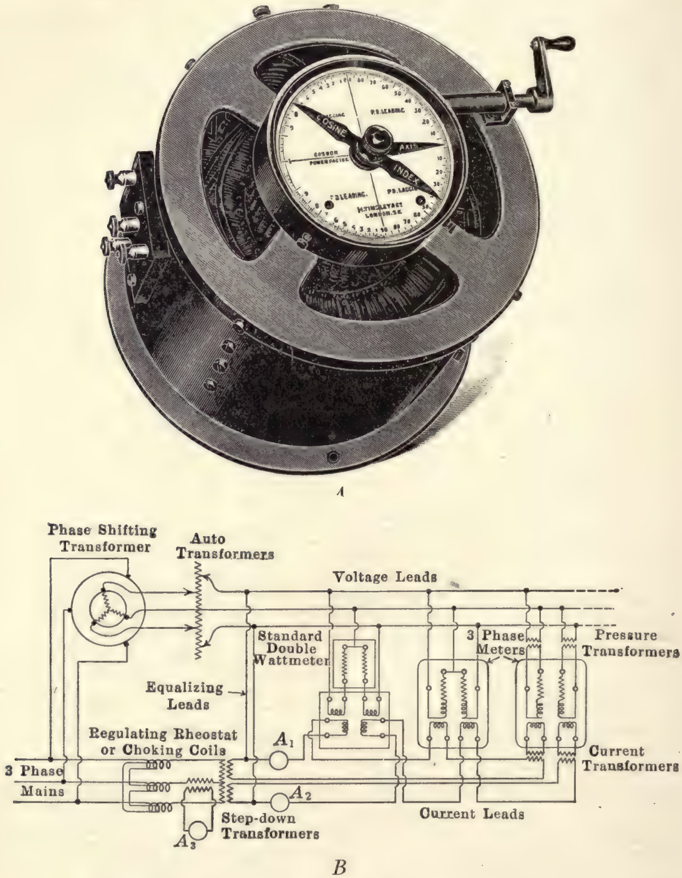


FIG. 289.—Drysdale phase-shifting transformer.

at low power factors, is greatly influenced by wave form. To obtain good wave forms, specially designed three-phase machines with Y-connected armatures are necessary.

A much simpler and less expensive device for accomplishing

the same purpose is the Drysdale phase-shifting transformer, the principle of which is explained on page 290.

This transformer as designed for meter tests, together with the connections necessary in testing three-phase meters, is shown in Fig. 289.

The phase-shifting transformer should be used on circuits which have sinusoidal voltage waves, otherwise the wave form in the secondary will change with the adjustment of the phase displacement.

In order to lag an induction meter it is necessary to operate it at two power factors and usually 1 and 0.5 are chosen. The double-motor generator set or the phase-shifting transformer previously described may be used, but these two particular power factors may be obtained from a three-phase circuit. Fig. 290 shows the connections.

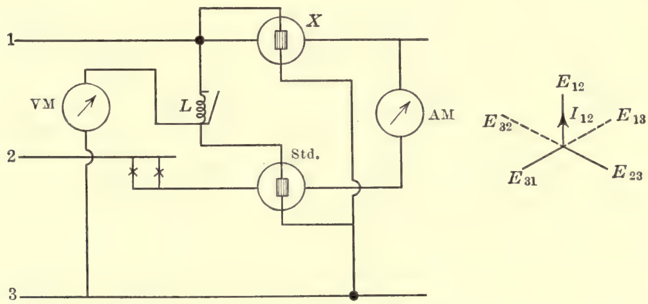


FIG. 290.—Arrangement for obtaining power factor 0.5 from a three-phase circuit.

As shown, the current is in phase with E_{12}, the voltage at the meters in phase with E_{13} and the power factor is 0.5 leading. To obtain 0.5 power factor with lagging current the voltage coils would be connected between leads 3 and 2.

To determine whether one is dealing with a lagging or leading current, a small inductance, L, of low resistance, may be included in the potential circuit of the standard dynamometer wattmeter; *normally this inductance is short-circuited.* If the current is lagging, the insertion of the inductance will slightly increase the apparent power factor and will decrease it when the current is leading.

A power factor of zero may be obtained from a balanced three-phase circuit, as shown in Fig. 291.

The currents I_{12} and I_{13} must be equal and the resistances non-reactive.

A power factor of zero may also be obtained from a two-phase circuit, the voltage being taken from one phase and the current, through a non-reactive resistance, from the other. It must be assured at the beginning that the two phases are really in time quadrature and that the inductances of the current coils do not cause an appreciable phase displacement. Where a two-phase

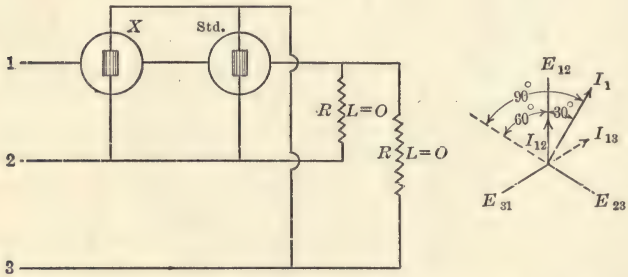


FIG. 291.—Arrangement for obtaining zero power factor from a three-phase circuit.

current is obtained from a three-phase circuit by Scott transformers, unless the wave forms of the primary supply are sinusoidal, the wave forms in the secondaries may be badly distorted, one being flat-topped, the other peaked.

With any of these phase-shifting devices it is important that the voltage and current waves be sinusoidal; for a 60° displacement of the fundamental in the current wave with respect to the fundamental in the voltage wave implies a 180° displacement of the third harmonics, a 300° displacement of the fifth harmonics and so on. A statement that the load has a power factor of 0.5 gives little idea of the conditions under which the watt-hour meter is operating. The changed phase relation greatly complicates the behavior of induction meters.

Testing Polyphase Induction Meters.—When a single-phase induction watt-hour meter is used on a non-inductive load, the error due to incorrect lagging is negligible. If a polyphase induc-

tion watt-hour meter is used on a three-phase load of power factor unity, the error due to incorrect lagging may be appreciable, for in this case, although the power factor of the load is unity, one of the elements of the meter operates at a power factor 0.866 leading while the other element operates at a power factor 0.866 lagging (see page 332).

For other three-phase power factors the conditions under which the elements operate are shown by Fig. 294.

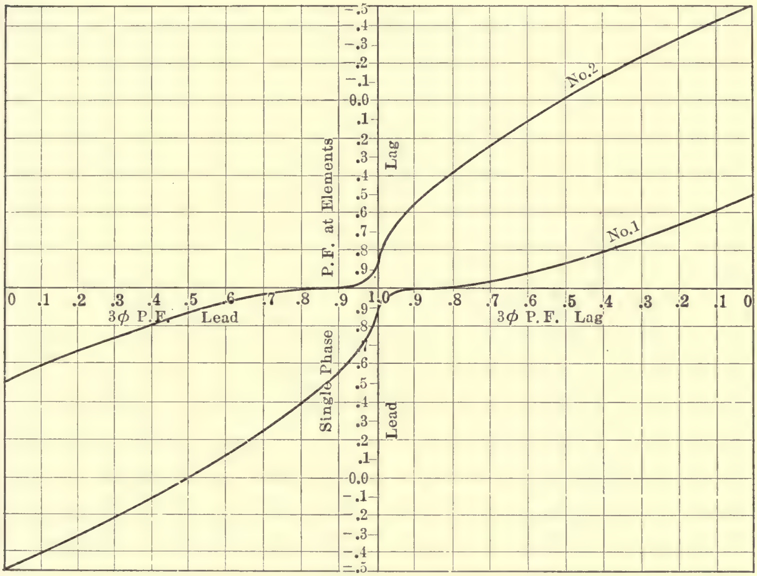


FIG. 294.—Showing the power factors at which the two elements of a polyphase watt-hour meter operate when the balanced three-phase load has different power factors.

Suppose the upper element is underlagged while the lower element is overlagged. Then, when the upper element operates at a lagging power factor and the lower element at a leading power factor, both elements tend to make the meter register too little. If the elements are interchanged, both tend to make the meter register too high.

Polyphase induction watt-hour meters, operated through instrument transformers, are often used in determinations of the

water rates of three-phase turbo-generators, water rheostat loads being employed. The three-phase power factor is then unity. In calibrating the meter, together with the transformers, a three-phase non-inductive load should be used and the connections so made that the element which operated with the lagging power factor during the test is traversed by a lagging current during calibration.

Testing of Large Direct-current Watt-hour Meters on Fluctuating Loads.[15]—On account of the great revenue per meter which may be involved, it is very important for both the supply company and the consumer that the meters by which large amounts of power are sold be kept in an accurate condition. The necessary tests must be made with the meters in place, and if they are used on a rapidly fluctuating load, such as a street-railway system, difficulties are experienced in making the tests and the necessary adjustments.

Owing to the large number of readings of the current which it is necessary to take in order to obtain a good average, the ordinary method of using a stop-watch and of measuring the line voltage and current is a time-consuming operation, and in some cases the fluctuations are so rapid that the use of the ammeter is quite out of the question. An alternative procedure is to take the meter out of service and to send through its coils the current from a storage battery (see page 500). This current may be controlled by resistors, so that tests at light load and up to about 500-amp. may be made without the apparatus being too unwieldy to be managed by two persons. For this purpose two Edison cells are convenient, being readily portable. It is, however, desirable to avoid taking the meter out of service, for the test may occupy an hour or more, and the loss of revenue is worth obviating; it may be as much as $5 to $10 for each hour the meter is out of service. Also it is desirable to make the test with the customer's regular load.

The very convenient portable standard watt-hour meters developed for alternating-current work naturally suggested similar devices for use on direct-current circuits. Their development, however, has been attended with much difficulty. Nevertheless the problem has been solved quite successfully, and the best of these instruments, when carefully used, are of great service where

load conditions are extremely variable. Such test meters are now made in capacities up to 150 amperes.

In railway work, it is frequently necessary to test meters of several thousand amperes capacity. The direct application of shunts to a portable standard watt-hour meter, of the commutating type, is not permissible on account of the change of the multiplying power of the shunt through heating, and the uncertainty due to bad contacts.

As it is desirable to retain this type of meter as a standard, methods have been devised whereby shunts are applied to the portable standard watt-hour meter in such a way that errors due to heating and to contact resistances are eliminated.

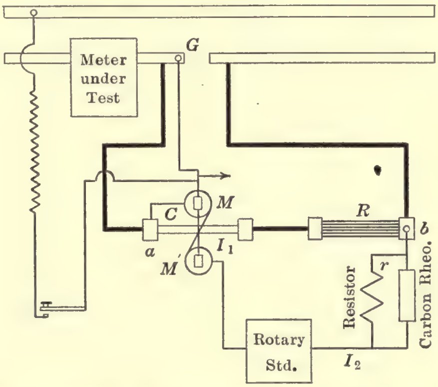


FIG. 295.—Showing connections for testing large direct-current watt-hour meters by the differential multiplier method.

One method is shown in Fig. 295, where for the sake of simplicity the potential connections to the meters are omitted. The arrangement may be called a differential multiplier, for by its use the range of the portable standard watt-hour meter is extended.

The station busbar is arranged so that it has a narrow gap at G. This gap is ordinarily closed by plates firmly bolted in position. The gap should be narrow and the leads so arranged that the field at the meter is not altered when the gap is opened. The test circuit is clamped to the bus bars and the gap opened without interrupting the service. The entire current then flows to a, where it divides, a comparatively small portion flowing through the fine wire coils of the multiplier MM', the rotary standard and the

adjustable resistor r, to b. The main portion flows through the "coarse coil" C of the multiplier, which is in this case a straight bar, then through the resistor R, which is of such a magnitude as to give the voltage drop required in the standard meter circuit. The fields due to the currents in MM' and C are opposed, and in the resultant field is placed an astatic movable-coil system, which is provided with pivots and a damping device. The movable member, in series with a suitable resistor, is placed across the line, and serves to show when the fields due to C and MM' are balanced. The adjustment is made by varying r. The ratio of the currents $\frac{I_1}{I_2} = K$, which is necessary for a balance, is determined in the laboratory, therefore the corrected watt-hours by the test meter are obtained by multiplying its indications by $K + 1$.

The standard meter is set up where it will be as free from stray fields as possible. The leads to it are flexible and readings are

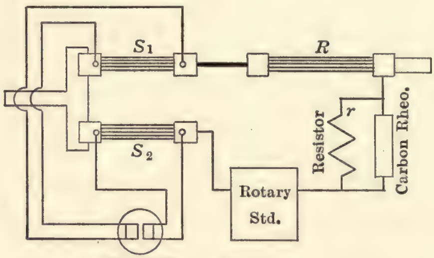


FIG. 296.—Arrangement for testing large direct-current watt-hour meters by use of two shunts and a differential millivoltmeter.

taken with the meter in four different azimuths 90° apart. This is usually sufficient; but conditions may arise where, owing to the change in the distribution of current between feeders which are at different distances from the test meter, this procedure would not eliminate the stray field errors. In such cases, a shielded instrument is desirable. The multiplier being astatic, with the centers of the upper and lower coils 2.5 in. apart, is not affected by uniform stray fields. Anything that produces a non-uniform stray field—for instance, a busbar close to the instrument—might, however, lead to a misinterpretation of the balance. So the apparatus should be set up at some distance from the switchboard. Fig. 296 shows a method where the

balance is not affected by stray fields. Two resistors, one in the circuit of the test meter, and the other in the parallel circuit, are employed. The potential drops in the two are made equal by the adjustable resistor r and this equality is indicated by a differential millivoltmeter of the D'Arsonval pattern. Any shunts which are suited to the purpose may be temporarily bolted together and used for S_1 and S_2. They should be free from thermal errors as these are troublesome in some cases.

The arrangement may be simplified and the differential millivoltmeter replaced by a pivoted D'Arsonval galvanometer if a special double shunt be constructed for the purpose. The connections are shown in Fig. 297. Inspection of the figure will

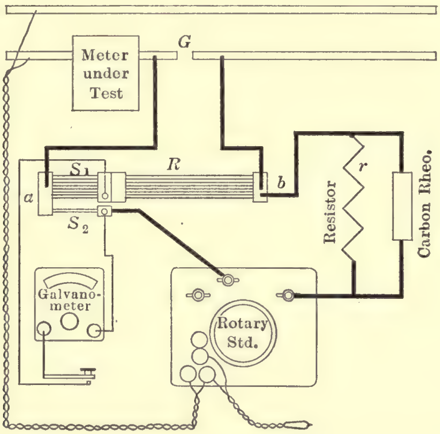


FIG. 297.—Connections for testing large direct-current watt-hour meters by the bridge method.

show that the arrangement has become a Wheatstone bridge, the low-resistance sides being composed of fixed resistors. One of the high-resistance sides is a resistor of fixed value; the other is made up of the rotary standard and the necessary adjustable resistor for maintaining the bridge in balance when contact and coil resistances change. The sections of the shunt (see Fig. 298) S_1 and S_2 have a common terminal at a. If the galvanometer stands at zero, then $\frac{I_1}{I_2} = \frac{S_2}{S_1}$, and the corrected reading of the test meter is its indication multiplied by $\frac{I_1 + I_2}{I_2}$. By the use of

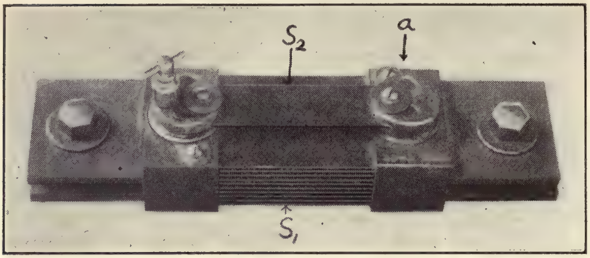


FIG. 298.—Special double shunt for use in bridge method of testing large direct-current watt-hour meters.

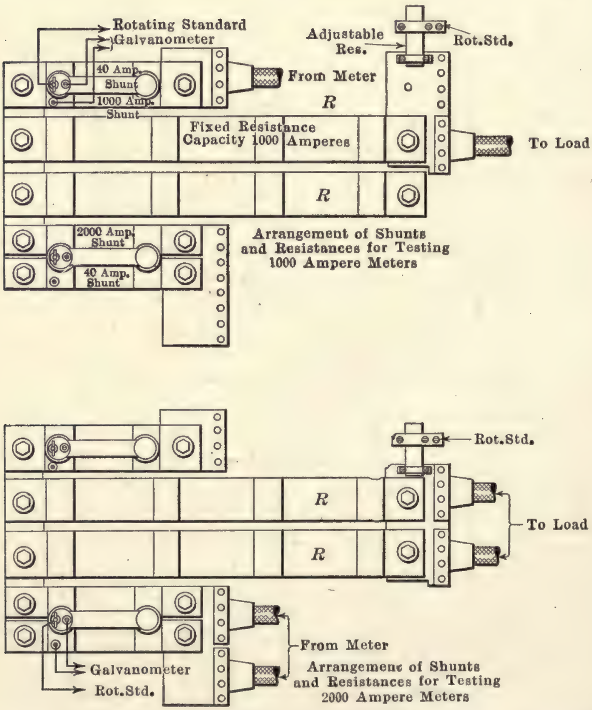


FIG. 299.—Double shunt and fixed resistances for use in bridge method of testing large direct-current watt-hour meters.

two potentiometers to measure $I_1 + I_2$ and I_2 when the galvanometer is balanced, the multiplying factor can be very accurately determined in the laboratory.

In a particular case the capacity of the test meter used is 40 amp. and there are two sets of shunts and auxiliary resistances, R, mounted on the same base, the ratings being 1,000 amp. and 2,000 amp. (see Fig. 299). The voltage drop in the shunts at full load is 100 millivolts, and in the resistor R it is 400 millivolts. The adjustable resistor, r, is a strip of Boker metal, the effective length of which can be altered by the use of screw clamps. To obtain a fine adjustment a carbon compression rheostat is placed in parallel with the strip.

DEMAND INDICATORS

The business of supplying electrical energy is peculiar because, broadly speaking, the product to be sold cannot be stored. It must be used as generated and the supply company must stand ready to furnish its product to customers at any hour.

The demand of the individual consumer for the company's product passes through a fairly well-defined daily and seasonal variation, naturally being the greatest when the days are the shortest. It is necessary to install generating machinery of sufficient capacity to carry safely the greatest aggregate demand, or the peak of the load as it is called, and provide a sufficient reserve. This means that machinery, representing a considerable investment, must stand idle for a larger portion of the time. This peculiarity of the business has led electrical companies to divide the cost of supplying their consumers into two parts, "fixed costs," which are independent of the amount of the product delivered to the consumer, and "running costs," which depend directly upon the amount of energy delivered.

On account of the large amount of time during which a portion of the machinery and the distribution system is idle, the "fixed costs" are large and efforts have been made to establish systems of rates which are in accordance with Hopkinson's maxim that "the charge for a service rendered should bear some relation to the cost of rendering it."

The investment necessary in order that a company may stand ready to supply any group of consumers is dependent on the

maximum demand which the consumers make for the company's product, and in certain systems of charging, maximum-demand indicators are used in conjunction with the watt-hour meters as an aid in apportioning the fixed costs among the consumers.

Demand indicators record the greatest *sustained* amount of current, or power, which the consumer uses. They are not supposed to indicate demands which are of such short duration that no serious burden is placed thereby on the generating machinery. The length of time during which the demand must be sustained in order that the indicator may register depends upon the character of the service; for power work some companies use a ½-hr. period, but a 5-min. period is not uncommon, especially with badly fluctuating loads.

Strictly speaking, to furnish adequate data for use in determining rates, a demand indicator should give not only the demand but the hour at which it occurs; for a consumer who takes a large demand at a time when the generating machinery and distribution system would otherwise be idle necessitates no additional investment and can be given a better rate than a consumer who makes the same demand at the time of peak load. It is only in the case of large consumers that a supply company is justified in installing an expensive form of demand indicator which will show the time at which the maximum demand occurs as well as its magnitude.

With small consumers, instruments such as the Wright or the General Electric Co. M-2 demand meters are used and the allowance for the fact that all the demands do not occur simultaneously is made, by use of the diversity factor, when the rates are originally determined. The diversity factor is defined as the ratio of the sum of the maximum power demands of the subdivisions of any system or part of a system to the maximum demand of the whole system or of the part of the system under consideration, measured at the point of supply.

Diversity factors can be determined only by actual observation of the consumers' maximum demands and the corresponding maximum demand on the station. They will be different for different classes of service.

In consequence of the detailed study of electrical rates now

being carried on, the demand indicator is an instrument of increasing importance and is being rapidly developed.

The Wright Maximum-demand Indicator.[13]—This indicator, which was the first maximum-demand instrument, may be looked upon as a registering differential thermometer, one bulb of which

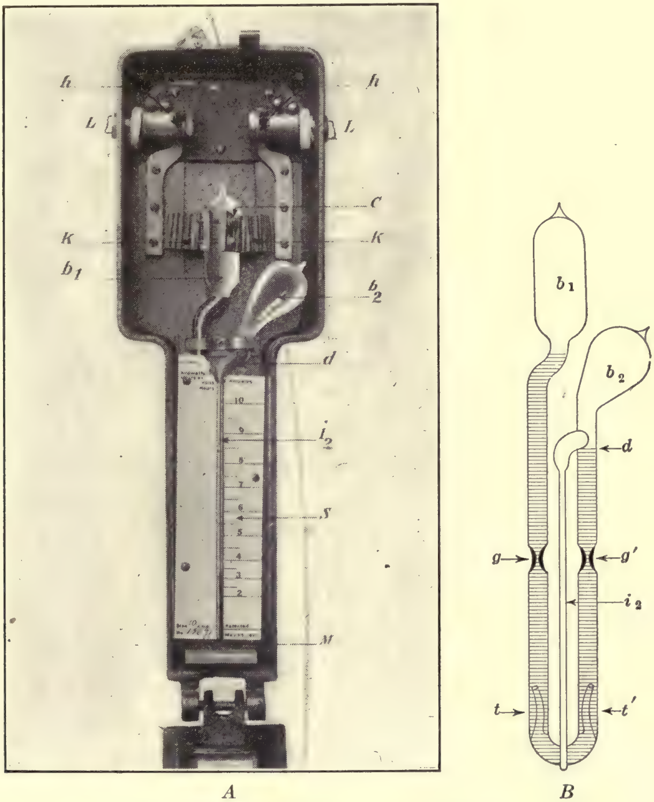


FIG. 300.—Wright demand indicator.

is heated by the passage of the current through a suitable heater coil which closely surrounds it. It registers the maximum *current.*

The internal appearance of the indicator as made in the smaller sizes, up to and including 25 amp., is shown in Fig. 300.

The essential working parts are the indicator tube, with its attached index tube i_2, the scale s, and the heater strips C.

In the small sizes (up to and including 25 amp.) the customer's entire current is taken in through the leads, LL, and to the heater strips *via* the spring hinges, h, and flexible connecting strips, k.

The indicator tube is of glass, annealed so that it will bear handling and not be subject to changes due to stresses in the glass; the two bulbs, b_1 and b_2, which are nearly equal in volume, contain air. The U-tube connecting them contains concentrated sulphuric acid in such an amount and so adjusted in the tube that when the indicator is cold and set ready to begin to operate, the level of the liquid is at d, so that it is just on the point of flowing into the index tube, i_2. Sulphuric acid is used because it "wets" the glass, is very heavy, flows readily, is hygroscopic, and expands comparatively little with rise of temperature. To prevent accidental transfer of air from b_1 to b_2, or *vice versa*, especially when the indicator is set, the tube is constricted to a capillary at g and g' and two traps are provided at t and t'.

The heater strips are of an alloy of high resistivity, which is but little affected by temperature; the strips are of very thin metal and are made to embrace closely the cylindrical glass bulb, b_1. by means of screw clamps. In the small-sized indicators, where it is necessary to carry the heater strips around the bulb a number of times, a non-inductive form is used. The object of this construction is to prevent the turns drawing together when a short-circuit occurs; if this should happen the strips would very likely be burned out or their intimacy of contact with the glass so altered that an error would be introduced. The corrugated copper terminals form somewhat flexible electrical connections to the heater proper. In indicators having a range of 35 amp. and above, shunts are used, the heater strips being of the 15-amp. type. In the shunted instrument, to insure permanency of calibration, it is essential that all the electrical joints be soldered.

The indicator is set by raising the lower end of the tube board, M, on which the above described members are mounted, until it is somewhat above the spring hinges, h, on which it is pivoted. This allows the liquid in the index tube i_2 to drain back into the U-tube; when thoroughly drained and the board is lowered to its normal position, the U-tube is filled with liquid up to d.

If a current is now sent through the heater strips, the air in b_1 is heated and expands, causing the liquid to flow slowly into the index tube i_2 which is in front of the graduated scale; the flow will continue until the permanent state of temperature corresponding to that particular current is reached.

Owing to the heat capacity of the strips, the glass bulb, etc., and the poor thermal conductivity of the glass, the response of the indicator to the increase of current is sluggish; this lag of the reading behind the increase of current is essential to the successful operation of any such device, for it must not take cognizance

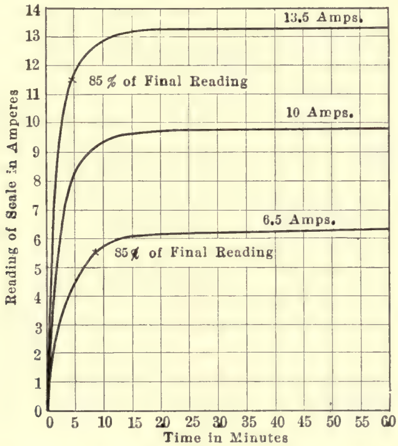


FIG. 301.—Showing characteristics of Wright demand indicator.

of currents which last for only a very short time. The indication desired is that due to the *sustained* maximum. Fig. 301 shows this gradual increase of reading when the current is kept constant. It is intended that approximately 90 per cent. of the full-load registration be accomplished in 4 min. and the entire registration in about 40 min.

It will be noted from Fig. 301 that the rise to a fair approximation to the final reading, say 85 per cent of it, occurs more abruptly when the indicator is worked at about its full capacity than when it is lightly loaded. Fig. 302 illustrates this point; a 50- and a 100-amp. indicator were tested in series at 45 amp. Of

course, each indicator has its characteristic rate of response to the current.

Another point may be noted: after the device has cooled down, owing to shutting off the current, there will be no increase of reading when a current slightly larger than that previously registered, is turned on until the larger current has been maintained for a time longer than the normal time lag of the indicator; for that time must elapse before there has been sufficient expansion of the air in b_1 to cause the liquid again to begin to flow into the

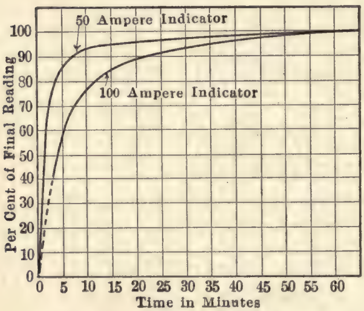


FIG. 302.—Illustrating the influence of the size of a Wright-demand indicator on the rate at which the final reading corresponding to a given current is attained.

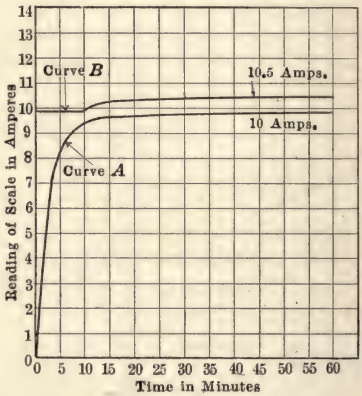


FIG. 303.—Showing the delay of a Wright-demand indicator in beginning registration after the indicator has cooled down.

index tube. This is illustrated in Fig. 303. Curve A shows the normal rise of the indication, when, after the device has been set, the current is maintained at 10 amp. After the current had been cut off and the indicator allowed to cool thoroughly, a run at 10.5 amp. gave curve B, the indicator *not* being reset.

Owing to differences in the tubes, it is impracticable to print the scales, for each must be graduated by experiment to fit the particular tube to which it is applied. It is usual to determine either four or five points by passing measured currents through the indicator for a sufficient time, and marking on the scale the corresponding heights of the liquid in the index tube i_2; the

subdivision is done mechanically. The 20 per cent load mark is the lowest one on the scale.

The readings at the lower end of the scale are considerably influenced by temperature. The effect of temperature becomes smaller as the readings increase. This is illustrated by the following tests: Two indicators, one of 5, the other of 10 amp. capacity, were used. They were placed in a suitable chamber in which were heating coils and a fan to circulate the air. The temperature being originally at 68°F. was raised to 104°F., no current being sent through the indicators. The rise of liquid in the index tube was for the 5-amp. indicator 0.38 in., for the 10-amp. indicator 0.50 in.

The temperature was maintained constant for 4 hr. at 104°F., the fan being kept running; at the end of this time the 20 per cent. load test was begun; the other tests followed as usual, and

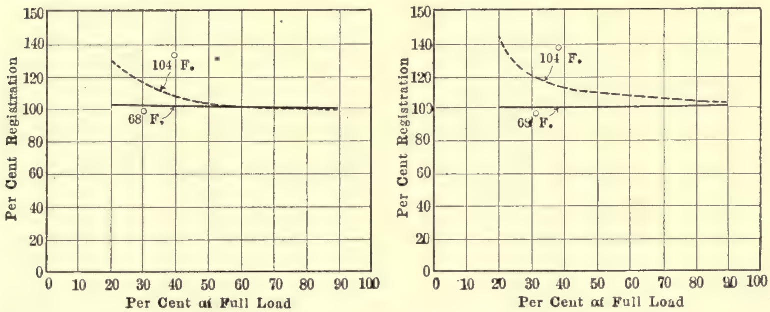


FIG. 304.—Illustrating the effect of temperature on two Wright demand indicators.

the results are shown in Fig. 304. It will be seen that the low readings are very considerably affected, that the percentage error decreases with an increase of load, and that each instrument has its own characteristic behavior.

The leads to the indicator influence its action by conducting heat away from the bulb; the error so introduced will depend on the size of the leads and the difference in temperature between the heater strip and the outside air.

In a two-wire service one indicator is installed on the customer's side of the watt-hour meter. According to American practice it is customary in a three-wire service to install two indicators, one

on each side of the circuit; in this case the average of the two readings is used. Obviously there is no guarantee that the two maxima occur simultaneously. The cheapness of the Wright demand indicator permits its use with the small consumer. In practice, immediately after the reading of the instrument has been taken, it is reset by the reader employed by the supply company. No trace of the indication remains, and no opportunity exists for its subsequent verification in case of a dispute.

General Electric Type W Watt Demand Indicator.—This device is made for use on alternating-current circuits and for polyphase work only; it is essentially a polyphase indicating wattmeter of the induction type, which is provided with an exceedingly strong electromagnetic damping system, so that its response to variations of the load is rendered very slow. The indications are given on a dial which is provided with two pointers, one of which indicates the load (subject to the time lag of the instrument); the other shows the sustained maximum to which the load has risen. Fig. 305 shows, in diagram, the essential features of the instrument. W_1 and W_2 are the two wattmeter elements which are essential to the measurement of power in the ordinary polyphase systems, for, as is usual in such measurements, the "two-wattmeter method" is here employed; D_1 is the disc in which currents are induced by the elements W_1 and W_2; these currents react with the magnetic fields set up by W_1 and W_2 and cause the indication of the instrument. The disc is made of brass in order that the effect of temperature changes may be minimized, since the electrical resistance of alloys like brass varies much less with changes of temperature than does that of pure metals, such as copper.

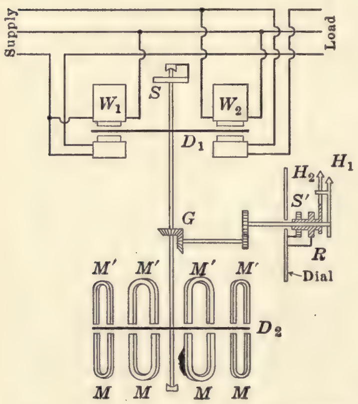


Fig. 305.—Diagram for General Electric type W watt demand indicator.

The controlling spring against which the movable system deflects is at S. In reality three springs are used in series in order

that the movable system may be enabled to make three complete revolutions without complications arising from the spring being twisted too tightly.

The damping disc D_2 is of copper and rotates between two sets of magnets M and M'. Each magnet is adjustable vertically, so that the strengths of the magnetic fields through which the disc moves may be varied. In this manner the strengths of the currents induced in the disc when it turns, and consequently the retardation experienced by it, may be altered and the rapidity with which the instrument responds to changes of load adjusted. It is intended that the magnets be so set that with a constant load, 90 per cent. of the registration is produced in 5 consecutive minutes.

The hand H_1 is driven from the spindle by a system of gearing, and moves over a dial graduated in kilowatts. As it moves

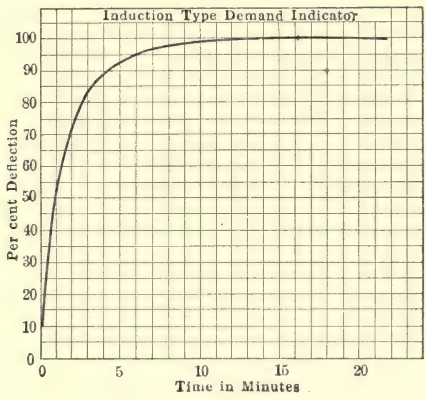


FIG. 306.—Characteristic of General Electric type W demand indicator.

it pushes before it the hand H_2, which is loose on the shaft and provided with a ratchet R and a light spring S', which tends to turn the hand back against the ratchet. The result is that H_2 is pushed up to the maximum by H_1 and left there, when, owing to the decrease of the load, H_1 returns toward zero. The instrument may be set by opening the case and raising the ratchet which allows S' to return the pointer H_2 to zero, H_1 being previously turned back to that point. It will be seen that this device gives the power which is being used at any time, as well as

the maximum demand in kilowatts. It gives no indication of the time when the maximum occurred.

Though the principle on which it works is entirely different, this instrument is purposely designed to have the same general operating characteristics as the Wright demand indicator. This will be seen by reference to Fig. 306, which shows the motion of the pointer from its zero position after turning on the current.

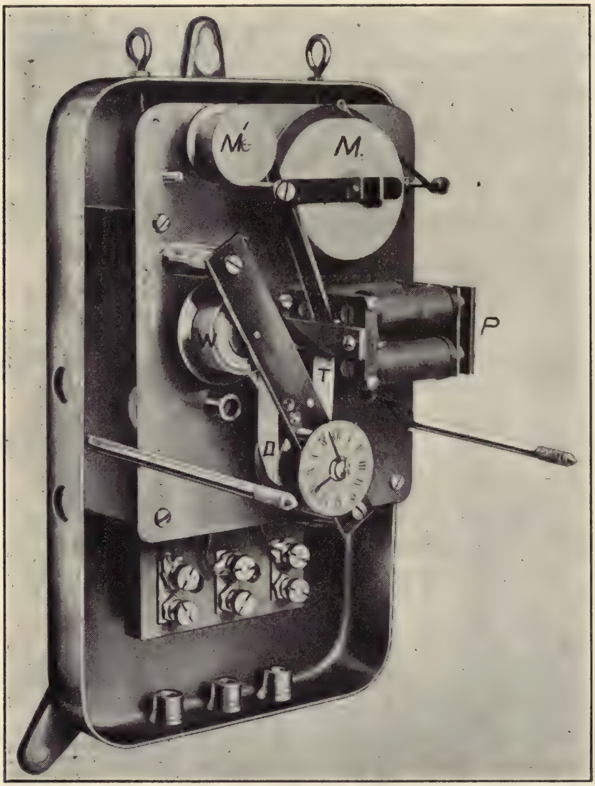


FIG. 307.—Ingalls relay demand indicator.

Ingalls Relay Demand Indicator.[13]—This device may be regarded as an auxiliary to the watt-hour meter by means of which the number of revolutions made by that instrument in a given time (half an hour) may be obtained from a record impressed on a uniformly moving paper tape. From this record, knowing

the disc constant of the watt-hour meter and the gear ratio of the contact arrangement which is described later, the demand may be calculated.

Fig. 307 shows the general appearance of one of these devices. A very powerful, double-spring clock or an electrically driven clock is used to drive the drum D over which the paper tape T is passed; to prevent slipping of the tape the drum is armed with needle points. By the clockwork the tape is drawn from the magazine M and caused to pass in front of the punch P. W is the take-up roll and is actuated by a friction drive from the clock. M' is the magazine roll for the strip of carbon paper used in impressing the record on the tape.

Whenever a current is passed through one of the magnets of the punch P the corresponding armature is drawn in and a mark is made. Once each hour a reference mark is printed on the tape.

To actuate the punch a contact arrangement is added to the counter of the watt-hour meter; it is shown diagrammatically in Fig. 308. The wheel g is driven by the counter and revolves once for each 100 revolutions of the meter disc. The contactor C and the weight W are in one piece, which is loose on the shaft. The pin P is long enough to engage with this piece and then to push it to the dotted position, when it suddenly falls forward. This causes the contactor C to connect b and b' for an instant, thus closing the circuit through the magnets of the punch, which then marks the tape. After 100 revolutions of the meter disc this operation is repeated. The appearance of the record thus obtained is seen in Fig. 309.

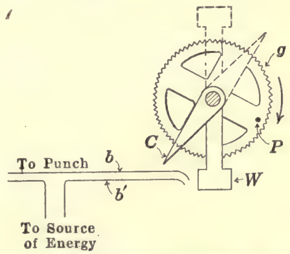


FIG. 308.—Diagram of contacts for Ingalls relay demand indicator.

The tapes are replaced once a week; when this is done the time of the beginning and the ending of the record is recorded on the tape.

To find the maximum demand the tape is examined and those parts where the marks appear to be closest together are selected for measurement. A scale having a length corresponding to the movement of the tape during ½ hour is applied, and the number

of spaces in this length is determined. Each one of the spaces corresponds to 100 revolutions. Thus the maximum number of revolutions made by the watt-hour meter in ½ hour is found. The ordinary formula for the watt-hour meter is

$$\text{Kilowatts} = \frac{N \times K \times 3{,}600}{t \times 1{,}000},$$

where N is the number of revolutions of the meter disc occurring in t sec. and K is the disc constant of the meter. The kilowatts demand corresponding to one space on the paper tape, that is, to 100 revolutions, if they occurred in ½ hour would then be

$$\frac{100 \times K \times 3{,}600}{30 \times 60 \times 1{,}000} = 0.2K.$$

If instead of one space in ½-hour, there be any other number, the above is simply multiplied by the number of spaces. To

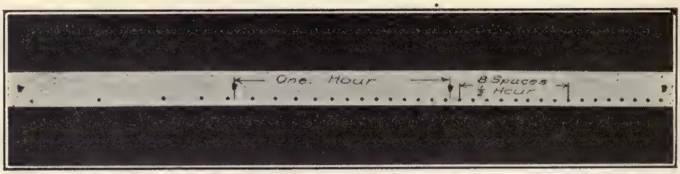


FIG. 309.—Record from Ingalls relay demand indicator.

illustrate: take the tape shown in Fig. 309 and assume that the disc constant, K, of the watt-hour meter is 25. Then the kilowatts demand is given by $0.2 \times 25 \times$ (maximum number of spaces in ½ hour).

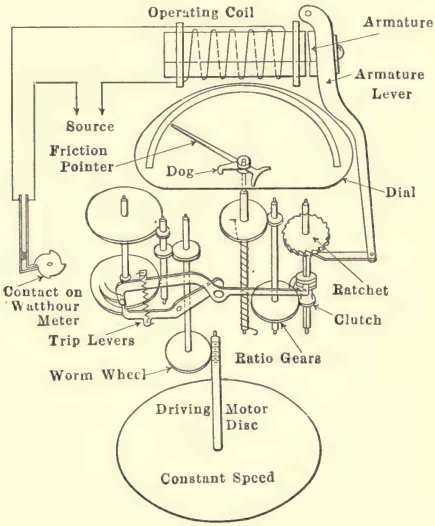
Where the marks are closest together there are eight spaces in ½ hour, so the demand is

$$0.2 \times 25 \times 8.0 = 40 \text{ kw.}$$

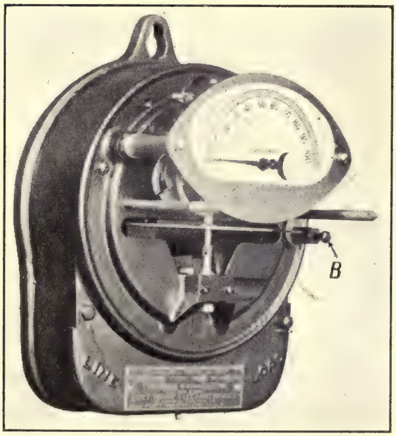
It will be noticed that the device gives information of value other than the maximum demand, for it tells just how the customer's load varies and gives the hour at which the maximum demand is reached. This may or may not be at the time of the peak of the load on the station.

The accuracy of this device depends: first, on the accuracy of the watt-hour meter to which it is applied; second, on the rate of the clock mechanism.

The General Electric M-2 Demand Indicator.—Fig. 310 shows the M-2 demand indicator made by the General Electric Co. for



A



B

Fig. 310.—General Electric Co. M-2 demand indicator.

use in conjunction with a watt-hour-meter. Referring to the diagram, which is for the instrument used on alternating-current

circuits, a contact on the registering train of the watt-hour meter closes the circuit of the operating coil every time the registration of a definite number of kilowatt-hours has been completed. When the armature is attracted, the dog is advanced by the ratchet and pushes the friction pointer before it.

To introduce the time element, the clutch (a sliding gear) is controlled by the trip levers which in turn are controlled by a cam arrangement driven through a system of gearing from a constant-speed motor. The trip levers are thus operated periodically, every half hour for instance, and the clutch thrown out of gear, allowing the spiral spring to return the dog to the beginning of its traverse. The friction pointer will thus be left at the end

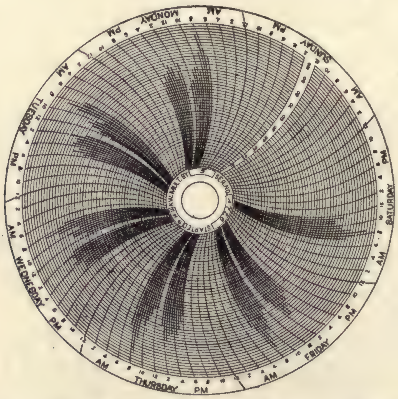


FIG. 311.—Record from General Electric registering demand indicator.

of the largest traverse of the dog and will indicate the maximum demand. Arrangements are made for regulating the speed of the motor in order that the time element may be adjusted.

When the indicator is used on a direct-current circuit the clutch is operated by an 8-day clock.

The General Electric Co. also manufactures a registering demand indicator which is based upon the same principle. An arm, corresponding to the friction pointer, carries a stylus which draws lines on a circular chart which are proportional, when properly scaled, to the demand during the consecutive 30-min. or other period for which the indicator is set. Fig. 311 shows such a record.

Printometer.—This is a device to be used in connection with watt-hour meters. It prints on a paper tape the equivalent of

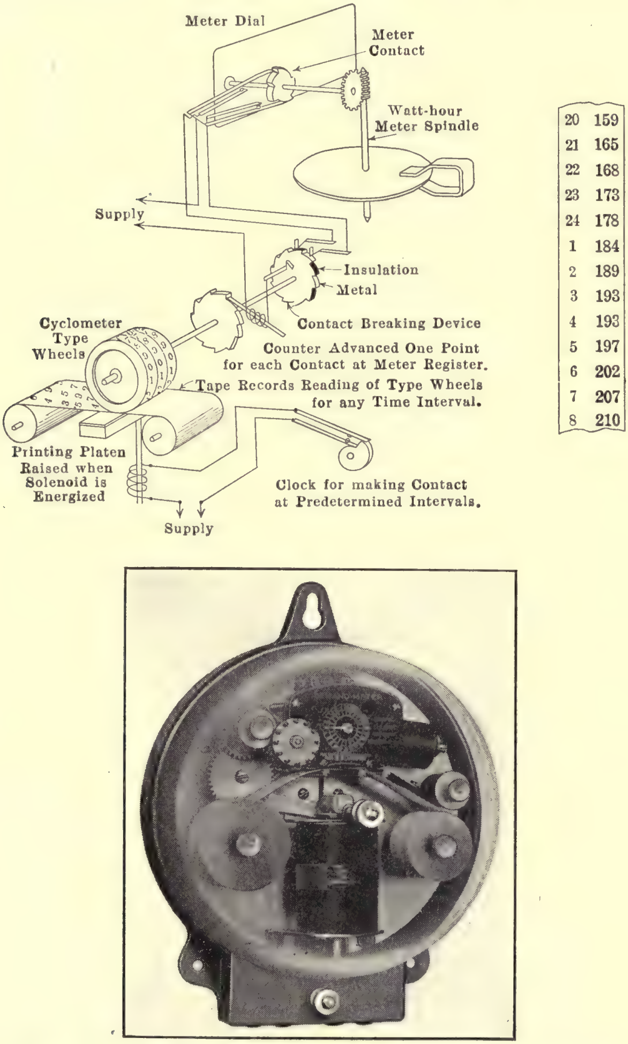


FIG. 312.—Printometer and record. General Electric Co.

the dial reading, together with the hour when the record was made. The instrument is shown in Fig. 312.

The essential portions are: a system of cyclometer type wheels actuated by a solenoid which is controlled by the watt-hour meter; an arrangement for automatically advancing the paper tape after each record (about 5/8 in.), and at the same time moving along the copying ribbon necessary for the printing; an electrically operated platen for taking the impression. The platen is controlled by a contact-making clock in a separate case and is operated every half hour.

In addition, there is a fourth type wheel, in line with the other three, which prints the hours. Every half hour the contact-making clock closes the circuit and causes the printing platen to be drawn up. At the same time it sets in motion a system of levers and gears by which the hour wheel is turned. At down stroke of the armature the paper tape and the copying ribbon are advanced.

To actuate the printometer, a contact device is placed on one of the shafts of the register of the watt-hour meter so that it closes the circuit to the cyclometer solenoid after the appropriate number of revolutions of the meter disc. The circuit is so arranged that it is made by the contact device and quickly broken by the plunger in the solenoid at the end of its stroke. The arcing is thus transferred to very substantial contacts and friction is avoided. The current is kept out of the solenoid except when it is actually operating.

The record obtained is shown in Fig. 312. By subtracting the successive readings the demand during any specified half hour may be obtained.

Westinghouse R.O. Demand Indicator.—This instrument is a combined watt-hour and watt-demand meter for use on alternating-current circuits. The kilowatt-hours are registered on four dials, as usual, while the demand is shown by a long pointer which moves over a fifth dial graduated in watts or kilowatts.

The mechanism is such that when a load is thrown on, the wattmeter attains the corresponding deflection only after a predetermined time, for example 15 min., the rate of increase of the deflection being controlled by the watt-hour meter.

Fig. 313 shows in a diagrammatic form the essential features of the registering mechanism. The main disc is that of an ordinary induction watt-hour meter. By means of the worm it actuates

the usual four-dial register, from which the kilowatt-hours are read. The auxiliary disc is a metal sector which forms the movable element of an induction wattmeter, the spiral spring furnishing the control. Both discs move in the same air gap, and so are actuated by the same set of current and potential coils (not shown in the figure).

The auxiliary disc is connected by means of the spindle and gears 1, 2, 3, 4, with the dog which drives the demand pointer. When this disc, and consequently the gear 4, deflects from the zero

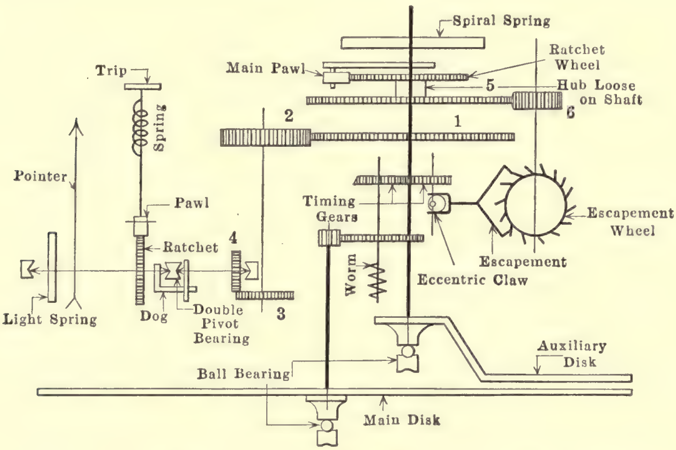


FIG. 313.—Diagram for Westinghouse R.O. demand indicator.

position the dog pushes the pointer up the scale. If the auxiliary disc returns toward zero, the driving arm leaves the dog and the pointer remains stationary, its position being maintained by the ratchet and light spring. The maximum deflection is thus registered.

The time element is introduced as follows: Attached to the spindle of the wattmeter element is an arm carrying the main pawl. This pawl rests on the ratchet wheel, the hub of which carries the gear 5 and is loose on the spindle. The gear 5 meshes with 6, and is thus connected to the escapement wheel. The escapement is oscillated by an eccentric, the rate of oscillation being controlled from the worm shaft by means of the timing gears.

The teeth of the escapement wheel are radial, so there is no interchange of power between the two elements.

When a load is thrown on, the main disc begins to revolve and the auxiliary disc tends to assume its ultimate position at once However, the escapement mechanism prevents this, and the deflection increases step by step at a rate dependent on the rapidity of oscillation of the escapement; that is, on the velocity of the main disc. If the load is doubled the escapement oscillates twice as fast, but as the pointer must move twice as far the ultimate deflection is attained in the same time. By changing the timing gears the maximum deflection may be reached in 1, 2, 5, 15 or 30 min. as desired. When the load decreases, the main pawl drags over the ratchet wheel and the driving arm moves away from the dog, leaving the pointer at its maximum deflection.

By raising the trip, which is protected by a separate seal, the pointer may be set back to zero without opening the meter.

During calibration the main pawl is raised, thus disconnecting the escapement. The instrument is then calibrated like an ordinary wattmeter by altering the length of the spiral spring and the zero adjustment.

The instrument is reset each month by the meter reader and leaves no record of its former indication, and there is no way of telling the hour of the day when the maximum occurred.

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CHAPTER X

PHASE METERS, POWER FACTOR INDICATORS, SYNCHROSCOPES AND FREQUENCY METERS

In the mathematical discussion of alternating currents, it is usual to assume sinusoidal waves, in which case,

$$\text{Power factor} = \cos\theta = \frac{\text{watts}}{\text{volt-amperes}}$$

where θ is the time-phase displacement of the current wave with respect to the e.m.f. wave; that is, the angular distance between the zero points of the waves. With non-sinusoidal waves the power factor is taken as the ratio of the watts to the volt-amperes. In this case θ is without significance.

The output of a generator is limited by the heating due to the currents in its coils, and the financial return on this output is primarily based on the true watts. For this reason alone then, it is highly desirable to operate the system supplied by the generator at as high a power factor as possible. Also, the power factor of the load influences the voltage regulation of the system. It is not unusual to employ some form of synchronous apparatus as a transforming device between the generator and the load. As its power factor may be controlled by varying the excitation, it becomes necessary to have on the switchboard a power-factor meter, or its equivalent, as an aid to the proper handling of this apparatus.

Idle Current Meters.—In any reactive circuit the current will either lag behind or lead the applied e.m.f. and may be resolved into two components, one the power component, in time phase with the voltage, the other the quadrature component which is wattless.

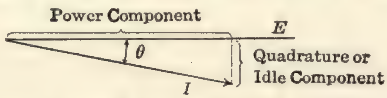


FIG. 314.—Showing power and quadrature components of current.

Evidently, for sinusoidal currents,

$$\text{Power component} = I\cos\theta$$
$$\text{Quadrature component} = I\sin\theta.$$

The component $I \sin \theta$ is sometimes called the idle or wattless current. It will be seen that operating a circuit at unity power factor is equivalent to so operating it that the idle current is zero.

A two-circuit electrodynamometer, with the movable circuit placed across the line and the fixed coils traversed by the line current, will give a deflection

$$D = KI_F I_M \cos \theta,$$

where θ is the phase difference of I_F and I_M. If the circuit of the movable coil is non-inductive the instrument is an ordinary wattmeter, but if this circuit could be made perfectly reactive the phase of I_M would be shifted 90° with respect to the line voltage and

$$D = KI_F I_M \cos (90° - \theta) = KI_F I_M \sin \theta.$$

At a constant voltage the deflection would be proportional to the idle current, $I \sin \theta$, or at any voltage, to the idle volt-amperes.

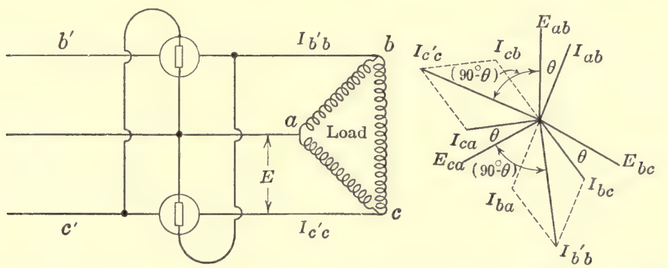


FIG. 315.—Connections for measuring idle volt-amperes in balanced three-phase circuit.

On account of the energy dissipated in the reactor, it is impossible to shift the phase of the potential-coil current by exactly 90°, and the phase shift will depend on the frequency.

Reference to the theory of the induction wattmeter will show that this instrument would be converted into an idle current meter if its potential circuit were made perfectly *non-inductive.*

The idle volt-amperes in a balanced three-phase circuit may be measured by the use of wattmeters if the coils be connected in circuit as shown in Fig. 315.

From the vector diagram it is seen that each wattmeter gives a deflection proportional to $EI \cos (90° - \theta) = EI \sin \theta$. If the two wattmeters in Fig. 315 are the two elements of a polyphase wattmeter, the reading of that instrument will give $2EI \sin \theta$. As the total idle volt-amperes in the load is $\sqrt{3}EI \sin \theta$,

$$\text{Idle volt-amperes} = (\text{reading}) \frac{\sqrt{3}}{2}.$$

The scale of the instrument may be graduated so that the idle volt-amperes may be read directly. (Compare with two wattmeter method for measuring three-phase power, page 331).

Tuma Phase Meter.—In America, power-factor meters are much more frequently used than idle current meters. Power-factor meters, as well as various forms of synchroscopes, are developments from the Tuma Phase Meter, the essential portions of which are shown in Fig. 316.

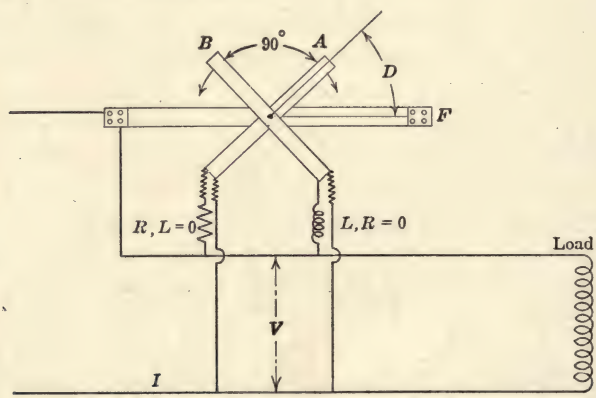


Fig. 316.—Diagram of Tuma phase meter.

In its original form the ideally perfect Tuma phase meter is applicable only to single-phase circuits and gives a deflection equal to the power-factor angle of the load. By a trifling alteration it may be adapted to polyphase circuits, as will be seen later.

The fixed coil F is traversed by the load current. The coils A and B are of equal magnetic strength and are firmly lashed together to form a single movable system which is pivoted in the field due to F; in the ideal instrument A and B are inclined at an angle of 90° to each other.

The current in coil A is supposed to be controlled by a pure resistance, and consequently is in phase with the applied voltage V. The current in coil B is supposed to be controlled by a pure inductance, and hence is in quadrature with the voltage V.

The currents are taken into the movable system at the pivot, through flexible connections of annealed silver foil which resemble ordinary controlling springs in appearance but which exercise no appreciable torque on the system.

When no currents are flowing, the crossed coils are perfectly neutral and will remain in any position to which they are turned. The position of the crossed coils from which the deflections are reckoned is that assumed by them when the device is applied to a load of power factor unity. In that case the planes of coils A and F will coincide, for as the currents in B and F are in quadrature, on account of the inductance L, the average turning moment on B is zero.

In general, on the passage of currents through the coils F, A and B, a field will be set up by F, and the coils A and B which form the movable element will both experience turning moments. As A and B are rigidly connected, the movable element will turn to such a position that the resultant moment acting on it becomes zero. The deflection, D, from the initial position occupied by the coils when the power factor is unity, will be equal to the power-factor angle of the load.

It will be assumed that the coil F is so large compared with A and B that the crossed coils move in a sensibly uniform field; also that the circuits of A and B are inductionless and resistanceless respectively. If the P.D. wave be taken as the datum for measuring phase displacements, the turning moment acting on coil A will be, at any instant,

$$K_A[I \sin(\omega t - \theta)]\left[\frac{V}{R} \sin \omega t\right] \sin D,$$

where K_A is a constant depending on the windings.

The turning moment on coil B will be, at any instant,

$$K_B[I \sin(\omega t - \theta)]\left[\frac{V}{L\omega} \sin(\omega t - 90°)\right] \sin[D + 90°].$$

The currents in the coils are such that A and B tend to turn in opposite directions. When the movable system has come to

rest, the average turning moment on coil A must equal that on coil B, so

$$\left[\frac{K_A IV}{R}\right] \sin D \frac{1}{T}\int_0^T \sin(\omega t - \theta)\sin(\omega t)dt =$$

$$\left[\frac{K_B IV}{L\omega}\right] \cos D \frac{1}{T}\int_0 \sin(\omega t - \theta)\sin(\omega t - 90°)dt$$

$$\left[\frac{K_A IV}{R}\right] \sin D \cos\theta = \left[\frac{K_B IV}{L\omega}\right] \cos D \sin\theta.$$

If, by the construction of the apparatus

$$\frac{K_A}{R} = \frac{K_B}{L\omega} \tag{a}$$

then

$$\tan D = \tan\theta$$

and

$$D = \theta.$$

That is, the movable system turns through an angle equal to the power-factor angle of the load.

The assumptions made in order to obtain this result are that the frequency is constant, that the coils A and B are small compared with F, that the planes of coils A and B are 90° apart in space, that the coils are traversed by currents which are 90° apart in time phase and that the current in coil A is in phase with the line voltage V. In practice, these current relations cannot be attained; the lag in the circuit B can never be exactly 90°. Nevertheless, by a proper adjustment of the angle between the coils, the instrument can be made to read correctly.

To investigate this matter, suppose the current in coil B lags $\Delta°$ behind V and that the mechanical angle between the coils is β instead of 90°.

Assuming that the crossed coils are alike and have the same number of ampere turns,

$$\sin D \cos\theta = \sin(D + \beta)\cos(\theta - \Delta).$$

The fiducial point on the scale corresponds to the reading when the power factor of the load is unity; in that case the deflection, D_0, will be given by

$$\cot D_0 = \frac{1}{\sin\beta\cos\Delta} - \cot\beta.$$

When the power-factor angle of the load is θ the reading will be given by

$$\cot D = \frac{1}{\sin\beta\cos\Delta + \tan\theta\sin\beta\sin\Delta} - \cot\beta.$$

The change of deflection is $D - D_0$,

$$\cot(D - D_0) = \cot\theta\left\{\frac{1 - \cos\beta\cos\Delta + \left[(\cos^2\Delta - \cos\beta\cos\Delta) + (\cos\beta\cos\Delta + \sin^2\beta\frac{\cos\Delta}{\cos\beta} - 1)(\tan\theta\cos\beta\sin\Delta)\right]}{\sin\beta\sin\Delta}\right\}$$

Inspection shows that if $\beta = \Delta$, this equation reduces to

$$\cot(D - D_0) = \cot\theta\,\frac{1 - \cos^2\beta}{\sin^2\beta} = \cot\theta$$

or

$$D - D_0 = \theta.$$

Consequently if the crossed coils be adjusted once for all so that the angle between their planes is equal to the electrical angle between their currents, the deflection from the initial position will be equal to the power-factor angle of the load.

An explanation of the action of the Tuma phase meter may also be based on the fact that the crossed coils set up a rotating field (see page 444), for in the original design of the instrument these coils are 90° apart in space and are traversed by currents differing 90° in time phase.

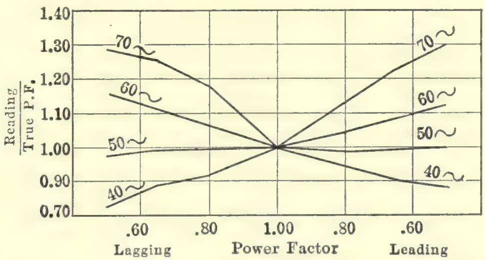


FIG. 317.—Showing effect of frequency on a single-phase power-factor meter.

Single-phase Power-factor Meters.—In the application of the principle of the Tuma phase meter to the construction of power-factor meters for use on single-phase circuits a difficulty is encountered. For though the windings and the angle between the crossed coils may be adjusted so that the instrument reads

correctly at the normal frequency, any change of frequency will render the readings inaccurate, because both the phase and the magnitude of the current in coil B are controlled by an inductance and, therefore, depend on the frequency. At low frequencies coil B carries too much current, at high frequencies too little current and condition (a) (page 534) is not fulfilled. The result of frequency changes on a single-phase power-factor meter is illustrated by Fig. 317. Single-phase power-factor meters are made but are not in common use.

Polyphase Power-factor Meters.—The indications of polyphase power-factor meters are correct only on balanced circuits. If the circuit is unbalanced the reading is without significance.

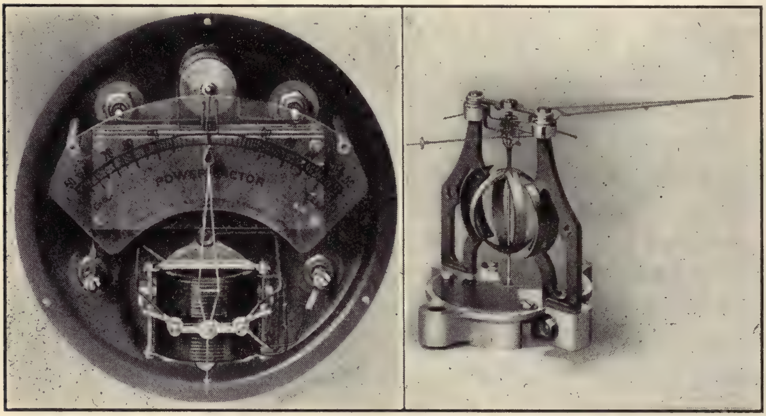


Fig. 318.—Indicating portion of Weston power-factor meter.

The application of the Tuma phase meter to a balanced two-phase circuit is obvious. The two crossed coils are placed 90° apart in space; their currents, 90° apart in time phase, are obtained by using a resistance in series with each coil and energizing a coil from each of the two phases. To adapt the instrument to balanced three-phase circuits it is to be remembered that the angle between the planes of the movable coils should be made equal to the electrical angle between the currents in these coils. The fixed coil is placed in one of the line wires, while the movable coils are connected from this wire through resistances to the other two wires of the circuit, see Fig. 319.

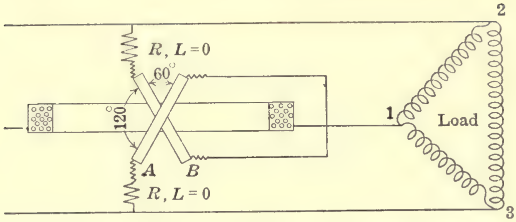


FIG. 319.—Connections for three-phase power-factor meter.

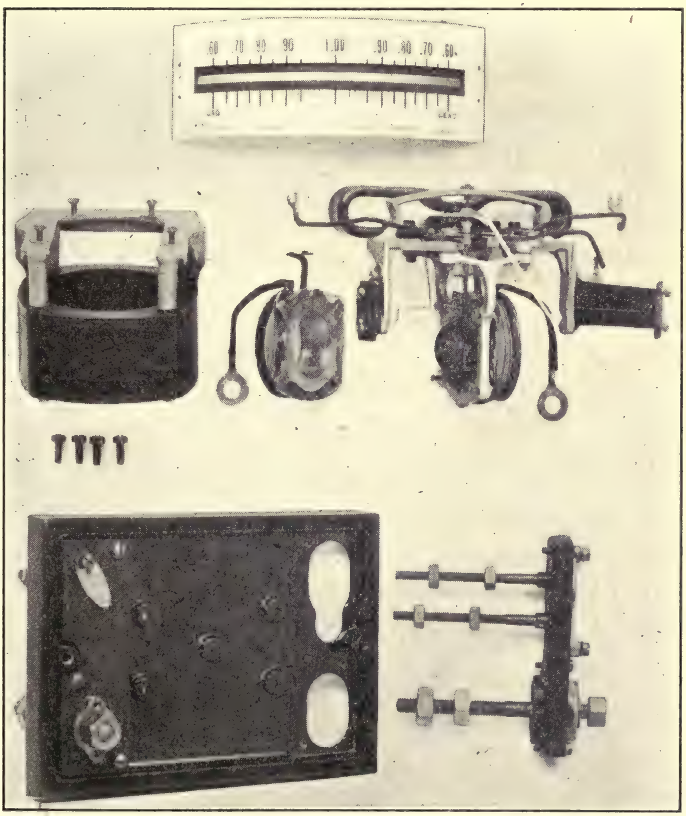


FIG. 320.—Showing parts of General Electric Co. power-factor meter with magnetic shielding.

Reckoning from lead 1 the currents in A and B are 60° apart. The fiducial position of the coils is given when the power factor of the load is unity; if the power factor is other than unity the current in lead 1 is shifted $\theta°$, where θ is the power-factor angle, and the crossed coils turn through an equal angle.

The polyphase instruments are independent of frequency, for no reliance is placed on the use of reactances to properly shift the phases of the currents in the crossed coils. On high-voltage circuits the meters are operated through instrument transformers.

In power-factor meters as actually constructed, see Figs. 318 and 320, the fixed coils are made to surround closely the movable system. Economy of space and of materials are thus attained. In this case the scale must be determined by calibration.

To avoid the use of movable coils, the Westinghouse Company in certain of its power-factor meters and synchroscopes uses the construction indicated in Fig. 321.

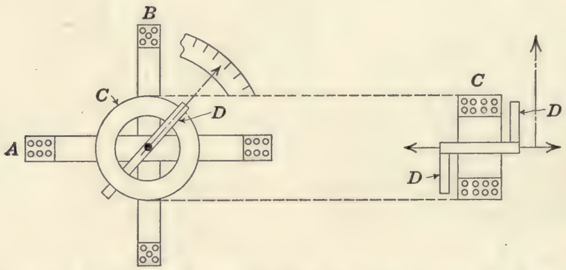


Fig. 321.—Westinghouse arrangement of coils for power-factor meter.

Within the stationary crossed coils A and B is a third fixed coil C with its axis perpendicular to the plane of the paper. This coil carries the line current and magnetizes the soft iron element D, which is mounted in jeweled bearings. The iron, D, thus forms the core of an alternating-current electromagnet and acts as if it were a pivoted coil carrying the line current.

Another arrangement of circuits, as used in the Punga power-factor meter, is shown in Fig. 322.

The movable system consists of three flat, rectangular coils lashed to the spindle so that they are 120° apart; they have a common electrical terminal at 0, that is, they are Y connected across the line; the currents are controlled by resistances.

At unity power factor the current in the fixed coil is in time phase with the voltage V_{ao} or the current I_{ao}, so taking V_{ao} as the datum for phase displacement,

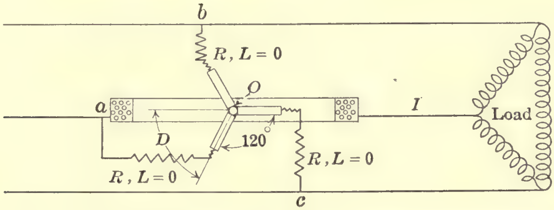


FIG. 322.—Connections for Punga power-factor meter.

Field due to fixed coil $= KI \sin (\omega t - \theta)$
Current in coil *ao* $= I' \sin \omega t$
Current in coil *bo* $= I' \sin (\omega t - 120°)$
Current in coil *co* $= I' \sin (\omega t - 240°)$.

For equilibrium the net turning moment due to the three coils must be zero, and

$$\sin D \frac{1}{T}\int_0^T \sin (\omega t - \theta) \sin \omega t dt$$
$$+ \sin (D - 120°) \frac{1}{T}\int_0^T \sin (\omega t - \theta) \sin (\omega t - 120°)\, dt$$
$$+ \sin (D - 240°) \frac{1}{T}\int_0^T \sin (\omega t - \theta) \sin (\omega t - 240°)\, dt = 0.$$

Integrating and substituting the values of the functions of 120° and 240°,

$$\tfrac{6}{4} \cos \theta \sin D - \tfrac{6}{4} \cos D \sin \theta = 0$$
$$\therefore\ D = \theta,$$

the same result as was obtained with the two crossed coils.

Power-factor Charts.—In tests of industrial plants, it is frequently important to gain an idea of the power factor under ordinary operating conditions. In three-phase work the two wattmeter method of measuring power will usually be employed. For a balanced load the power indicated by the two wattmeters is

$$P_1 = EI \cos (\theta + 30°)$$
$$P_2 = EI \cos (\theta - 30°).$$

It is convenient to calculate and plot once for all a curve such as is shown in Fig. 323.

The ordinates of the curve are power factors or values of $\cos\theta$; the abscissæ are values of the ratio

$$\frac{\text{Smaller reading}}{\text{Larger reading}}=\frac{\cos\,(\theta+30^\circ)}{\cos\,(\theta-30^\circ)}=R.$$

The relation between the power factor ano the ratio R is

$$\text{Power factor}=\frac{1}{\sqrt{1+3\left(\frac{R-1}{R+1}\right)^2}}.$$

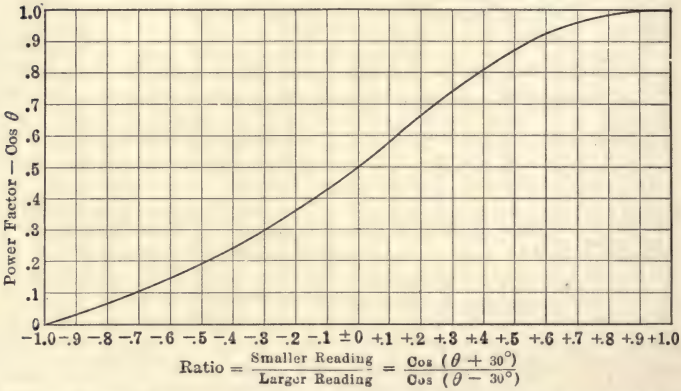


Fig. 323.—Power-factor chart, two-wattmeter method, balanced three-phase load.

Synchroscopes, Synchronizers.—When alternators are operated in parallel, it is necessary in putting a machine on the system that the voltage of the incoming machine be equal in magnitude and opposite in phase to the voltage of the busses. To avoid accidents and injurious stresses in the machines, it is necessary to have some instrument which will show when the proper phase relation has been attained and in the case of large machines, whether the speed of the incoming machine must be increased or diminished before the main switches are closed.

The Lincoln Synchroscope.—The Lincoln and kindred forms of synchroscopes furnish the switchboard attendant with the desired information. This instrument is in principle a Tuma phase meter with the slight modification that the currents are carried to the crossed coils by brushes and slip rings so that the

movable element can rotate continuously. The essential electrical connections are indicated in Fig. 324.

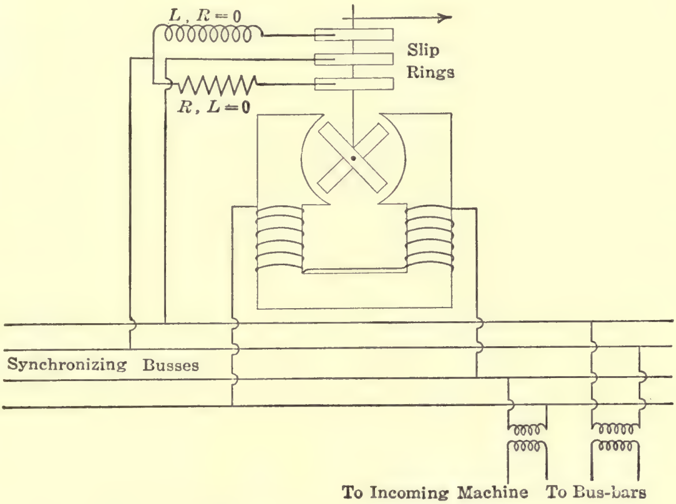


FIG. 324.—Diagram for Lincoln synchroscope.

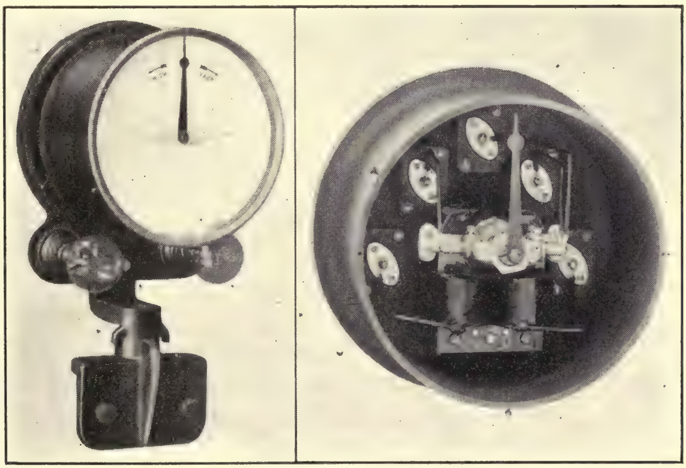
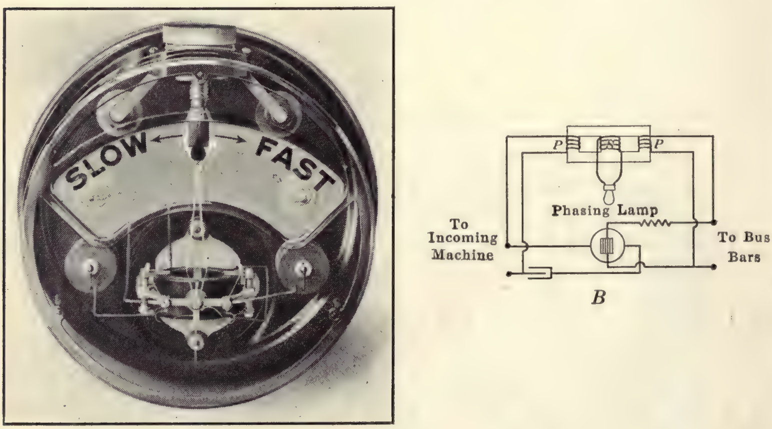


FIG. 325.—General Electric Co. synchroscope.

In the actual instrument, Fig. 325, the shaft carrying the slip rings and the movable element is mounted in ball bearings and

as shown is perpendicular to the plane of the paper. To increase the turning moment acting on the movable system and thus reduce the effect of the brush friction, both the field and movable coils are wound on laminated iron cores. The similarity of the arrangement to the Tuma phase meter is at once apparent.

The index tries to point to the angle of phase difference between the machines, and its rate of movement is dependent on the difference of the machine speeds. It will move forward or backward or come to rest depending on whether the incoming machine is running faster, slower or at the same speed as the other machine. The pointer may come to rest in any position on the dial. This merely means that both machines are running at the same speed, though not necessarily in phase. The speed of the incoming machine must be altered very gradually and the main switch closed as the pointer slowly drifts across the index mark.



A

FIG. 326.—Weston synchroscope.

Weston Synchroscope.—This instrument is an electrodynamometer with a spring control. The fixed coils are connected across the station bus-bars, in series with a suitable resistor, while the movable coil, in series with a condenser, is placed across the incoming machine.

If the machines are in synchronism or 180° out of synchronism, the currents in the two coils will be in quadrature and the index

will stand at the reference mark. If they are running at the same frequency, but are not in the proper phase relation, the pointer will come to rest at a position depending on the phase difference of the machines. If the frequencies are nearly, but not exactly the same, the pointer will "beat," or move back and forth over the scale. This arrangement is supplemented by a synchronizing lamp, as indicated. The lamp is behind the pointer and the transparent scale and is arranged to be bright when the machines are in synchronism. The incoming machine is to be connected to the bus-bars when the dark pointer coincides with the reference mark and both appear on the light field due to the glowing of the synchronizing lamp.

Hartmann and Braun Synchroscope.—The firm of Hartmann and Braun has devised a synchroscope based on the vibrating reed frequency meter, page 548. This instrument, with a diagram of its circuits, is shown in Fig. 327.

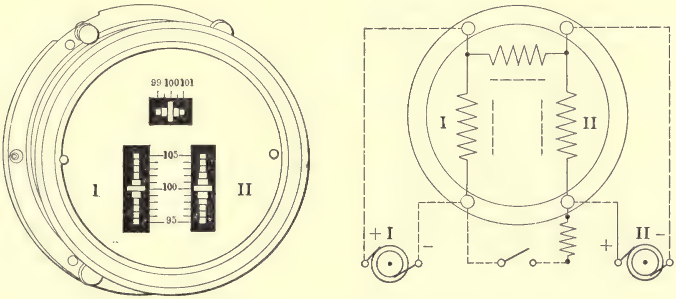


FIG. 327.—Hartmann and Braun synchroscope.

The frequency at the bus-bars and that of the incoming machine are shown on the two vertical banks of reeds. On closing the switch, the upper set of reeds is put in series with the two machines and will be acted upon by the net voltage around the machine circuit. The synchroscope circuits are so arranged that when both machines are running at the normal frequency, shown on banks I and II, and are in the proper phase relation, the reed in the upper set which corresponds to the normal frequency will vibrate continuously with its maximum amplitude.

Phasing Lamps.—Before the invention of such synchronizing devices as have been described, it was customary to

depend upon synchronizing lamps. With small low-voltage machines it is sufficient to place an incandescent lamp across the gap of the single pole switch by which the incoming machine is to be connected to the bus-bars. If, when the voltage has been adjusted, the incoming machine is not running at the proper frequency, the lamp will be alternately light and dark. As the frequency of the incoming machine is brought toward its proper value, the flicker of the lamp becomes slower and slower. The proper time for closing the switch is when the lamp remains dark, for then the voltages on the machine circuits are in opposition.

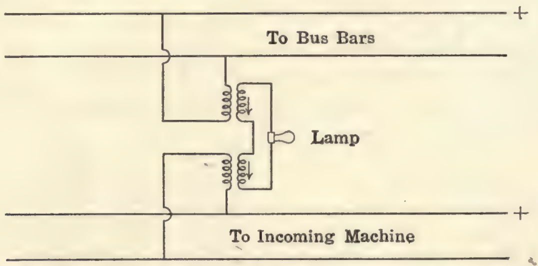


FIG. 328.—Arrangement of phasing lamp actuated by two transformers.

When high voltage machines are used, it becomes necessary to employ transformers. They may be connected so that the proper time for closing the main switch is shown either when the lamp is dark or when it is at full brilliancy. The latter is the better practice as it avoids mistakes due to the failure of the lamp. The two transformers may be combined into one with two primaries wound on two different branches of the magnetic circuit and a single secondary wound on a third branch as shown in Fig. 326.

A fault of these arrangements of phasing lamps is that they give no indication as to whether the speed of the incoming machine should be increased or diminished.

Siemens and Halske Arrangement of Phasing Lamps.—The operation of the arrangement will be understood from the simplified diagram, Fig. 329, where only three lamps are shown, and the transformers necessary on a high voltage system are omitted.

The dotted connection simply denotes that the two neutral points are at the same potential. The noticeable feature of the

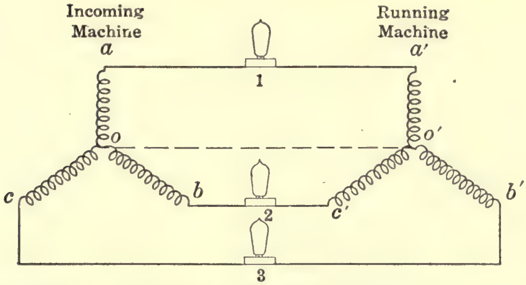
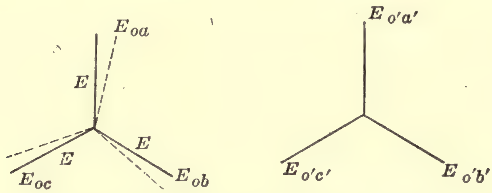
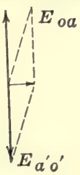


FIG. 329.—Diagram for Siemens and Halske arrangement of phasing lamps.



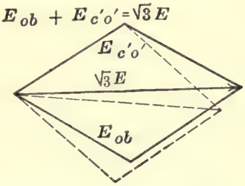
The e.m.f. acting on lamp No. 1 will be:

$$E_{oa} + E_{a'o'} = O$$



The e.m.f. acting on lamp No. 2 will be:

$$E_{ob} + E_{c'o'} = \sqrt{3}\,E$$



The e.m.f. acting on lamp No. 3 will be:

$$E_{oc} + E_{e'o'} = \sqrt{3}\,E$$

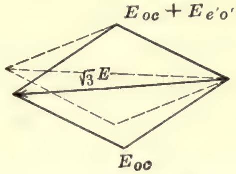


FIG. 330.—Vector diagrams, Siemens and Halske phasing lamps.

arrangement is that phase *oc* is connected to *o'b'* and phase *ob* to *o'c'*.

Suppose the two machines are in synchronism. The vector diagrams for the electromotive forces are shown in Fig. 330.

Synchronism is indicated when lamp No. 1 is dark and lamps 2 and 3 are glowing. If the machines are not in phase, the vectors being in the dotted position, lamp No. 1 begins to glow, No. 2 grows dimmer and No. 3 grows brighter, so the dark lamp is passed from position No. 1 to position No. 2. If the phase displacement is in the other direction, lamp No. 1 begins to glow, No. 2 grows brighter and No. 3 is dimmed, so that the dark lamp is passed from position No. 1 to position No. 3.

The lamps are arranged on the switchboard at the corners of an equilateral triangle; by noticing whether the order of brilliancy of the lamps proceeds around the triangle in the right-hand or the left-hand direction, one can tell whether the speed of the incoming machine must be increased or diminished.

Frequency Meters.—Instruments of this class should be independent of wave form and also of variations of the line voltage. Because of the latter requisite, certain forms of frequency meter are constructed so that the controlling and deflecting moments acting on the movable system *both* depend on the current through the instrument, that is, on the line voltage.

Resonating Frequency Meters.—Under the usual operating conditions the range of frequencies which must be covered by any form of frequency meter is small, and the normal frequency of the current has a definite fixed value. It thus becomes natural to employ in these instruments the principle of resonance, either electrical or mechanical.

In the General Electric Company's resonating frequency meter, advantage is taken of the action of circuits containing inductance and capacity in series. When such a circuit is properly adjusted, if the periodicity of the applied P.D. be varied, the maximum value of the current will be sharply defined, provided the energy losses in the circuit be small. Fig. 331 shows the circuits of this particular instrument. The movable element consists of two crossed coils set very nearly at right angles to each other; it is pivoted and free to move, there being no controlling springs.

For a 60-cycle instrument one main circuit is tuned to reso-

nate at 68 cycles, the other at 52 cycles; each circuit is connected to one of the crossed coils which tend to turn the system in opposite directions. The fixed coil carries the total current.

If the frequency is high, in the neighborhood of 68 cycles, the 68-cycle member of the system will carry a large current while the current in the 52-cycle coil will be small. The effect of the 68-cycle coil will preponderate and it will turn toward the left, carrying the 52-cycle coil with it. As it turns, the 68-cycle coil moves to a less advantageous position while the 52-cycle coil is moved toward the position where its effect will be a maximum. The movable element thus arrives at an equilibrium

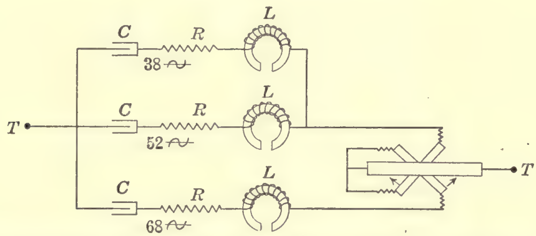


Fig. 331.—Diagram for General Electric resonating frequency meter.

position which depends on the frequency. A high degree of sensitiveness may be attained, so that the full scale of a 60-cycle instrument extends from 55 to 65 cycles. At abnormally low frequencies, the currents in both coils would be very small and the pointer might drift back on the scale and thus give rise to errors; for this reason a circuit tuned to 38 cycles is added, its effect being to keep the index off the scale at low frequencies.

The inductances are wound on laminated iron cores provided with air gaps; the gaps are necessary, for in order that the tuning may be accomplished the power factors must be low and wave distortion must be avoided. The iron must be worked at a low flux density and the hysteresis reduced to a minimum. These instruments are made both in the indicating and in the curve drawing forms.

In 1888 Ayrton suggested that it was possible to determine the frequency of an alternating current by employing the principle of mechanical resonance, and of late years this suggestion has been developed into commercial forms of frequency meters. In

these modern instruments there is a bank of steel reeds so tuned that the natural periods of successive reeds differ by one alternation. This bank of reeds is acted upon by an electromagnet traversed by current taken from the line. Only the reeds very nearly in tune with the frequency of the circuit respond visibly, the reed most nearly in tune showing the maximum amplitude. If it is exactly in tune, the amplitude is very large. This is well illustrated by Fig. 332, which shows the amplitude of vibration of a reed tuned to a frequency of 90 alternations when currents

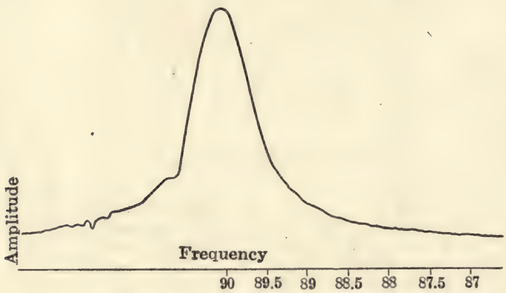


Fig. 332.—Showing effect of frequency on the amplitude of vibration of a reed in a frequency meter.

of various frequencies are sent through the magnet. A variation from 90 to 89.5 alternations reduces the amplitude over 50 per cent. (compare with the vibration galvanometer, page 434).

In order to insure reliability and long life the butts of the reeds must be firmly held. The arrangement adopted by the firm of Hartmann and Braun and one form of the complete instrument are shown in Fig. 333.

If the reeds are unpolarized they will be drawn toward the magnet at each alternation, so that the reed having twice the frequency of the current responds. If they are polarized, by either permanent or electro-magnets, the reed having the same frequency as the circuit will have the maximum amplitude of vibration.

The reeds are approximately tuned by making them of different lengths, the final tuning being effected by altering their weights by filing away drops of solder which are at their outer

ends just behind the white indexes. In the instrument of this class made by Siemens and Halske, all the reeds are fixed to a single metallic bar which is so mounted on a spring support that it may be gently vibrated by the action of an electromagnet excited from the circuit. The particular reed which is in tune with the circuit responds with the maximum amplitude.

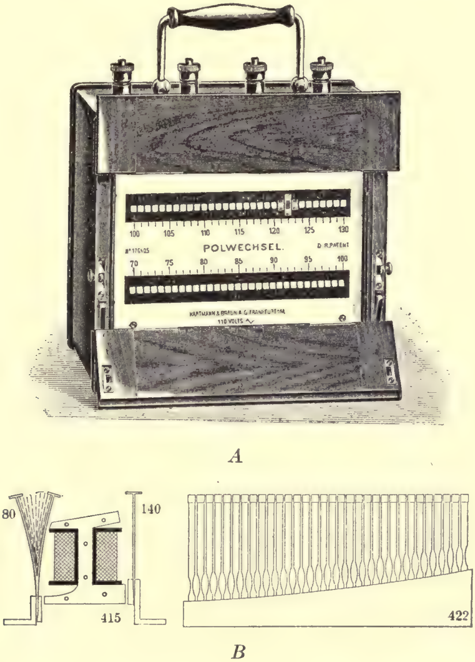


FIG. 333.—Hartmann and Braun frequency meter.

These frequency meters may be used as speed indicators. In this case, a toothed wheel or some other form of rotary key is attached to the shaft of the machine and serves to interrupt a direct current which is sent through the meter. Knowing the number of contacts per revolution of the shaft and the number of impulses as read from the scale, the revolutions may be calculated.

The Siemens and Halske arrangement in a modified form is used as a revolution indicator for steam engines and other machinery.

In this case, the bank of reeds is supported from the base of the machine in question. Owing to the lack of exact balance in the moving parts of the machine, a vibration exists which is sufficient to set the reeds in motion and thus indicate the speed.

Induction Frequency Meter.—The essential features of the induction frequency meter made by the Westinghouse Company are shown in Fig. 334.

At *A* and *C* are two shaded-pole motor elements which tend to rotate the movable element *B* in opposite directions. *A* is in

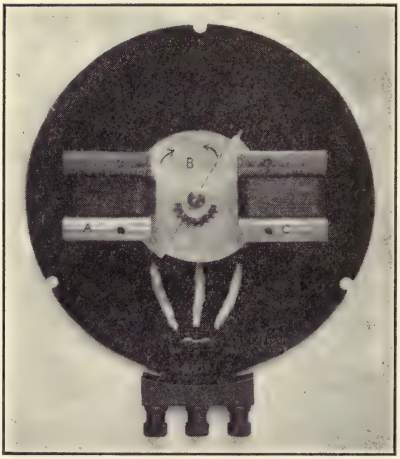


FIG. 334.—Westinghouse induction frequency meter.

series with an inductance, *C* in series with a resistance. The movable element is a flat plate of aluminum; its boundary above the dotted diametral line is a semicircle with its center in the axis of rotation, while below the same line it is practically a semicircle with its center shifted a little upward along the line. The torque exerted by each element is proportional to the frequency and to the square of the current. If the voltage of the circuit varies, both elements are affected and the movable system is not disturbed, but if the frequency rises, less current will flow through the inductance and the effect of the other element will preponderate. The disc will begin to turn toward the left but on account of the shape of the disc this brings less of it under the influence of the element *C*, so that it takes up a new equilib-

rium position. There is no moving wire and consequently no necessity for taking current to and from the movable element.

Magnetic Vane Frequency Meter.—The arrangement adopted in the Weston frequency meter is shown in Fig. 335.

The crossed coils A and B are fixed and in their field is pivoted a long thin needle of soft iron, N, which carries the pointer. There is no controlling spring. The inductances (L) and resistances (R) are so proportioned that the combined action of the two

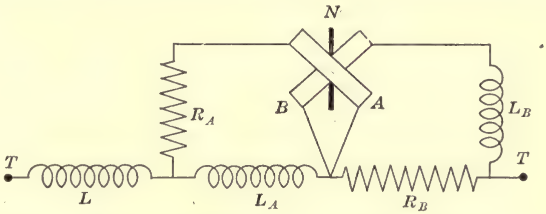


FIG. 335.—Diagram for Weston magnetic vane frequency meter.

coils sets up an elliptical rotating field. The needle takes up the direction of the longer axis of the field, the angular position of which changes as the frequency alters. If the frequency rises, for the same total current in the circuit the current in coil A is increased while that in coil B is diminished. This shifts the direction of the axis of the field and the pointer is thus carried over the scale.

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CHAPTER XI

GRAPHIC RECORDING OR CURVE-DRAWING INSTRUMENTS

Graphic recording instruments are those which automatically record their indications on a uniformly moving strip, or circular sheet, of paper. Continuous and permanent records of the quantity which the instruments are adapted to measure are thus obtained.

Such instruments are extremely useful in investigating the power conditions in factories and in studying the cycle of operations of single machines. In many cases the load fluctuates so rapidly that to obtain equivalent data by using indicating instruments would necessitate the observers remaining continuously at their posts taking frequent readings at noted times. Afterward, these readings must be plotted and the best representative curve drawn. This is a time-consuming operation.

Continuous records are frequently important in central-station work. For instance, a registering ammeter in a feeder gives a record of the current and shows, if the clock be of good quality and properly regulated, when the feeder is put in and taken out of service, as well as the time when any abnormal conditions arise. Such data, if systematically kept, may be of great importance as evidence in adjusting disputes arising from accidents. Again, a continuous record of the potential on a lighting circuit contributes to the life of the incandescent lamps by directing the attention of the operator toward constancy of voltage. The records obtained by a registering wattmeter show the customer's power consumption throughout the day and are useful in determining rates.

Fig. 337 illustrates the application of an instrument of this class to the study of a particular machine. It shows the current taken by a direct-current motor driving a roughing lathe. The cycle of operations is to be referred to Fig. 336 which shows

the piece which is being turned. Corresponding points in the two figures bear the same letter.

The variation of power with depth of cut and the time required for each operation are clearly shown.

A simple form of recording ammeter which has been in use for many years is shown in Fig. 338. The current flows through the coil *A* giving rise to an attraction on the soft iron disc *B*

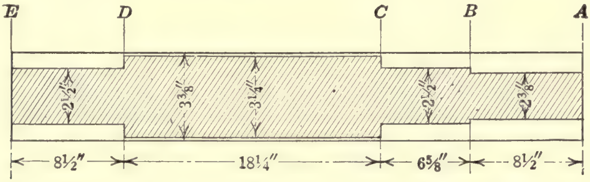


FIG. 336.—Piece to which cycle shown in Fig. 337 applies.

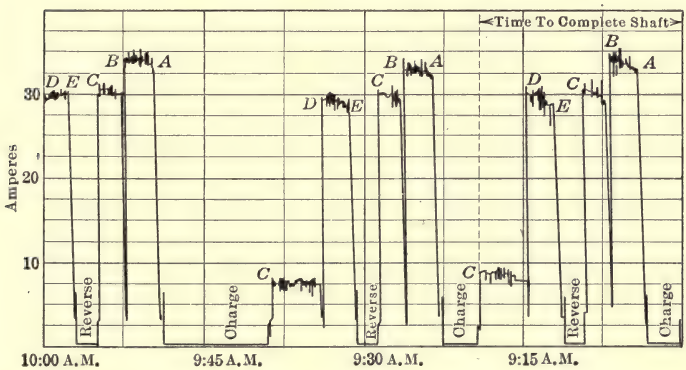


FIG. 337.—Curve showing typical cycle on a roughing lathe.

carried by the rod *CD* which passes freely along the axis of the coil. The rod is supported on knife edges by two flat springs *CC* and *DD* which are fixed at their lower ends; *DD* carries the pointer *E* to which the pen is attached. The record is made on a circular sheet of paper which is rotated at a uniform rate by the clockwork. Ordinarily for central station work the records are for a 24-hour period. For special work this may be varied by using the appropriate clockwork. The pen, which rests on the paper continuously, is a little V-shaped trough cut away at one end so that only a fine point at the apex of the V drags on

the paper. The trough holds a few drops of an aniline-glycerine ink which is carried to the paper by capillary action. As the reservoir holds but little ink, frequent attention by the operator is necessary.

Another instrument of the same class is shown in Fig. 339*A*. A counterbalanced soft iron core, consisting of a tube which projects perpendicularly from a disc of the same material, is attracted into the coil against the action of a spiral spring. By a

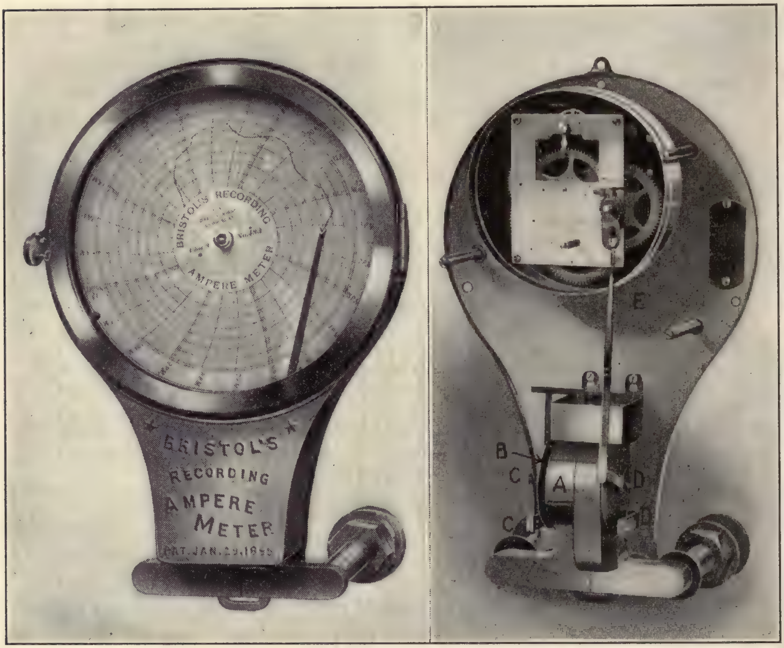
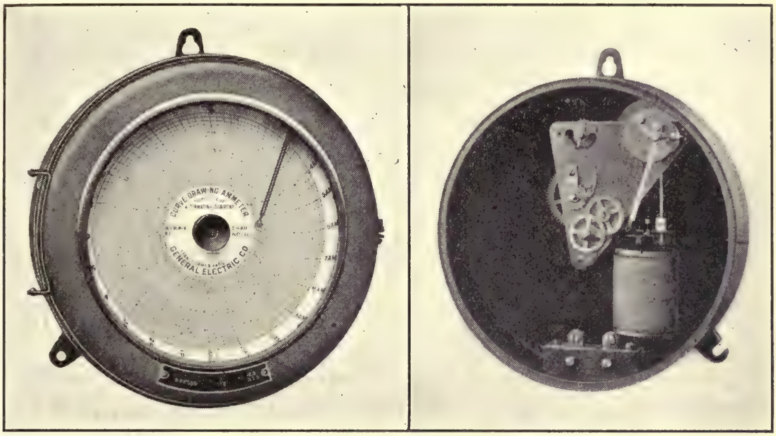


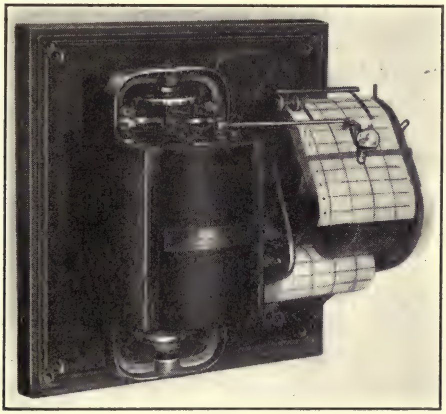
Fig. 338.—Bristol curve-drawing ammeter.

simple lever the motion of the core causes the pen to travel over the chart. The tube and disc are slit to reduce eddy currents. An adjustable iron plug in the lower part of the solenoid allows the deflection at the upper end of the scale to be adjusted. The needle is damped by an aluminum damping disc of the usual form actuated by gearing from the pivot carrying the index. The clock is ordinarily arranged so that the chart is for either a 12- or a 24-hour period.

Instruments like those just described are useful when a moderate accuracy will suffice; for instance, on a set of feeder panels where many instruments must be installed and consider-



A



B

Fig. 339.—Curve drawing ammeters, General Electric Co.

able expense is not justified. If a high degree of accuracy is desired, more complicated arrangements must be used.

A registering instrument should be capable of operating for a considerable period without attention, for 1 or 2 weeks in

many cases. The clock should be of good quality, preferably self-winding, the motion of the paper positive, and the time scale uniform.

The special difficulties in the design of accurate instruments of this sort come from the pen friction which impedes the motion of the pointer. It is best that the scale be uniform and the records given on rectangular coördinates so that they may readily be integrated by a planimeter. A uniform time coördinate may be attained by driving the paper by a metal drum having projecting pins which engage in perforations at the edges of the record paper.

In the better class of instruments the effect of pen friction is minimized or eliminated:

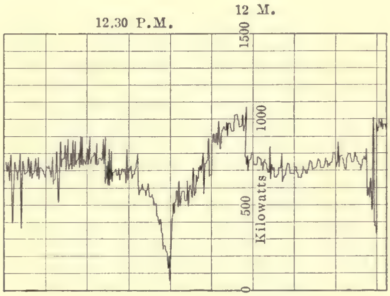
1. By giving the movable system a high torque and employing a very strong controlling spring. By using soft-iron instruments of proper design, ammeters and voltmeters may thus be constructed in which the pen-friction error is reduced to 1 or 2 per cent of the full-scale deflection, without an unduly great consumption of power. (50 watts in a voltmeter; 7 watts in an ammeter.)

2. By providing the pointer with a stile that ordinarily swings clear of the paper but which is periodically pressed against it by an electromagnet and imprints a dot. This arrangement is useful where the phenomena under investigation vary slowly. It has proved of great service in those forms of registering thermo-electric pyrometers which are in reality registering millivoltmeters. A modification is to have an arrangement by which a high-tension spark is periodically caused to pass from the stile through the paper, thus again giving the record in the form of dots.

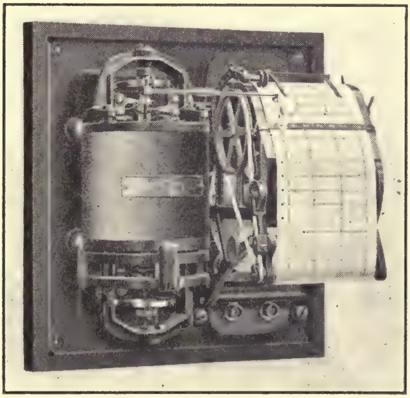
3. By using the relay principle; in this case the movable system has only to control the position of the pen, the power necessary to move it being supplied from an external source. Relay instruments are somewhat complicated, but the wattmeters and the direct-current ammeters as usually designed have a uniform scale and thus give records on rectangular coördinates.

A direct-action registering wattmeter for polyphase circuits is shown in Fig. 340. Here the friction is overcome by the high torque of the movable systems.

The large vertical cylinder at the left is a laminated soft iron shield. The working parts consist of two substantial dynamometer wattmeters, both movable coils being rigidly attached to the same stem which is suspended by a steel torsion wire. The lower end of the stem is centered by a small steel pin which



A



B

FIG. 340.—Curve-drawing polyphase wattmeter and record, General Electric Co.

passes freely through a ring jewel. Magnetic damping is employed, the magnets and damping disc being just below the shield. Strong non-magnetic controlling springs giving a full-load torque of 500 millimeter-grams are used.

The pen consists of a glass reservoir containing a week's supply of ink; into it dips a capillary tube with an iridium tip. The

tip is very hard, takes a high polish and does not corrode. The pen friction is thus reduced. The pen is carried by a jointed arm attached to the movable stem. The two members of the arm are nearly perpendicular and the pen is pressed against the paper by a small weight attached to a bell-crank lever carried by the first member; the unweighted arm of the bell crank is attached to the second member by a light cord.

FIG. 341.—Pen and ink reservoir for General Electric Co. curve-drawing meters.

Relay Instruments.—The essential features of a registering relay wattmeter for three-phase circuits are shown in Fig. 342.

The two dynamometer elements are shown at 8 and 9. The spindle carries the outer end of the flat spiral spring, 13, the inner end of which, 15, is attached to an arbor which is turned by the motor 18. The motor is controlled by a circuit through the relay points, 22, 23, 24, so that the torque on the coil is kept balanced by that of the spring. The excursion of the pen on the chart is proportional to the twist in the spring and the resulting diagram is on rectangular coördinates.

A relay voltmeter is shown in Fig. 343. The six coils are arranged as in the Kelvin balance. The power for moving the pen is obtained from the auxiliary source B by means of the solenoids P and P_1, one of which moves the pen to the right, the other to the left. The controlling spring S connects the link M and the balance arm, the latter being provided with a contact finger which plays between the contacts at D. On the passage of the current the finger is brought in contact with the lower stop, thus energizing the solenoid P and moving the pen to the right until the tension on S is sufficient to break the contact; P then becomes inactive, the linkage tends to return to the zero position

and the contact is reëstablished, the result being that the pen remains practically stationary if the voltage be constant. If the voltage falls, the upper contact and P_1 act to bring the pen to its proper position. Sparking at the contacts is eliminated by use of the condenser KK and the resistance R.

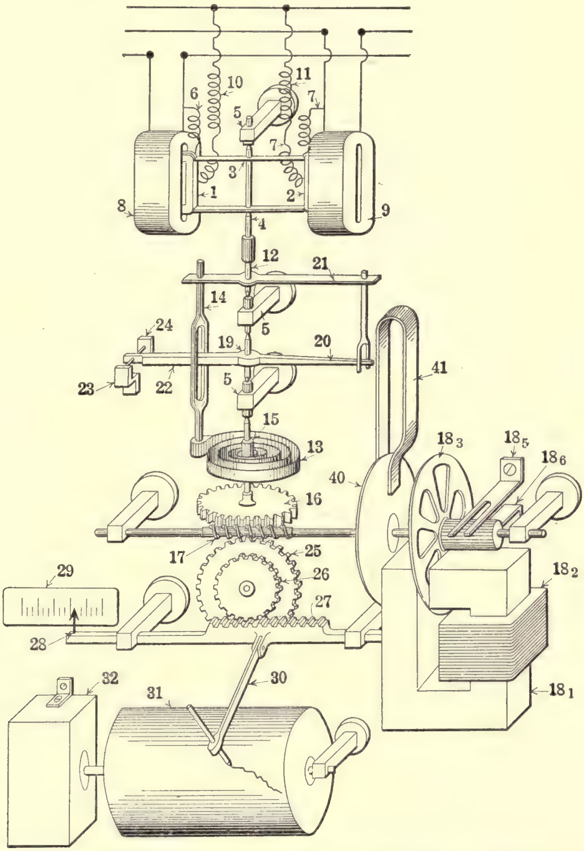
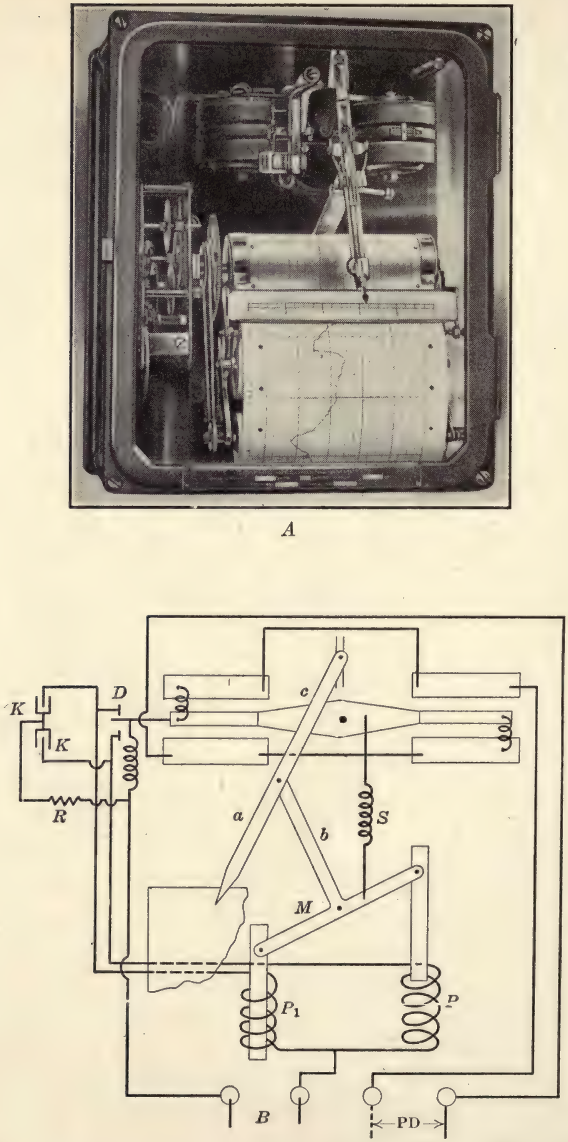


FIG. 342.—Elements of Arconi curve-drawing wattmeter.

Rectilinear motion of the pen is obtained by making the arms of the linkage, a, b and c of equal length. The upper end of c slides in a vertical slot.

Registering instruments are made for both direct- and alternating-current circuits, to measure current, voltage, power,



A

B

FIG. 343.—Relay curve-drawing voltmeter, Westinghouse Co.

frequency, and power factor. Special types have been developed for use in tests of electric cars and locomotives.

In selecting curve-drawing instruments it should be kept in mind that in many of those on the market the inertia of the moving parts is so great that rapidly varying phenomena will not be correctly depicted.

References

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"Investigating Manufacturing Operations with Graphic Meters," C. W. Drake, *Electric Journal*, vol. 7, 1910, p. 536.

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CHAPTER XII

INSTRUMENT TRANSFORMERS

In the course of development of high-voltage alternating-current systems of transmission and distribution it has been found necessary to remove the various instruments, as well as the devices used to actuate the switching gear, from direct contact with the line circuits and to operate them by means of properly constructed transformers, since direct connection between the high-tension lines and the devices on the front of the switchboard must be avoided. This method of operation through transformers reduces to a minimum the possibility of personal injury to the station attendants and enables them, especially in emergencies, to operate the apparatus with confidence, thus contributing to maintaining continuity of the service.

Again, it is frequently necessary to meter very large currents in circuits of only moderate voltage; and as it is highly desirable to avoid the expense of carrying heavy leads to the switchboard, current transformers are used.

By properly choosing the transformers, it is possible to use instruments and switchboard devices wound for 5 amperes and 110 volts, for installations of all capacities. This reduces the instrument cost and is now the accepted American practice.

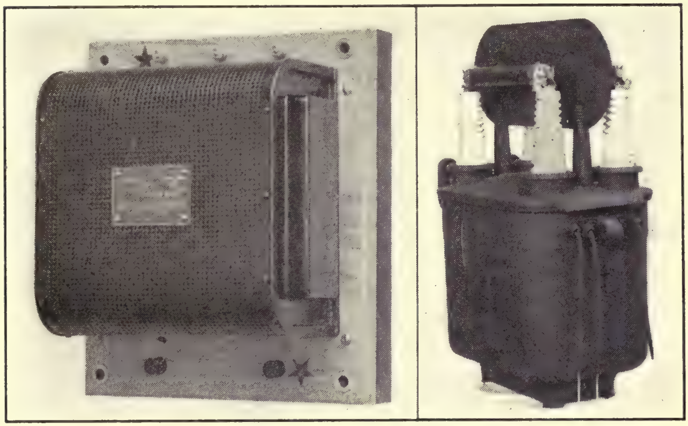
Potential Transformers.—Where it is necessary to measure a high voltage, a potential transformer is used to reduce this voltage to a more convenient and safe value for measurement. Fig. 344 shows two such transformers; they are connected in the circuit as shown in Fig. 345.

The transformer in Fig. 345 is diagrammatic only; as constructed, the primary and secondary windings are superposed.

As potential transformers are usually operated under practically fixed conditions of applied voltage, frequency, and number and character of the instruments in the secondary circuit, one would expect them to be instruments of precision, and experience

shows this to be the case. They are much more permanent than the instruments they actuate. When used for voltage measurements, only the ratio of transformation is important, and this should be constant under the varying operating conditions. The line voltage is given by

$$V = (\text{Ratio}) \times (\text{Reading of voltmeter}).$$



For 2,300-volt circuit. For 13,000-volt circuit.

FIG. 344.—Potential transformers, General Electric Co. Note stars on base of left-hand transformer indicating polarity.

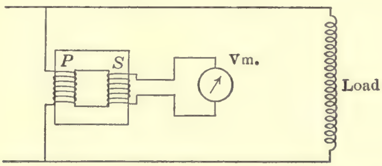


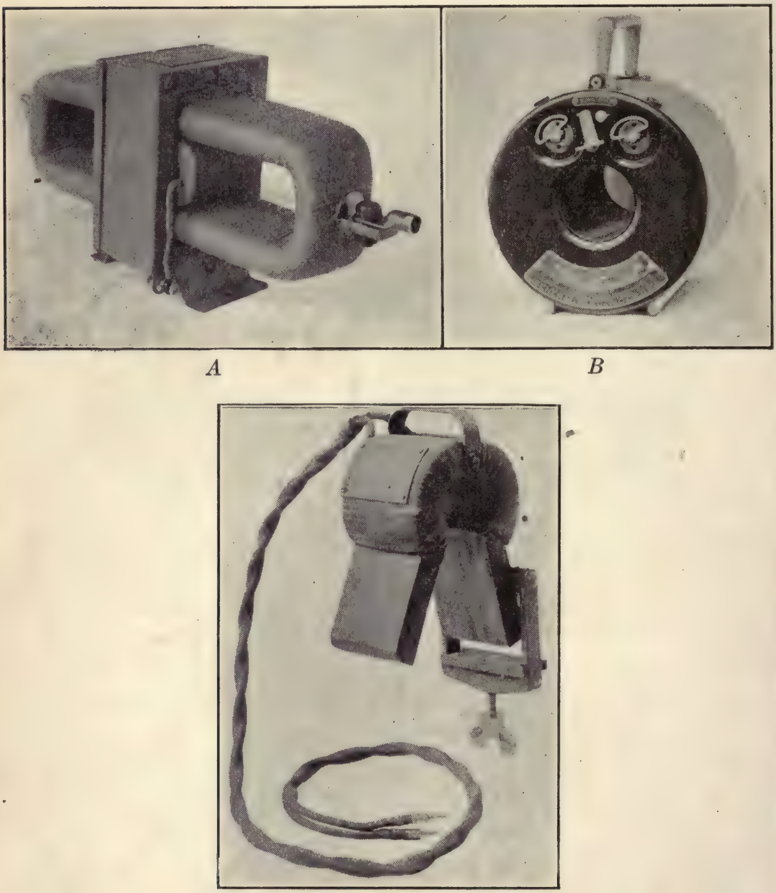
FIG. 345.—Showing manner of using potential transformer.

Switchboard voltmeters are graduated so that the line voltage is read directly from the dial. If the ratio is not constant, the combination of transformer and voltmeter may be calibrated as a unit.

Current Transformers.—The current transformer is used in cases where very large alternating currents must be measured and also where the current coils of instruments must be isolated from high-voltage lines.

Different designs of this instrument are shown in Fig. 346.

Transformer *A* is for use on installations of from 2,500 to 15,000 volts. The distance between the primary and the secondary terminals and frame is to be noted. Transformer *B* is a portable



A *B* *C*

Fig. 346.—Current transformers, General Electric Co.

instrument designed for general testing purposes. The ratio is variable, for the primary is formed by thrusting a flexible cable through the central opening, the number of primary turns being thus readily altered. Transformer *C* has its iron core made in

two parts which are hinged together so that the magnetic circuit can be opened when the screw clamp is loosened. This allows the transformer to be placed around a cable and permits the current in a single conductor cable to be measured without interrupting the service.

Fig. 347 indicates the connections for a simple current measurement. For obvious reasons this device is sometimes called a series transformer in distinction from a potential transformer, which is frequently called a shunt transformer.

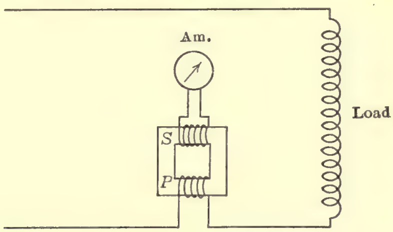


Fig. 347.—Showing manner of using current transformer.

For a current measurement,

$$I = (\text{Ratio}) \times (\text{Reading of ammeter}).$$

Convenience dictates that the ratio be a constant. This involves a difficulty, for, as the load changes, the transformer must work under widely varying conditions. Experiment shows that the ratio is not constant, being to a certain extent dependent on the strength of the current which is being measured, and also on the number and the character of the instruments in the secondary circuit.

Power Measurements.—In power measurements on high-voltage circuits, it is necessary to use both current and potential transformers. As shown in Fig. 348, the connections are such that the current and voltage as well as the power are measured.

With the connections as shown, the power, to a fair degree of approximation, is given by

$$P = (\text{Ratio of current transformer}) \times (\text{Ratio of potential transformer}) \times (\text{Reading of wattmeter}).$$

Another difficulty is here encountered. In the discussion of power measurement, it was repeatedly emphasized that for ac-

curate work, the currents in the fixed and movable coils of a dynamometer wattmeter must have the same phase relation as the current and voltage of the load.

It is one of the imperfections of instrument transformers that they introduce false phase relations. With the potential transformer the voltage at the secondary terminals is not in *exact* opposition to the voltage applied to the primary, the departure from exact opposition being small, to be sure, and of the order of magnitude of 10′ of arc under normal operating conditions.

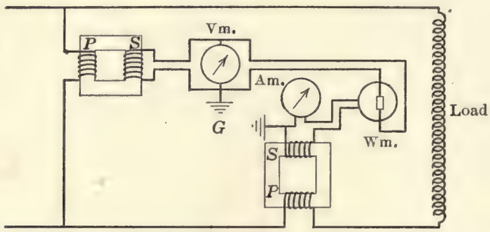


FIG. 348.—Showing connections for measuring power, voltage, and current, using instrument transformers.

This phase angle may be either a lag or a lead, and depends on the frequency as well as on the number and character of the instruments in the secondary circuit.

The current transformer is subject to a much greater phase-angle error than the potential transformer. The displacement of the secondary current from exact opposition to that in the primary may, at low loads, sometimes amount to as much as 3°. This displacement depends on the magnitude of the primary current, on the frequency and on the number and the character of the instruments in the secondary circuit.

It will be seen that the errors introduced into power measurement by the use of transformers are those due to the variation of ratio of both the current and potential transformers, as well as those due to the phase displacement in both instruments.

It is possible to determine the ratio and phase angle and to make the corresponding correction so that accurate results may be obtained even at low power factors, where the phase-angle errors are most pronounced. These matters are of great practical importance, for instrument transformers are used in con-

nection with wattmeters in all sorts of acceptance tests of alternating-current apparatus, as well as in connection with watt-hour meters on all high-capacity alternating-current circuits.

General Considerations.—In all instrument transformers, the primary must be thoroughly insulated from the secondary and from the core and case, so that there is little chance of puncturing the insulation. In addition, the secondary circuit should be grounded so that the operator is protected even though the insulation between the primary and secondary breaks down. Grounding the secondary circuit prevents errors due to accumulation of electrostatic charges on the instruments. The coils must be held in place so firmly that there is no chance of mechanical injury when short-circuits occur. The primary and secondary terminals must be so far apart that there is no liability of an arc forming between the two circuits when the line circuit is violently disturbed.

The ordinary vector diagram for a transformer is shown in Fig. 349. It is not drawn to scale, however, and gives no idea of relative numerical magnitudes.

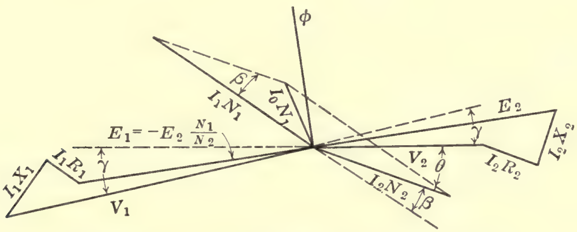


FIG. 349.—Vector diagram for transformer.

By means of the diagram, a general explanation of the phenomena occurring in instrument transformers may be obtained.

For the potential transformer, the ratio which is used is

$$\frac{V_1}{V_2}.$$

It differs in magnitude and in the phase of its components from $\frac{E_1}{E_2}$, which is the ratio of the number of turns, or the true "ratio of

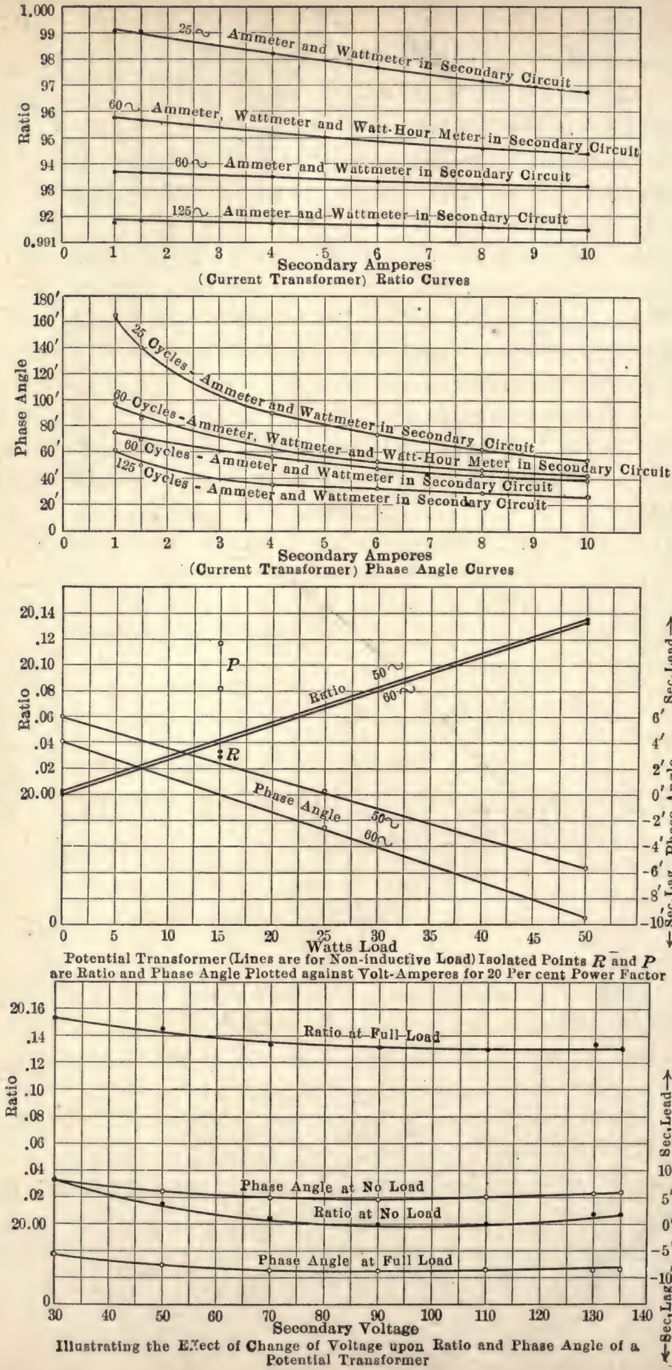


FIG. 350.—Showing characteristics of instrument transformers.

transformation." The phase angle of the potential transformer is designated on the diagram by γ. This angle is the departure from exact opposition of V_1 and V_2. It is usually very small and in reality may be either an angle of lead or of lag, according to circumstances. The phase angle of the current transformer will be denoted by β.

Fig. 350 shows the results of experimental determinations of the constants of certain commercial current and potential transformers. It gives an idea of the order of magnitude of the changes to which the ratios and phase angles are subject. Similar curves, giving representative values of ratios and phase angles, which have been determined by testing a few transformers all made in accordance with the same specifications, may be obtained from the makers of such instruments and are sufficiently accurate for much commercial work.

In tests where the greatest accuracy is desired, the ratios and phase angles for the transformers should be determined at the frequency and voltage and with the same connected burden of instruments and leads as are to be used in the subsequent work.

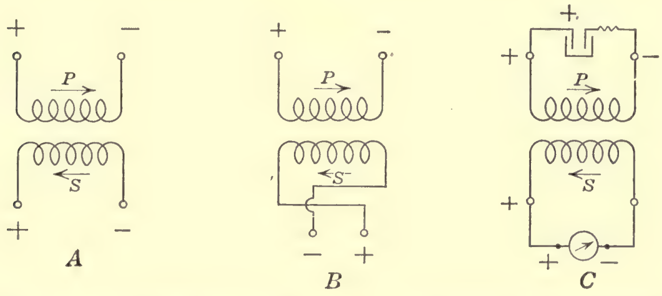


FIG. 351.—Pertaining to polarity tests of instrument transformers.

When using instrument transformers in connection with wattmeters and power-factor meters it is necessary to know the relative polarities of the secondaries of the transformers with respect to their primaries, otherwise the meters may be so connected that they will not read properly.

Some manufacturers arrange the internal connections so that the corresponding terminals of the primary and of the secondary

are always of like polarity. Thus in Fig. 351*A* the + and − signs indicate the polarities at some particular instant. The primary and secondary currents are shown diagrammatically as flowing in opposite directions.

The + on the primary is the terminal at which the current enters and the + on the secondary is the terminal by which the current leaves the transformer to enter the external circuit.

Some manufacturers cross the secondary connections giving the relative polarity shown in Fig. 351*B*.

A simple method of testing the polarity is as follows: connect a direct-current voltmeter to one of the windings (Fig. 351*C*) noting which terminal is connected to the + post of the voltmeter. Touch for an instant the terminals of a dry cell to the terminals of the other winding, making the polarity such that the meter reads up the scale on closing the circuit. The two corresponding + terminals will be the carbon of the cell and the + post of the voltmeter. A very small current should be used, otherwise the iron may be left in a highly magnetized condition and the ratio and the phase angle of the transformer altered from their normal values.

When stating the conditions under which instrument transformers are used (especially current transformers) it is highly desirable that the inductance and resistance of the external secondary circuits be specified, for in that case there can be no misunderstanding of the conditions under which the transformer is operating.

If the conditions are such that the readings of the 5-ampere wattmeters and ammeters, which are commonly used in the secondary circuits of current transformers, are in the lower parts of the scales there is a temptation to substitute lower range instruments (for instance, 3 amperes) in order to obtain a good scale reading. It must not be forgotten that such instruments will heavily tax the current transformer and alter both the ratio and the phase angle, for the burden placed on the transformer by the 3-ampere equipment is such that the voltage at the transformer terminals must be increased to approximately 2.8 times its original value.

Resistance and Inductance of the Current Coils of Typical Alternating-Current Instruments

Range	Ammeters		Wattmeters	
	Resistance	Inductance	Resistance	Inductance
3 amp.	0.37 ohm	0.00076 henry	0.36 ohm	0.00046 henry
5 amp.	0.133 ohm	0.00028 henry	0.119 ohm	0.00016 henry

Current Transformers.—It is important that when the transformer is being operated the secondary circuit always be kept closed. If it is opened, there will be no demagnetizing effect due to the secondary, and as the primary current is fixed by the load on the line, the flux will rise to a high value. This will increase the iron losses to such an extent that the insulation may be injured by the heat so that at some subsequent time it may be punctured by a moderate voltage, or perhaps burned out. Again, the voltage at the secondary terminals will be large, and the secondary insulation may be injured. There is also the possibility of disagreeable, if not fatal, shocks.

Opening the secondary circuit when the transformer is being operated may alter both the ratio and the phase angle, for the circuit opening may occur when the iron is fully magnetized. In the subsequent use of the instrument, the iron will not be put through its normal hysteresis cycle, and the exciting current will therefore be altered. At low loads, the alteration may amount to several per cent. For the same reason, direct current, used for the purpose of calibrating the instruments, must never be sent through either the primary or secondary of the transformer. Under ordinary operating conditions, these changes in the magnetic state of the core will persist, since, in instrument transformers, the magnetic circuit is unusually good. The core may be demagnetized in the usual manner, an alternating current being sent through the primary, and gradually decreased from its full-load value to zero, the secondary being open.

Owing to the necessity of having considerable insulation between the primary and the secondary coils and of having the terminals of the coils widely separated there may be a pronounced stray field in the neighborhood of current transformers. This is the case with the type shown in Fig. 346*A*. Instruments, if not

shielded, should be used at a distance of at least 6 or 8 ft. from the transformer.

Theory of the Current Transformer.—To examine the theory of the current transformer, transfer from Fig. 349 those parts of the diagram which are shown in Fig. 352.

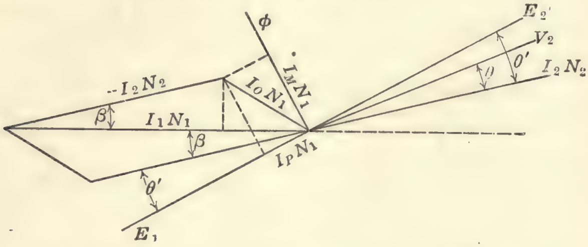


FIG. 352.—Diagram for current transformer.

Let:

N_1 and N_2 = number of turns in primary and secondary respectively
I_1 and I_2 = primary and secondary currents.
I_0 = total exciting current.
I_M = magnetizing component of exciting current.
I_P = power component of exciting current.
θ' = angle between E_2 and secondary current.
β = phase angle of transformer.

Reference to the figure will show that both the ratio and the phase angle are dependent on the exciting current, for obviously if I_0 were zero, $I_1N_1 = I_2N_2$, and β, the phase angle of the transformer or the departure of the primary and secondary currents from exact opposition, would also become zero. Such an ideal transformer can never be realized, for in any case there must be enough ampere-turns to give the requisite flux through the core, and with the iron core, the exciting current must have an energy component sufficient to account for the hysteresis and the eddy-current losses. When an iron core is used, I_0N_1 may be resolved into two components, the magnetizing component I_MN_1 along the flux, and the power component I_PN_1 along E_1.

Referring to Fig. 352 and projecting I_2N_2 and I_0N_1 on the line I_1N_1,

$$I_1N_1 = N_2I_2 \cos\beta + I_0N_1 \cos\left[90 - \theta' - \beta - \sin^{-1}\frac{I_P}{I_0}\right]$$

$$\therefore \frac{I_1}{I_2} = \frac{N_2}{N_1}\cos\beta + \frac{I_M \sin(\theta' + \beta) + I_P \cos(\theta' + \beta)}{I_2}. \qquad (a)$$

But β is a small angle; its cosine is, therefore, very nearly unity, and the second member on the right-hand side of the equation is virtually a correction term.

Hence:

$$\text{Ratio} = \frac{I_1}{I_2} = \frac{N_2}{N_1} + \frac{I_M \sin \theta' + I_P \cos \theta'}{I_2} \text{ approximately.}$$

The expression for the phase angle is determined as follows. From the diagram,

$$\tan \beta = \frac{I_0 N_1 \sin \left[90 - \theta' - \beta - \sin^{-1} \dfrac{I_P}{I_0}\right]}{I_2 N_2 \cos \beta}$$

$$= \frac{I_M N_1 \cos (\theta' + \beta) - I_P N_1 \sin (\theta' + \beta)}{I_2 N_2 \cos \beta}. \qquad (b)$$

As β is a small angle,

$$\beta = \frac{N_1}{N_2}\left[\frac{I_M \cos \theta' - I_P \sin \theta'}{I_2}\right] \text{ approximately.}$$

In the practical case I_2 leads I_1 reversed. This is important in power measurements.

The dependence of the ratio and the phase angle on the properties of the core are clearly shown in (*a*) and (*b*). Evidently both I_M and I_P should be reduced to a minimum if both the ratio and the phase angle are to be made as nearly independent of the secondary current and the character of the secondary load as possible.

In the current transformer I_0 varies with the saturation of the core, *i. e.*, with consumers' load current. To reduce I_M the core must be of high permeability and of large cross-section. I_P is rendered small by choosing for the core an iron of small hysteresis loss and working the iron at a very low flux density. The impedance of the instruments forming the load should be small so that the requisite secondary e.m.f. is furnished by a small flux.

As there is iron in the magnetic circuit the current wave form in the secondary cannot be an exact reproduction of that in the primary. But with periodic phenomena the distortion, while measurable by refined methods, is so small that it is of no practical moment even though the wave form be very complicated.

Owing to the action of the iron, however, large currents of a transient nature, such as occur in short-circuit tests of fuses.

switches, etc., and which rise to values much higher than those for which the transformer was designed are not correctly reproduced.

Fig. 353 shows the results of measurements of the magnetizing and the power components of the exciting current, I_0, made by use of a quadrature dynamometer.[13] The tests were made at 25 and 60 cycles on the current transformer to which the data

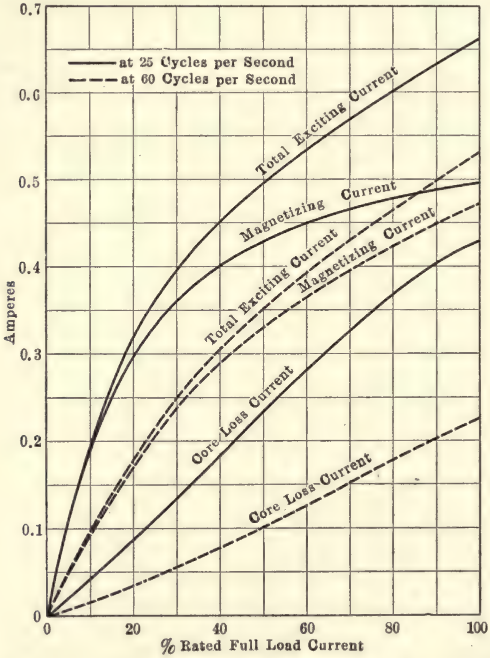


FIG. 353.—Showing components of total exciting current in current transformer.

given below apply, and Fig. 354 shows how well calculations based on these results agree with direct measurements of the ratio and phase angle.

In order to give an idea of the magnitude of the quantities involved, reference may be made to the following data which pertain to the current transformer whose characteristics are given in Figs. 353 and 354:

$$\text{Nominal ratio} = \frac{I_1}{I_2} = \frac{8}{1}.$$

$$\text{Primary turns} = N_1 = 25.$$

$$\text{Secondary turns} = N_2 = 196.$$

Rated full-load currents of primary and secondary, 40 and 5 amp., respectively.

Secondary resistance, 0.51 ohm.
Resistance of connected load, 0.17 ohm.
Inductance of connected load, 0.08 millihenry.
Max. flux density at 60 cycles, 290 gausses.
Max. flux density at 25 cycles, 700 gausses.

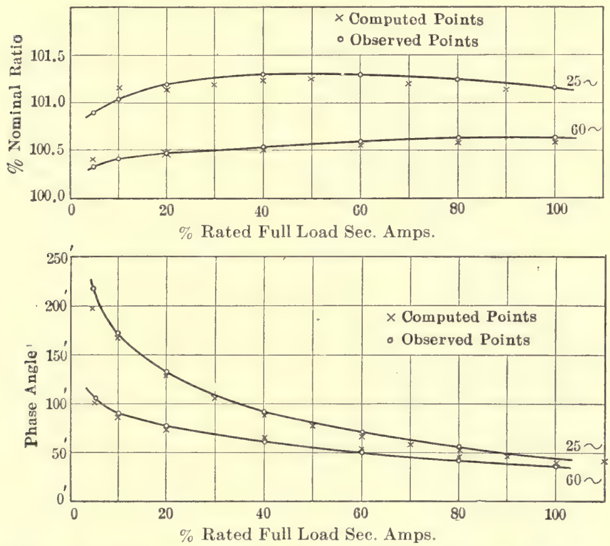


FIG. 354.—Showing agreement of computed and observed values of the ratio and phase angle of a current transformer. The form of the ratio curve is abnormal; the ratio usually decreases with increase of load.

It will be noted that in order to bring the ratio to its rated value a few secondary turns are left off. Attention is called to the low flux densities at which the core is worked.

Theory of Potential Transformer.—In the theory of the potential transformer the two most important quantities are the equivalent reactance and resistance as determined by the usual short-circuit test. The exciting current and the reactance and resistance of the primary windings must also be taken into account but their combined influence is much less than that of the equivalent impedance.

Referring to Fig. 355,

Let:

N_1 and N_2 = number of primary and secondary turns, respectively.
R and X = equivalent resistance and reactance of the transformer.
R_1 and X_1 = resistance and reactance of the primary windings.
I_0 = exciting current.
V_1 and V_2 = primary and secondary terminal voltages.
θ = power factor angle of the secondary load.
δ = angle between I_0 and V_2 reversed.
γ = phase angle of the transformer, or the angle between V_1 and V_2.

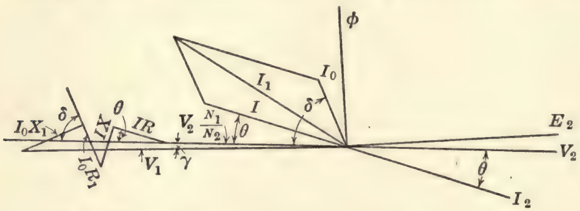


FIG. 355.—Diagram for potential transformer.

The component of the primary current which is in opposition to the secondary current is $I_2\dfrac{N_2}{N_1} = I$. To determine the ratio, projecting V_1 on V_2 and adding up the components of the projection, gives

$$V_1 \cos\gamma = V_2\frac{N_1}{N_2} + IR\cos\theta + IX\sin\theta + I_0R_1\cos\delta + I_0X_1\sin\delta.$$

$$\therefore \frac{V_1}{V_2} = \frac{1}{\cos\gamma}\left[\frac{N_1}{N_2} + \frac{IR\cos\theta + IX\sin\theta + I_0R_1\cos\delta + I_0X_1\sin\delta}{V_2}\right].$$

In potential transformers γ is usually considerably less than 1°, so $\cos\gamma = 1$ very closely, and

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} + \frac{IR\cos\theta + IX\sin\theta + I_0R_1\cos\delta + I_0X_1\sin\delta}{V_2}.$$

To determine the phase angle, projecting on a line perpendicular to V_2, gives

$$V_1\sin\gamma = IR\sin\theta - IX\cos\theta + I_0R_1\sin\delta - I_0X_1\cos\delta$$

but for small angles $\sin\gamma = \gamma$ approximately, so

$$\gamma = \frac{1}{V_1}\Big[IR\sin\theta - IX\cos\theta + I_0R_1\sin\delta - I_0X_1\cos\delta\Big].$$

In the potential transformer the small change of exciting current, I_0, between no-load and full-load produces a negligible effect on the results.

Application of Corrections for Ratio and Phase Angle.—It is customary to express the results of tests of instrument transformers in the form of curves similar to those in Fig. 350. Reference to the curves having the proper load characteristics will give the ratio and phase angle to be used in any particular tests.

When power measurements are made, besides the ratios, there are to be considered:

1. The phase displacement in the potential transformer.
2. The phase displacement in the current transformer.
3. The phase displacement in the potential coil of the wattmeter.

These phase relations may be indicated as in Fig. 356.

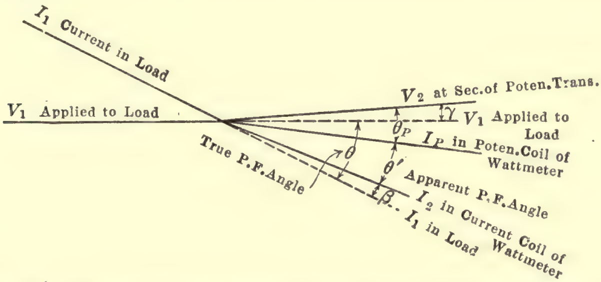


FIG. 356.—Phase diagram for power measurement using instrument transformers.

The phase angle, γ, of the potential transformer is shown as a lead, though V_2 may either lag behind or lead V_1 according to circumstances. If γ is a lag angle, its algebraic sign is to be reversed.

The wattmeter gives an indication proportional to the mean product of the current in its two coils, or to cos θ'. The apparent power factor of the load is

$$\cos \theta' = \frac{\text{Wattmeter reading}}{\text{Volt-amperes}}.$$

The true power-factor angle is

$$\theta = \theta' + \beta + \theta_P - \gamma$$

and the true power factor is

$$\cos \theta = \cos (\theta' + \beta + \theta_P - \gamma) = \frac{\text{True watts}}{\text{Volt-amperes}}.$$

$$\therefore \text{True watts} = \frac{\cos \theta}{\cos \theta'} \times (\text{wattmeter reading}) =$$

$$[1 - \tan \theta' \tan (\beta + \theta_P - \gamma)] \times (\text{Reading}), \text{approximately.}$$

The true watts must be multiplied by the appropriate transformer ratios to give the power in the circuit. Obviously, the effect of the phase angles increases as the power factor of the load decreases.

To illustrate the foregoing, consider the following data:

The losses in the instruments are neglected.
Inductive load, 25 cycles.
Reading of ammeter, corrected for calibration, 1.5 amp.
Reading of voltmeter, corrected for calibration, 110.5 volts.
Reading of wattmeter, corrected for calibration, 50.0 watts.
θ_P is negligible.
Nominal ratio of current transformer, 8 : 1.
Ratio of current transformer from its curve, 1.0125×8.
Phase angle β, from current transformer curve, 1°.79.
Nominal ratio of potential transformer, 10 : 1.
Ratio of potential transformer from its curve, 0.995×10.
Phase angle γ from potential transformer curve, 0°.12.
V_2 lags behind $-V_1$.

Apparent power factor $= \cos \theta' = \dfrac{50}{110.5 \times 1.5} = 0.3016.$

Apparent power-factor angle, 72°.45.

True power-factor angle $= \theta = 72°.45 + 1°.79 + 0°.12 = 74°.36.$

True power factor $= \cos \theta = 0.2696.$

True value of the load $= \dfrac{0.2696}{0.3016} \times 50 \times 8.1 \times 9.95 = 3{,}602$ watts,

which is the corrected reading of the wattmeter multiplied by the proper transformer ratios.

Effect of Phase Angles in Three-phase Power Measurements.—The case which will be considered is that of a balanced three-phase load where the power is measured by the two-wattmeter method using transformers of the same characteristics in both phases. When the power factor of the load is low the wattmeter, in the coils of which the currents are the more nearly in phase, indicates the larger part of the load. The reading of the other wattmeter, which works under much more adverse conditions as to the phase displacement of the currents in its coils, is small so that even if the percentage error in its readings be large, the percentage error introduced by it into the measurement of the power will be small.

It has just been shown that the power in a single-phase circuit is given by

$$P = \frac{\cos\theta}{\cos\theta'}\cdot(\text{Reading}).$$

Assume that the transformer ratios are 1:1, then in the two-wattmeter method the power which *should* be indicated by the two instruments is

$$P_1 = VI\cos(30° + \theta).$$
$$P_2 = VI\cos(30° - \theta).$$

So

$$P = VI[\cos(30° + \theta) + \cos(30° - \theta)] = VI\sqrt{3}\cos\theta.$$

The effect of the phase angles of the transformers and that of the potential circuit of the wattmeter is to reduce the phase difference of the currents in the fixed and movable coils of the wattmeters by the angle $\beta + \theta_P - \gamma$; the readings become:

$$(\text{Reading})_1 = VI\cos[30° + \theta - \beta - \theta_P + \gamma] = VI\cos[30° + \theta'].$$
$$(\text{Reading})_2 = VI\cos[30° - \theta + \beta + \theta_P - \gamma] = VI\cos[30° - \theta'].$$
$$(\text{Reading of meters}) = VI\sqrt{3}\cos\theta'.$$

So $P = (\text{Reading})\,\dfrac{\cos\theta}{\cos\theta'}$.

That is, the fractional error due to the phase angles is the same as that occurring in a single-phase measurement at the same power factor.

If the load is not balanced the readings of each instrument should be corrected as in a single-phase measurement.

Use of Transformers with Watt-hour Meters.—It is customary to use instrument transformers in connection with induction watt-hour meters. In this case, especially at low power factors, an additional complication is introduced, for both the phase angles of the transformers and the adjustments of the phase relations of the fluxes in the potential circuits of the meters affect the measurements.

To be ideally perfect an induction meter, when used with a current transformer, would have to be lagged so that the time-phase angle between the potential coil flux and the current in the secondary of the current transformer plus the power-factor angle of the load would be 90°. This suggests that the watt-hour meter and the transformers be treated as a unit when the

lag adjustment is made. However, a perfect adjustment is not possible, for both the ratios and the phase angles vary with the load.

In careful industrial tests, the combination of watt-hour meter and instrument transformers may be calibrated in the laboratory without undue expenditure of power by using fictitious loads (see page 500). The test conditions may then be reproduced, *wave form excepted.*

An attempt is sometimes made to correct for the variation of the ratio of the current transformer, which is most troublesome at light loads, by altering the light-load adjustment of the meter. If the ratio increases at the light loads the adjustment is set so that the meter runs a little fast, creeping being avoided. There is no means of making even an approximate adjustment for the variation of the phase angle.

DETERMINATION OF THE RATIOS AND PHASE ANGLES OF INSTRUMENT TRANSFORMERS

From what has preceded it is evident that if accurate measurements are to be made, it is necessary to know both the ratios and the phase angles of the transformers. Among the various methods which have been devised for their determination, a few of those based on the potentiometer principle have become generally accepted as being the best. They give results of high accuracy and are convenient because they do not require currents and voltages to be held at fixed values.

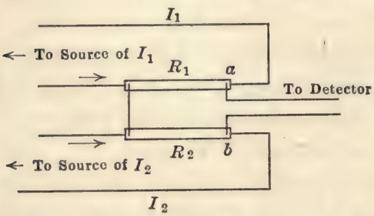


FIG. 357.—Connections for determining the ratio of two direct currents.

Ratio and Phase Angle of Current Transformer.—The determination of the ratio of a current transformer is simply the determination of the ratio of two currents. To make such a measurement using direct currents the connections shown in Fig. 357 are used.

If the detector stands at zero,

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}.$$

When two alternating currents are to be compared, the resistances R_1 and R_2 should be non-inductive; I_1 and I_2 must have the same frequency and wave form and either be in time phase or have a fixed phase difference.

In applying this general method to the testing of current transformers, I_1 and I_2 are the currents in the primary and secondary circuits; they will ordinarily be in phase to within 2° or less. As this time-phase difference exists, it is impossible by any adjustment of the resistances R_1 and R_2 to balance the two IR drops. In order to bring the detector to zero, unless it be separately excited, it is convenient to inject into the detector circuit an e.m.f. in quadrature with I_2. This may be done by the method

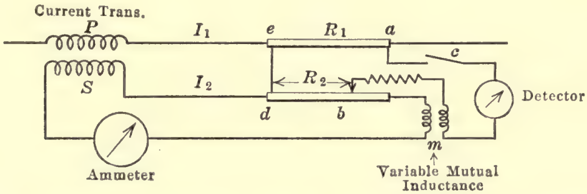


FIG. 358.—Connection for determining characteristics of current transformers.

used by Hughes and by Heaviside in their inductance bridges, by employing a variable mutual inductance, or air-core transformer, m, see Fig. 358.

The arrangement of apparatus shown diagrammatically in Fig. 358 has been used by a number of experimenters, the chief differences being in the form and manner of using the detector.

Sharp and Crawford use a D'Arsonval galvanometer, the current being rectified by a synchronous reversing key driven by a synchronous motor. The brushes or their equivalent are so mounted that the time phase of the commutation may be altered, for it must be matched with the time phase of the potential difference between a and b.

Agnew and Silsbee use a vibration galvanometer of special design.

Let the circuits be as in Fig. 359.

Then by a double adjustment of R_2 and m the vibration galvanometer may be brought to zero.

Using the mesh currents as indicated,

$$I_G(Z_1 + Z_G + Z_2) - XZ_1 - YZ_2 + j\omega mY = 0$$

$$I_G = \frac{XZ_1 + YZ_2 - jm\omega Y}{Z_1 + Z_G + Z_2}.$$

At balance $I_G = 0$ or

$$XZ_1 + YZ_2 = + j\omega mY.$$

In this case $X = -I_1$, $Y = I_2$,
also

$$Z_1 = R_1 + j\omega L_1$$
$$Z_2 = R_2 + j\omega L_2.$$

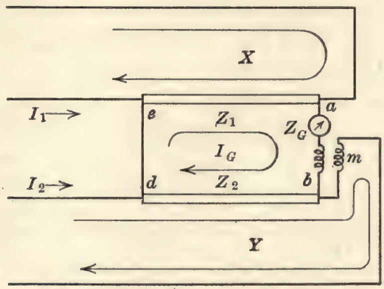


FIG. 359.—Mesh diagram for Fig. 358, pertaining to determination of characteristics of current transformers.

Substituting,

$$I_1 = \left[\frac{R_2 + j\omega(L_2 - m)}{R_1 + j\omega L_1}\right] I_2 = \left[\frac{R_1R_2 + \omega^2 L_1(L_2 - m)}{R_1^2 + \omega^2 L_1^2}\right] I_2 - j\omega\left[\frac{L_1R_2 - R_1(L_2 - m)}{R_1^2 + \omega^2 L_1^2}\right] I_2.$$

The value of I_1^2 in terms of I_2^2 and the constants of the circuit is

$$I_1^2 = \left[\frac{R_1^2R_2^2 + \omega^2L_1^2R_2^2}{(R_1^2 + \omega^2L_1^2)^2} + \frac{\omega^2R_1^2(L_2 - m)^2 + \omega^4L_1^2(L_2 - m)^2}{(R_1^2 + \omega^2L_1^2)^2}\right] I_2^2.$$

Ratio =

$$\frac{I_1}{I_2} = \sqrt{\frac{R_2^2 + \omega^2(L_2 - m)^2}{R_1^2 + \omega^2L_1^2}} = \frac{R_2}{R_1}\sqrt{\frac{1 + \dfrac{\omega^2(L_2 - m)^2}{R_2^2}}{1 + \dfrac{\omega^2L_1^2}{R_1^2}}}$$

$$\therefore \frac{I_1}{I_2} = \frac{R_2}{R_1} \text{ approximately.}$$

$$I_2 \text{ leads } I_1 \text{ by } \tan^{-1}\frac{\omega L_1R_2 - \omega R_1(L_2 - m)}{R_1R_2 + \omega^2L_1(L_2 - m)}.$$

Naturally the inductances L_1 and L_2 are made as small as possible; if they could be entirely neglected,

$$\tan\ \beta = \frac{\omega m}{R_2}.$$

The same results may be more simply derived by use of a vector diagram, Fig. 360. Take I_2 along the horizontal axis, and assume that the condition of balance has been attained.

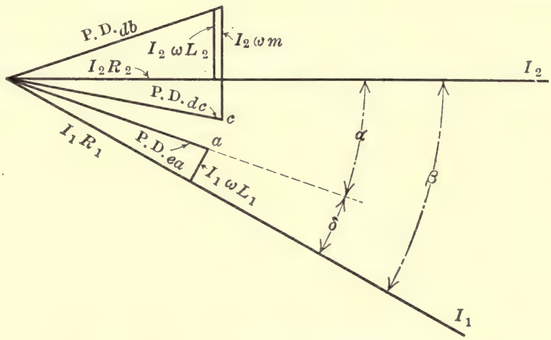


FIG. 360.—Vector diagram for Fig. 358.

At balance the P.D.$_{ea}$ must equal the P.D.$_{dc}$ (see Fig. 358); that is, the points a and c in Fig. 360 must coincide; so

$$I_2{}^2R_2{}^2 + I_2{}^2\omega^2(L_2 - m)^2 = I_1{}^2R_1{}^2 + I_1{}^2\omega^2L_1{}^2$$
$$\therefore \frac{I_1}{I_2} = \sqrt{\frac{R_2{}^2 + \omega^2\,(L_2 - m)^2}{R_1{}^2 + \omega^2L_1{}^2}}.$$

When a and c coincide,

$$\tan\ \beta = \tan\ (\alpha + \delta).$$
$$\tan\ \alpha = -\frac{\omega\,(L_2 - m)}{R_2}.$$
$$\tan\ \delta = \frac{L_1\omega}{R_1}.$$
$$\tan\ \beta = \frac{-\dfrac{\omega\,(L_2 - m)}{R_2} + \dfrac{L_1\omega}{R_1}}{1 + \dfrac{\omega^2\,(L_2 - m)\,L_1}{R_1R_2}} = \frac{\omega L_1R_2 - \omega R_1\,(L_2 - m)}{R_1R_2 + \omega^2L_1\,(L_2 - m)}.$$

With good transformers the quantity m is small; both R_1 and R_2 are non-inductive, so called. Though reduced to a minimum, L_1 and L_2 may be of importance when phase angles are determined.

A variable mutual inductance designed for this measurement is shown in Fig. 205, page 346.

The arrangement shown in Fig. 358 is an alternating current potentiometer of limited range. The underlying idea may be developed so that an instrument generally applicable to measurements with sinusoidal currents results.

Ratio and Phase Angle of Potential Transformers.—The ratio of a potential transformer for moderate voltages may be determined by voltmeter measurements using two similar instruments, one of them being supplied with the proper multiplier, so that the readings of the two voltmeters do not differ greatly. In this case uncalibrated instruments may be used.

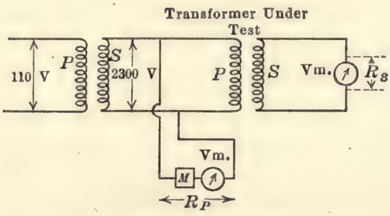


FIG. 361.—Pertaining to determination of ratios of potential transformers by voltmeters.

It is frequently convenient to use a second potential transformer, as shown in Fig. 361, to raise the ordinary supply voltage to that required for the test. The *total* resistances of the voltmeter circuits are R_P and R_S.

Two sets of readings are made, the first with the connections as shown; call the readings, in volts, of the primary and secondary instruments, D_P and D_S.

A second set of readings is made after putting the two voltmeters in series and bringing the indications up to D_P and D_S, as nearly as is practicable. If the multiplier is large, this is accomplished sufficiently well by placing the secondary voltmeter in the primary circuit, in series with the other instrument and its multiplier. During the second set of readings, the same current flows through both instruments. Call the readings, in volts, D'_P and D'_S and the current I; then $\frac{I}{D'_P}$ and $\frac{I}{D'_S}$ are the currents per scale unit of the two instruments.

In the first case,

$$V_1 = \frac{I}{D'_P} D_P R_P$$

and

$$V_2 = \frac{I}{D'_S} D_S R_S.$$

So

$$\frac{V_1}{V_2} = \frac{D'_S D_P R_P}{D'_P D_S R_S}.$$

With a slight modification, Poggendorf's method of comparing an e.m.f. and a P.D. may be applied to the determination of the voltage ratios and phase angles of potential transformers.

For comparing two direct potentials the connections shown in Fig. 362 may be used.

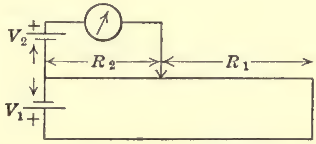


Fig. 362.—Connections for determining the ratio of a direct current P. D. to a direct current e.m.f.

If the detector stands at zero,

$$\frac{V_1}{V_2} = \frac{R_1 + R_2}{R_2}.$$

In testing potential transformers, V_1 and V_2 are replaced by the primary and secondary terminal voltages, and either R_1 or R_2 must contain an adjustable reactor by which the P.D. across R_2 may be brought into time phase with V_2. This is necessary on account of the phase angle of the transformer. If the adjustable reactor were not used, the detector could be brought to a minimum but not to zero.

Except as mentioned, the resistances should be non-reactive, that is, free from both inductance and capacity effects. With very high voltages it may be necessary to use shielded resistances[21] to avoid the last. Either R_1 or R_2 must be adjustable. The detector may well be a vibration galvanometer.

The connections for the potential transformer test are shown in Fig. 363.

To find the condition of balance, the mesh equations are

$$X(Z_1 + Z_2) - YZ_2 + V_1 = 0$$
$$Y(Z_G + Z_2) - XZ_2 - V_2 = 0.$$

The galvanometer current is

$$Y = \frac{V_2(Z_1 + Z_2) - V_1Z_2}{Z_G(Z_1 + Z_2) + Z_1Z_2}.$$

In this, as in other similar cases involving networks, the equation for Y might have been written at once from the solution for the direct-current case, simply replacing the resistances by impedances in the vector notation.

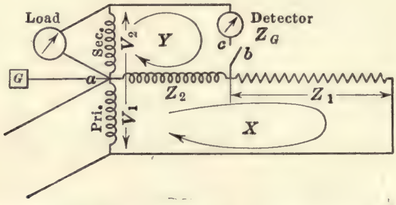


FIG. 363.—Connections for determining characteristics of potential transformers.

For balance, $I_G = Y = 0$ and

$$V_1 = \frac{(Z_1 + Z_2)}{Z_2}V_2.$$

Let Z_1 be without reactance, then,

$$Z_1 = R_1,$$

Z_2 must then be reactive, so

$$Z_2 = R_2 + j\omega L_2$$

$$V_1 = \left[\frac{(R_1 + R_2) + j\omega L_2}{R_2 + j\omega L_2}\right]V_2.$$

$$V_1 = \left[\frac{(R_1 + R_2)(R_2 - j\omega L_2)}{R_2^2 + \omega^2L_2^2}\right]V_2 + j\left[\frac{(\omega L_2)(R_2 - j\omega L_2)}{R_2^2 + \omega^2L_2^2}\right]V_2.$$

The value of V_1^2 in terms of V_2^2 and the constants of the circuit is

$$V_1^2 = \left[\frac{(R_1+R_2)^2R_2^2 + (R_1 + R_2)^2\omega^2L_2^2}{(R_2^2 + \omega_2L_2^2)^2} + \frac{R_2^2\omega^2L_2^2 + \omega^4L_2^4}{(R_2^2 + \omega^2L_2^2)^2}\right]V_2^2.$$

$$\text{Ratio} = \frac{V_1}{V_2} = \sqrt{\frac{(R_1 + R_2)^2 + \omega^2 L_2{}^2}{R_2{}^2 + \omega^2 L_2{}^2}} =$$

$$\frac{R_1 + R_2}{R_2}\sqrt{\frac{1 + \dfrac{\omega^2 L_2{}^2}{(R_1 + R_2)^2}}{1 + \dfrac{\omega^2 L_2{}^2}{R_2{}^2}}} =$$

$$\frac{R_1 + R_2}{R_2}, \text{ approximately.}$$

$$V_2 \text{ leads } V_1, \text{ and } \tan\gamma = \frac{\omega L_2 R_1}{R_2(R_1 + R_2) + \omega^2 L_2{}^2}.$$

As the phase angle may be either positive or negative, it is necessary to be able to give Z_2 either a positive or negative reactance. This may be accomplished if it is made up as shown in Fig. 364. The condenser C_2 is shunted by a variable non-inductive resistance r_2; L_2 is an inductance,

$$Z_2 = \left[R_2 - r_2 + \frac{r_2}{1 + \omega^2 C_2{}^2 r_2{}^2}\right] + j\omega\left[L_2 - \frac{C_2 r_2{}^2}{1 + \omega^2 C_2{}^2 r_2{}^2}\right].$$

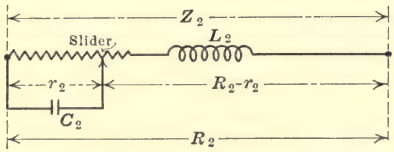


FIG. 364.—Arrangement for obtaining positive and negative reactances in potential transformer tests.

If $\omega^2 C_2{}^2 r_2{}^2$ is negligible compared to unity,

$$Z_2 = R_2 + j\omega\,[L_2 - C_2 r_2{}^2].$$

That is, the circuit acts as if it had a resistance R_2 and an inductance $(L_2 - C_2 r_2{}^2)$. The resistance r_2 may be varied by the slider. The inductance L_2 may have a fixed value; if so, it can be placed at a distance from the vibration galvanometer and other apparatus, thus avoiding trouble from stray fields.

The vector solution for the ratio and phase angle, when the connections shown in Fig. 363 are used, is obtained as follows.

Referring to Fig. 365, the direction and magnitude of the line ab which represents the drop across Z_2 may be controlled by adjusting R_2 and L_2, and the point b may be made to coincide

with c. When this has been done, that is, at balance, if L_2 is the effective inductance of Z_2,

$$V_2^2 = I^2(R_2^2 + \omega^2 L_2^2)$$

and

$$V_1^2 = I^2(R_1 + R_2)^2 + I^2\omega^2 L_2^2.$$

$$\frac{V_1}{V_2} = \sqrt{\frac{(R_1 + R_2)^2 + \omega^2 L_2^2}{R_2^2 + \omega^2 L_2^2}} = \frac{R_1 + R_2}{R_2} \text{ approximately.}$$

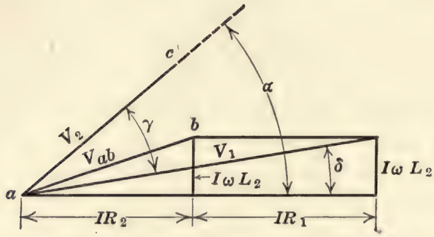


FIG. 365.—Vector diagram for Fig. 363.

To obtain the phase angle,

$$\gamma = \alpha - \delta.$$

At balance, $\tan \alpha = \dfrac{L_2\omega}{R_2}$ and $\tan \delta = \dfrac{L_2\omega}{R_1 + R_2}$.

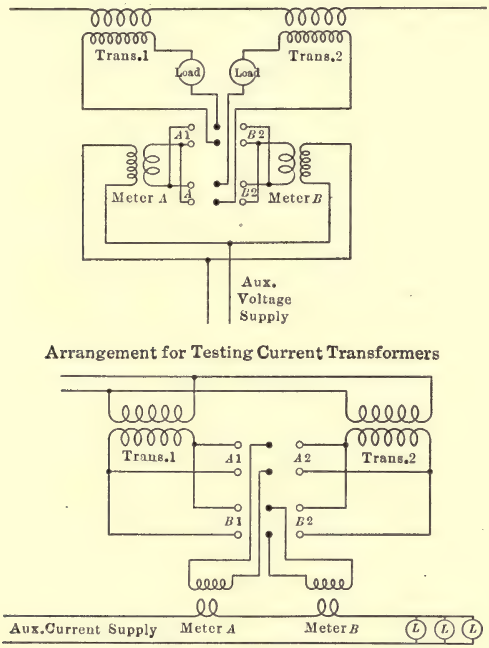
V_2 leads V_1 by

$$\tan^{-1} \frac{\dfrac{L_2\omega}{R_2} - \dfrac{L_2\omega}{R_1 + R_2}}{1 + \dfrac{L_2^2\omega^2}{R_2(R_1 + R_2)}} = \tan^{-1} \frac{\omega L_2 R_1}{R_2(R_1 + R_2) + L_2^2\omega^2}.$$

Comparison Tests of Instrument Transformers.[11]—Current and potential transformers may be compared with standard transformers of the same rating whose constants are known, by a method developed by Agnew.

When dealing with current transformers the two primaries are connected in series, while the two secondaries are connected to the current coils of two induction watt-hour meters whose potential coils are excited from a common source. The phase difference between the P.D. applied to the meters and the current in the primaries of the transformers should be adjustable. Fig. 366 shows the connections for testing both current and potential transformers by this method. If the applied voltage and current were in phase and the meters were perfectly accurate, the relative value

of the transformer ratios would be found by taking the ratio of the numbers of revolutions made by the two watt-hour meters in the same time. If the power factor were other than unity, the ratio of the revolutions would be affected by an amount dependent on the difference in the phase angles of the transformers.



Arrangement for Testing Current Transformers

Arrangement for Testing Voltage Transformers

FIG. 366.—Connections for Agnew method of comparing instrument transformers by use of watt-hour meters.

To eliminate differences in the calibration of the meters two runs are made, the meters being interchanged.

Let θ = power factor angle or the phase difference of the current (or voltage) in the primary of the transformer and the auxiliary voltage (or current) applied to the watt-hour meter. θ is taken + for lagging current.

m_A and m_B = the rates of meters A and B respectively, that is, the values of the ratio $\dfrac{\text{Watt-hours registered}}{\text{True watt-hours}}$.

K_h = watt-hour constant of the meters as marked on the discs.

$(N_A)_1$ and $(N_A)_2$ = number of revolutions made by meter A when connected to transformers 1 and 2, respectively.

$(N_B)_1$ and $(N_B)_2$ = number of revolutions made by meter B when connected to transformers 1 and 2, respectively.

β_1 and β_2 = phase angles of the transformers 1 and 2, respectively, taken + when the current or voltage in the secondary lags the primary current or voltage reversed.

R_1 and R_2 = ratios of the transformers 1 and 2.

Suppose that meter A is connected to transformer No. 1, and that the phase of the auxiliary voltage is adjusted so that θ is large. The watt-hours registered on the meter dials are given by $(K_h)(N_A)$ which, when corrected for the rate of the meter, is $(K_h)(N_A)\left(\frac{1}{m_A}\right)\cdot$ As the meter is operated through a transformer of ratio R_1., this quantity must be multiplied by R_1. giving $(K_h)(N_A)_1(R_1)\frac{1}{m_A}\cdot$ To obtain the true watt hours by meter A this result must be multiplied by $\frac{\cos\theta}{\cos\theta'} = \frac{\cos\theta}{\cos(\theta+\beta)}\cdot$ (see page 577). Therefore from meter A,

$$\text{corrected watt-hours} = (K_h)(N_A)_1(R_1)\frac{1}{m_A}\frac{\cos\theta}{\cos\ (\theta+\beta_1)}\cdot$$

Similarly for the meter B connected to transformer 2,

$$\text{corrected watt-hours} = (K_h)(N_B)_2(R_2)\frac{1}{m_B}\frac{\cos\theta}{\cos\ (\theta+\beta_2)}$$

$$\therefore (N_A)_1(R_1)\frac{1}{m_A}\frac{\cos\theta}{\cos\ (\theta+\beta_1)} = (N_B)_2(R_2)\frac{1}{m_B}\frac{\cos\theta}{\cos\ (\theta+\beta_2)}, \quad (1)$$

and when the meters are interchanged,

$$(N_A)_2(R_2)\frac{1}{m_A}\frac{\cos\theta}{\cos\ (\theta+\beta_2)} = (N_B)_1(R_1)\frac{1}{m_B}\frac{\cos\theta}{\cos\ (\theta+\beta_1)}\cdot \quad (2)$$

If the test is made at unity power factor, $\theta = 0$, and since β is a small angle,

$$(N_A)_1(R_1)m_B = (N_B)_2(R_2)m_A \quad (1a)$$

$$(N_A)_2(R_2)m_B = (N_B)_1(R_1)m_A \quad (2a)$$

$$\therefore \frac{R_1}{R_2} = \sqrt{\frac{(N_B)_2(N_A)_2}{(N_B)_1(N_A)_1}}.$$

To determine the difference of the phase angles of the transformers, a test is made at low power factor.

Using the relation

$$\frac{\cos\,\theta}{\cos\,(\theta+\beta)} = \frac{1}{\cos\,\beta\,(1-\tan\,\theta\,\tan\,\beta)}$$

and remembering that β is a small angle whose cosine is very nearly unity, 1 and 2 give

$$\frac{(N_A)_2(N_B)_2}{(N_A)_1(N_B)_1}\left(\frac{R_2}{R_1}\right)^2 = \frac{1-2\tan\,\theta\,\tan\,\beta_2}{1-2\tan\,\theta\,\tan\,\beta_1} =$$

$$1+2\tan\,\theta\,(\tan\,\beta_1-\tan\,\beta_2)\text{ approximately.}$$

$$\therefore \tan\,\beta_2-\tan\,\beta_1 = \frac{1}{2\tan\,\theta}\left[1-\frac{(N_A)_2(N_B)_2}{(N_A)_1(N_B)_1}\left(\frac{R_2}{R_1}\right)^2\right].$$

Rotary standard watt-hour meters are convenient for making the tests, for the revolutions may be accurately read from the dials.

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CHAPTER XIII

THE CALIBRATION OF INSTRUMENTS

This chapter deals chiefly with indicating electrical instruments intended for use in engineering work.

Accuracy and Precision.—Every experimenter must form his own estimate of the accuracy, or approach to the absolute truth, obtained by the use of his instruments and processes of measurement. He must remember that a high precision, or agreement of the results among themselves, is no indication that the quantity under measurement has been accurately determined. For instance, under favorable conditions one may be certain of the *reading* of a voltmeter (at about 100 volts) to $\frac{1}{10}$ of 1 per cent. At the same time the instrument may be several per cent. in error, and the true value of the measured P.D. will remain unknown until the instrument has been calibrated.

Generally speaking, in any piece of experimental work one should aim at as high a degree of accuracy in the final result as can be attained without undue labor and expense. This conduces to discipline among the observers, as it discourages slipshod methods of observation. It also aids in the collection of reliable engineering data for use on other occasions. But it is to be remembered that the expense and the labor increase very rapidly as the required degree of accuracy is raised. Consequently the determination of what the required degree of accuracy shall be, becomes an economic problem.

Clear ideas concerning the distinction between accuracy and precision are especially important to those beginning experimental work.

During the progress of many tests the obvious thing, and therefore the factor on which the beginner's attention is likely to be fixed, is the uncertainty in determining the best representative values of the readings of his instruments, for frequently the circuit conditions are fluctuating so that close attention in reading

and the averaging of many observations are necessary. After the experimental work has begun these things may distract the beginner's attention from insidious constant errors and errors of method so that these are scarcely thought of though they may be of vastly greater importance than the errors of reading. For this reason it is necessary to study carefully any proposed test or method of measurement in order that, as far as possible, all constant errors and errors of method may be eliminated before the experimental work is begun.

Another result of this preliminary study should be a proper division of the labor among the various component measurements. Generally there are some measurements whose effects on the final result are small, and therefore labor expended in making numerous readings is wasted. This, however, does not imply that when they are taken, the readings giving these less important components may be made in a slovenly manner.

In an investigation, the preliminary study is frequently a very difficult part of the work, involving as it does a careful analysis of the workings of the apparatus, a wide range of theoretical knowledge and an accurate understanding of the behavior and sources of error in many different kinds of instruments.

In a complicated experimental investigation, to complete the elimination of constant errors and errors of method, a second determination of the quantity under measurement should, if possible, be made by *an independent method*, using other apparatus.

The beginner must keep in mind that it is scarcely possible to measure any of the electrical magnitudes as he finds them in engineering practice, without changing the conditions of the circuit in which the measurement is made and thus altering the very thing which is to be determined. An ammeter when introduced into a circuit alters the current, and an electromagnetic voltmeter alters the potential difference to which it is applied. In the vast majority of cases, these effects are negligible, but the possibilities of error due to the alterations of circuit conditions must not be lost sight of.

Calibration Before and After Tests.—In making careful acceptance tests of electrical machinery, the indicating instruments should be calibrated before the test and a check calibration made at the conclusion of the work. This allows the various runs to

be worked up while the test is in progress, which is highly desirable, as it insures that all necessary data are being taken and that the procedure of the observers is correct. The check calibration eliminates questions as to the accuracy of the instruments and as to whether or not they have been tampered with or injured in any way.

Choice of Instruments.—In selecting the instruments for a particular piece of work those should be chosen which will give good deflections; that is, deflections in a favorable part of the scale, and of such a magnitude that the required *precision of reading* is readily attained. The choice, therefore, involves a preliminary study of the conditions of the test in order to determine approximately the magnitudes of the quantities involved. It must then be decided whether or not the desired, and obtainable, degree of accuracy in the final result is such that careful calibrations are necessary.

Often one knows from previous experience that his instruments are correct to within 1 or 2 per cent, and instances are continually arising where the conditions are such that an accuracy of 2 or 3 per cent is sufficient. In such cases, where differences of nearly equal quantities are not involved, there is no point in calibrating the instruments to 0.2 per cent, for example. Again, there are many cases where the magnitudes to be determined cannot be estimated *a priori*, and a certain amount of rough preliminary work is necessary to determine the most advantageous ranges of the instruments. In such a case the calibrations should be deferred until the proper instruments have been selected.

Attention to these simple matters may save the beginner much valuable time, and may possibly prevent his arriving on the ground for a test without the proper equipment.

Sources of Error in Instruments

The various sources of error which have been referred to in discussing particular instruments will be recapitulated.

Errors of Reading.—In general, the construction of the pointer and the graduation of the scale should be such that under steady conditions the position of the pointer may be read, by estimation, to one-tenth of a scale division. This is readily attained

in direct-current instruments of the moving-coil type and in wattmeters over the larger part of the scale. Alternating-current ammeters and voltmeters have scales on which the graduations are crowded together at the lower end and possibly the upper end also, so this precision of reading may be obtained between 25 per cent and about 90 per cent of the full-scale reading.

Under commercial conditions, one must expect irregular fluctuations in the readings. If the fluctuations are not too great, the readings may be averaged mentally, but it is generally best to record a series of readings taken at regular intervals and calculate the average. In industrial testing, it is surprising how closely the averages from various runs made under the same general conditions will check one another, even though there are great momentary fluctuations.

In selecting instruments for industrial testing, attention must be given to the damping (often very defective in alternating-current instruments), otherwise the swinging of the pointer in its own natural period will be superposed on the deflection due to the load, and will render it quite impossible to obtain the true reading. If several instruments have to be read simultaneously, their times of vibration and damping should be such that they will keep step as the load varies.

MECHANICAL ERRORS

Friction.—The effects of friction at the jewels and pivots should be reduced to a minimum. This means that the construction must be of the best and that the ratio of torque to total weight of the moving parts must be high. Various writers assign values ranging from $\frac{1}{20}$ to $\frac{1}{6}$ for the ratio,

$$\frac{\text{Torque in gram-centimeters, at full-scale deflection}}{\text{Weight of moving element in grams}}.$$

Before taking any instrument from the laboratory for use on a test, one should satisfy himself as to the pivot friction and freedom of motion of its movable element by putting it in circuit and *slowly* carrying the reading over the whole scale, stopping at several points. If there is undue friction, it will be made

evident by a sudden change of reading when the instrument is tapped. Excessive friction may be due to a cracked jewel, or to other injury, due to dropping the instrument. Again, the freedom of motion of the movable system may be impeded by the buckling of the paper scale, due to dampness, or to projecting fibers of the paper, which cause the pointer to stick. Air dampers, which have very small clearances may also give trouble by getting out of adjustment and lightly dragging on the damping box. In direct-current moving-coil instruments, trouble may be due to dust—possibly magnetic, in which case it is hard to dislodge—in the air gap.

Springs.—The assumption that the deflections of all sorts of instrument springs are always proportional to the deflecting moment is not tenable. The exact fulfilment of Hooke's law depends on the shape of the spring and on the method of mounting. In deflectional instruments the peculiarities of the springs are taken care of in the initial calibration and introduce no trouble unless the spring is subsequently deformed in some manner; but when an equally divided scale and a torsion head are used, as in the Siemen's dynamometer, the instrument must be tested at several points and a calibration curve drawn.

Zero Shift.—This is due to a gradual yielding of the spring when the instrument is kept at a large deflection for a considerable time, an hour or several hours. On breaking the circuit the pointer does not return at once to its original zero position, but will gradually assume it.

The magnitude of the zero shift depends on the design and material of the spring and on the nearness with which the elastic limit is approached. Springs which are used in high-resistance circuits, such as those of voltmeters and wattmeters, may be made of a material having good elastic properties, such as a bronze, for no limit is set as to spring resistance. Low-resistance springs, for millivoltmeters, have a high percentage of copper and show a larger zero shift than the bronze springs.

Temperature Coefficients of Springs.—With a rise of temperature, the elasticity of the springs decreases about 0.03 or 0.04 per cent per degree C. This, if uncompensated, would cause an increase of like amount in the deflection. In many cases electrical and magnetic changes afford a partial compensation.

Balancing.—Accurate balancing of the movable system is essential, for commercial instruments should not require careful levelling. As a test the instrument should be tilted from its normal position in various directions and the pointer observed. If lack of balance is found to be present and the instrument must be used, it should be set up, using a level, the same precaution being taken during the calibration. The rebalancing should be done by an experienced person.

Scale Errors.—The cardinal points on the scale are supposed to be laid off, for each particular instrument, by comparison with a standard. However, the subdivisions are frequently very carelessly made and their irregularities are often apparent on inspection. The calibration curves for such irregular scales are "lumpy" and the calibration points must be taken near together.

Corrosion.—Hard-rubber covers and instrument bases which are imperfectly vulcanized may give trouble. The free sulphur attacks delicate wires, controlling springs, and suspensions, causing gradual deterioration and finally total failure. The effect on the indications of the instrument is, of course, progressive. Fine wires insulated with soft-rubber tubes, as is common for internal connections, may suffer in the same way.

ELECTRICAL AND MAGNETIC ERRORS

Shunts.—Care must be taken that shunts suffer no mechanical injury. In making connections for a test they should be firmly bolted into the circuit, all contacts being clean. Imperfect contact at one end of the shunt may result in unequal heating and a consequent thermo-electromotive-force error; but the over-zealous application of the monkey wrench should be avoided, for if the shunt is not properly supported some of the soldered joints where the resistance strips are sweated into the terminal blocks may be broken, and though no damage is visible the shunt may be rendered entirely untrustworthy and the test useless. The current leads should be of ample size, so as to assist rather than to hinder the dissipation of the heat from the shunt. During calibrations, especially where high-capacity shunts are involved, current connections identical with those of regular service must be used, so that the current may be properly distributed before the potential terminals are reached.

Millivoltmeter Leads.—External shunt ammeters *must* be calibrated with the same set of leads connecting the shunt and the millivoltmeter that is to be used in the subsequent test. Frequently special leads 30 or 40 feet long must be used in order to remove the millivoltmeter from stray fields, or to allow it to be placed where it can be easily read. Such leads should be of large diameter to reduce the resistance, and should be provided with proper terminals.

In all cases when connecting the millivoltmeter and the shunt, care must be taken that all contacts are clean and firmly set up. The leads should be carefully examined to see that they are not broken inside the insulation or where they are soldered to the terminals.

Thermo-electromotive Forces.—The material of the resistance strips used in shunts should have a small thermo-electromotive force when opposed to copper. Manganin is the best. The existence of a thermo-electromotive force may be demonstrated by sending full current through the shunted instrument for a considerable time and then breaking the main circuit. The millivoltmeter will not return at once to zero. This effect may be differentiated from the zero-set by breaking the millivoltmeter circuit. Switch-board shunts are likely to be defective in this respect, and should not be used in very careful work until they have been investigated.

Effect of External Temperatures.—Variations of room temperature produce only small errors in soft-iron ammeters with a spring control, for as the spring weakens, the permeability of the iron decreases in such an amount as practically to compensate for it. In the voltmeter there is an additional source of error in the alteration of the resistance. The windings will be of copper and the series resistance of a material with a zero temperature coefficient. The net effect will depend on their relative magnitude; it will be small in high-range instruments.

The only effect on current dynamometers, with the coils in series, is to alter the spring, 0.03 or 0.04 per cent per degree C., causing the instrument to read too high. The effect on dynamometer voltmeters is on the resistance as well as on the spring.

With a rise of temperature, the magnets of a moving-coil voltmeter decrease in strength, the springs weaken, and the

total resistance increases somewhat. In a 150-volt instrument the net effect is negligible, but lower-range instruments, made by using the same galvanometer element and a smaller series resistance, will be affected by an amount increasing with the diminution of the range, 0.4 per cent per degree being the extreme value, for then the copper of the moving coil becomes relatively more important. Therefore, in accurate work very low-range instruments should be used with care. Frequently, in laboratory voltmeters, a thermometer is inserted in the case as an aid in making the temperature corrections.

From the standpoint of external temperature effects the shunt and the millivoltmeter in shunted ammeters should be of the same material, so that they may have practically the same temperature coefficient. As the shunt is best made of manganin, this implies that a resistance also of manganin be used in series with the copper moving coil, so as to obtain an approximation to the ideal condition. This means that the drop in precision ammeters is considerable—150 to 200 millivolts at full load. The drop in switchboard shunts is about 50 millivolts.

Internal Heating Errors.—In shunted ammeters, errors may arise from the unequal percentage increase of the resistances of the shunt and the millivoltmeter parts, due to the passage of the current. In old instruments, with internal copper shunts, this error is very pronounced. For instance, in a certain 150-ampere instrument, it was found to be 4 per cent at full-scale deflection. In modern high-resistance precision ammeters, this error ceases to be troublesome.

In voltmeters, if they are kept in circuit, there will be heating of the series resistance and movable coil due to the passage of the current, but on account of the low net temperature coefficient, the resulting error will not be great. High resistance multipliers should be properly ventilated.

In direct-current instruments (150-volt) the expenditure of energy is small, about 1.5 watts at full-scale deflection. In wattmeters and alternating-current voltmeters, together with their accompanying multipliers, much more heat must be dissipated on account of the lower resistances, about 7 watts in a 150-volt instrument at full-scale deflection. In any case the construction should be such that the heat is kept away from the

springs and the copper movable coil. Both instruments and multipliers should be properly ventilated.

Stray Fields.—One of the most troublesome sources of error in industrial testing is due to the stray fields from busbars, feeders, motors, masses of iron, etc. These may so modify the strength of the field in which the movable coil swings that the indications of the instrument are entirely untrustworthy. Especial care must be exercised when working near switchboards.

Stray fields due to ordinary working conditions are not likely to produce permanent alterations in the instruments, but violent short-circuits may cause permanent changes in the strength of the magnets and occasion very large errors. This is especially true of direct-current watt-hour meters. In this case the stray field is that due to the current coils. Fig. 266 shows the normal distribution of magnetism in the drag magnets as well as the distribution after a short-circuit. Strong alternating stray fields, due to short-circuits, may also greatly modify the strength of any permanent magnets in their neighborhood. If such an accident has happened with either alternating or direct currents, no reliance should be placed on the instruments until they have been tested and found to be correct.

Direct-current stray fields of ordinary strength cause a percentage change throughout the scale in the indications of moving-coil ammeters and voltmeters. Of course, they produce no effect on alternating-current instruments. Alternating stray fields of ordinary strength have no effect on direct-current moving-coil voltmeters and ammeters, but will affect ammeters, voltmeters, and wattmeters in which the current is of the same frequency as the field.

The effect on dynamometer instruments will depend on the angular position of the movable coil with respect to the direction of the stray field, being a maximum when the plane of the coil is in the direction of the field and zero when it is perpendicular to it. Assuming that the stray field is fixed in direction, dynamometer instruments with torsion heads should be set up in such a position that the movable coil is perpendicular to the field. The proper position is found by sending full current through the movable coil alone and turning the entire instrument in azimuth until the deflection disappears.

To detect the presence of a stray field, the instrument should be read and then immediately turned through 180° and read again, the circuit conditions being maintained as constant as possible. If no stray field is present, the readings will check. If a wattmeter is being used, current may be sent through the potential circuit alone and the instrument slowly turned in azimuth. Any deflection observed will be due to the stray field.

The stray fields due to heavy currents in the leads to the instruments themselves must not be neglected. The leads must be free from loops and coils, should run straight away and should be twisted. Especial care must be exercised when testing direct-current watt-hour meters *in situ*, that the stray fields from the temporary connections, such as the necessary jumpers, do not vitiate the results, especially at light loads. Do not set moving-coil instruments on sheets of "tin," which are tinned iron, and do not place instruments too near together. As the field strength in alternating-current instruments is small, they are more susceptible to these errors than direct-current instruments of the moving-coil type. In general, one cannot assume that stray fields are constant in either magnitude or direction.

Careful attention must be given to these points when deciding on the location and arrangement of apparatus for a test; for in any case where results are called in question, unless one can prove that there were no stray-field errors, the measurements have no standing. Shielded instruments obviate these troubles.

Electrostatic Attraction.—Electrostatic attraction between the fixed and movable members may cause erroneous deflections; for instance, in wattmeters which are operated from instrument transformers. In this case the remedy is to connect the current and potential circuits by a bit of the finest fuse wire. Again, glass and hard-rubber covers sometimes give trouble. They should not be rubbed immediately before a reading is taken. The surface charges may be dissipated by breathing on the instrument. High range instruments having metal covers which are supported by insulating bases are likely to give trouble. The secondary circuits of instrument transformers should be grounded.

Eddy Currents.—Eddy currents induced in massive coils, in metal frames supporting the coils or in metal covers, may be a

source of error in alternating-current work. These effects may be pronounced in wattmeters when working at low power factors; of course they are absent when direct currents are used.

Current Distribution.—Distribution errors may be met with in alternating-current instruments with massive coils, the current not distributing itself uniformly over the cross-section of the conductor as it does when direct current is used. This also may cause the alternating- and direct-current calibrations to differ.

Frequency and Wave Form.—There is a possibility of error in dynamometer voltmeters and wattmeters and in soft-iron voltmeters, due to frequency, if the reactance of the instrument becomes unduly high in proportion to the resistance. Especially should one be on his guard in investigation work where abnormal frequencies are sometimes employed. An instrument which is commercially correct at 60 cycles may be much in error at 500 cycles. Eddy-current effects are much accentuated at high frequencies.

Soft-iron instruments may be subject to wave-form errors arising from saturation effects, but with good modern instruments no trouble is likely to be experienced.

Induction instruments have errors all their own, due to the fact that when the compensation has been adjusted to suit the fundamental frequency, it will in general be incorrect for the various harmonics. These instruments are designed for use under definite conditions as to voltage, frequency, and wave form, and though they are serviceable on distribution systems where these things are fixed, it is unsafe to apply them indiscriminately in general testing. The wattmeters and watt-hour meters may have very serious frequency and wave-form errors, especially at low power factors.

Use of Transformers.—The use of instrument transformers introduces errors, due to ratio and phase angle, which vary with the load. These are discussed on pages 577, 578.

METHODS OF CALIBRATION

The dates of all calibrations should be recorded and inserted in the legends of the calibration plots, together with the numbers of the instruments.

Direct-current Instruments.—As electrical measuring instruments cannot be relied upon to give absolutely accurate results, it is necessary to have methods for calibrating them. It should be possible to assemble the apparatus necessary for dealing with direct-current ammeters and voltmeters from the instruments found in any laboratory devoted to general electrical measurements.

Voltmeter Calibration by Standard Cell.—The method here given is an application of Poggendorf's method (see page 269).

An electromagnetic voltmeter is in reality a galvanometer in series with a high resistance. The scale of the galvanometer is graduated, not in current strengths, but in the voltages which it is necessary to apply to the terminals of the instrument in order to obtain the various deflections; that is, in values of $I_V R_V$ where I_V and R_V are the current through, and resistance of, the

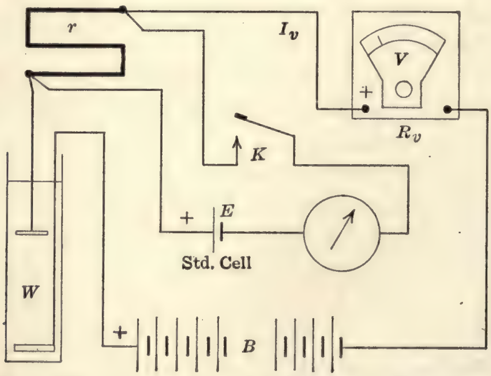


FIG. 369.—Connections for voltmeter calibration.

voltmeter. Therefore, if R_V and I_V have been measured, one may find the true value of the P.D. applied to the instrument, and this may be compared with its nominal value as read from the scale. The difference will be the correction to be added to the observed reading to obtain the true value of the P.D.

The resistance of the voltmeter is determined by a Wheatstone bridge.

The necessary connections for the measurement of I_V are shown in Fig. 369.

B is a battery capable of giving the desired current; W is a water rheostat by which the current I_V, and consequently the reading of the voltmeter V, may be varied; r is a known and variable resistance. The standard cell has a voltage denoted by E. *One must be sure that the cell is properly inserted so that in the galvanometer circuit its e.m.f. will oppose the P.D. due to the drop in r.* If the cell be so inserted and $I_V r = E$, the galvanometer will remain undeflected when the key K is depressed.

$$I_V = \frac{E}{r}$$

and the P.D. across the terminals of the voltmeter will be given by

$$\text{P.D.} = \frac{E}{r} R_V.$$

In practice it is usually desired to calibrate at or near certain predetermined points, at readings of 10, 20, 30 volts, and so on. It is, therefore, necessary to know the value of r which must be inserted in order that when the galvanometer is balanced the voltmeter reading may be that desired. This is readily determined, for suppose that the instrument is to be calibrated at or near a reading of 30 volts, that $E = 1.0186$ and $R_V = 17{,}000$ ohms. The proper value of r would be,

$$r = \frac{1.0186 \times 17{,}000}{30} = 577 \text{ ohms.}$$

577 ohms are to be inserted at r, and by the water rheostat, W, the reading of the voltmeter is to be brought to 30 volts. It may be necessary to change the number of battery cells. The key K should be depressed cautiously, and released immediately the deflection appears. In general there will be a deflection, for the voltmeter will probably have a slight error, so that although it reads 30 volts, the true P.D. between the terminals will differ from that value; also, the resistance r is not exactly the value corresponding to 30 volts for r is adjustable to single ohms only. An exact balance is obtained by varying the water rheostat, and the voltmeter is read immediately thereafter. The reading and the corresponding value of r must be recorded. The difference between the true P.D. at the instrument terminals and the reading gives the correction, the quantity which must be added to the reading to obtain the true P.D.

It is important to take the zero reading, for it is subject to variation; and if taken and separately allowed for, a calibration may retain its value, if the scale be equally divided, even though the zero reading alter.

Ammeter Calibration.—To calibrate an ammeter it is necessary to have a method for measuring the current which will be free from such instrumental errors as may affect the indications of even the best direct-reading standard instruments. Such calibrations may be carried out by the aid of standard cells and accurately adjusted resistances, as follows.

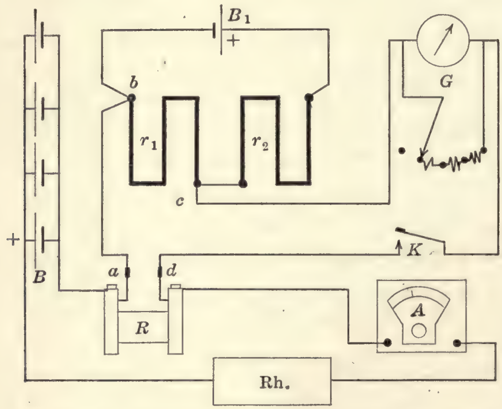


FIG. 370.—Connections for ammeter calibration.

Referring to Fig. 370, R is a known resistance so constructed that it will not heat appreciably with the passage of the current. This is inserted in series with the ammeter to be calibrated, A. Rh is a rheostat for controlling the current, and B is a battery of storage cells to furnish the steady current necessary in such work.

To measure the ammeter current it is necessary to determine the P.D. between the potential terminals of R. This may be done by the method of projection of potentials, an application of Poggendorff's method (page 269). To apply this it is necessary to have an auxiliary battery, B_1, of *constant* e.m.f. and capable of furnishing a small current, 0.001 amp., continuously; and two resistance boxes, r_1 and r_2, of a total resistance of about 10,000 ohms

each, which should be capable of adjustment in single ohm steps. In addition, a suitable galvanometer and key are required. The wire *ab* causes the points *a* and *b* to assume the same potential. If the current through the ammeter be denoted by I, the potential at *d* differs from that at *a* by IR. If the potential difference at the terminals of the battery B_1 be P.D., that between *b* and *c* will be,

$$\text{P.D.}\,\frac{r_1}{r_1 + r_2}.$$

When the potentials at *c* and *d* are the same,

$$IR = \text{P.D.}\,\frac{r_1}{r_1 + r_2},$$

and if they are the same, the galvanometer will not deflect when the key is depressed. Consequently, if r_1 or r_2 has been adjusted so that the galvanometer gives no deflection,

$$I = \frac{\text{P.D.}}{R} \times \frac{r_1}{r_1 + r_2}.$$

One cell of storage battery is the most satisfactory for use at B_1. In order that its P.D. may remain constant, the cell should be partially discharged. A standard cell cannot be used at B_1 because it is incapable of supplying even a small current without alteration of its e.m.f. through polarization.

The first step in carrying out this test is to determine the P.D. of the auxiliary battery B_1. This is to be done by Poggendorff's method.

The connections shown in Fig. 370 are then made; it is necessary that B_1 and R be connected + to +.

In general, on closing the key there will be a deflection which is to be brought to zero by adjusting r_1 or r_2. When a balance is obtained, the ammeter is to be read immediately.

If $r_1 + r_2$ greatly exceeds the battery resistance of B_1, P.D. will be approximately the e.m.f. of the cell. It is well to remember that imperfect connections to the cell and excessive lead resistance have the same effects on the results as high battery resistance at B_1, and that this battery resistance should be negligible both during the test by Poggendorff's method and the subsequent use of the arrangement in determining the current.

By using proper resistances at R, currents of all magnitudes can be measured by this method.

Calibration by Means of Potentiometer.—Direct-current ammeters and voltmeters are most readily calibrated by means of the ordinary or the deflectional type of potentiometer. Currents are determined by measuring the P.D. between the terminals of standard resistances. These resistances should be certified by the Bureau of Standards. This should be done once a year, since the resistances are subject to slight changes.

For voltage measurements, the range of potentiometers may be extended upward from 1.5 volts to any desired extent by the use of volt boxes.

As a source of current for ammeter calibrations, a 4-volt storage battery is most convenient. The cells may be charged in series and discharged in parallel. Variations in the current are obtained by the use of rheostats. These may have metal grids for the large steps and a carbon compression rheostat for the fine adjustments.

For potential differences a storage battery should also be employed. The cells must be large enough so that they may be properly taken care of. A drop wire furnishes the most convenient means of regulating the P.D. at the instruments.

Alternating-current Ammeters and Voltmeters.—The larger part of the alternating-current voltmeters in daily use for engineering work are based on the electrodynamometer principle. Such instruments may be calibrated with direct currents, using either a standard direct-current voltmeter, whose errors are accurately known, or a direct-current potentiometer and volt box. On account of the effect of the local field, it is essential that two readings be taken at each point, first with a voltage in a noted direction, and then with it reversed. The two results should be averaged. This procedure ignores the existence of any frequency error. The magnitude of this error may be calculated from the measured inductance and resistance of the instrument.

Except in the case of the thermal instruments, which give the same reading with both direct and alternating currents, and regular electrodynamometers, with the two coils in series, it is necessary to use alternating currents when calibrating alternating-current ammeters; for generally they are soft-iron instru-

ments, whose indications on direct-current circuits are complicated by the effects of residual magnetism in the iron vane. Also there may be errors due to wave form. These same remarks apply to soft-iron voltmeters which are intended for alternating currents. Induction ammeters and voltmeters must be calibrated with alternating current.

The calibrations may be made by the alternating-current potentiometer, used in connection with non-reactive volt boxes and shunts, but this instrument is not yet in common use. It would not be serviceable if wave-form errors were being investigated.

The arrangement shown in Fig. 371 does very well for ammeters.

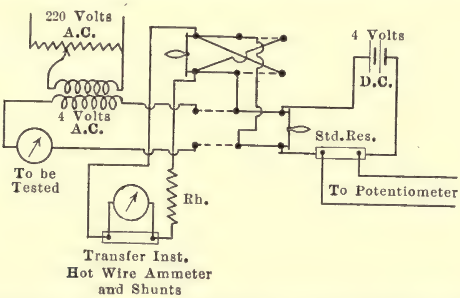


FIG. 371.—Connections used in calibrating alternating-current ammeter.

The hot-wire ammeter, provided with an appropriate set of shunts, is used as a transfer instrument. The ammeter under calibration is compared with it, and then by means of the double-throw switch the hot-wire instrument is transferred to the direct-current circuit and calibrated at the proper point by means of a potentiometer and standard resistances. This procedure avoids all questions as to the permanence of the calibration of the hot-wire instrument. A difficulty is that the range of a hot-wire ammeter is short, the deflection depending upon the square of the current. Consequently, the scale is of such a nature that even with care only about the upper 60 per cent of it is readable with sufficient accuracy. Therefore the millivoltmeter part should be sensitive, and the range extended by numerous shunts, so that the deflection may be kept in the upper part of the scale.

Soft-iron voltmeters may be compared with a dynamometer instrument which has been calibrated with direct current.

Northrup Alternating- and Direct-current Comparator.—The indicating portion of the Northrup comparator may be described as a differential hot-wire millivoltmeter (or milliammeter). The general features of its construction will be evident from Fig. 372. The "hot wires" are shown at a, d, c, and a', d', c'; they are supposed to be of the same diameter, resistance, and coefficient of expansion, and to be placed in similar environments. They are placed in a horizontal position, and shielded from all drafts by a suitable case.

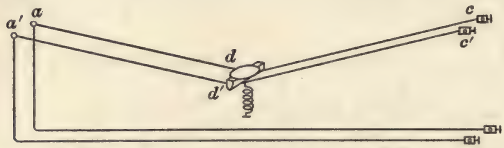


FIG. 372.—Diagram for Northrup comparator.

At the middle of their lengths they are connected by an insulating bridge piece, which carries a mirror and is drawn downward by a spring, so that both wires are taut. A telescope and scale are used to observe the angular deflection of the mirror. In the ideal case, if the same current flows through the two wires, they heat and therefore expand equally. In this case the mirror moves back a little without being tilted, and the scale reading is unchanged. If, however, the currents are not the same, the wires are heated and expand unequally, and a deflection will be observed, which may be reduced to zero by altering one of the currents. When the instrument is in use one wire is traversed by direct, the other by alternating current, and a means is thus afforded of telling when the currents are of equal strengths. After the currents have been adjusted to equality, the direct current is measured by any convenient and accurate means. With the appropriate auxiliary devices, the comparator may be used for the calibration of alternating-current ammeters and voltmeters.

Wattmeters.—When two wattmeters are to be compared, the current coils are placed in series and the potential coils in parallel. If both instruments are of the dynamometer type direct current may be used, reversals being taken to eliminate the effects of the local field.

When calibrating high-capacity wattmeters, in order to save power and bring the work within the range of the apparatus found in a well-equipped laboratory, it is necessary to resort to fictitious loading; that is, to supplying the current and potential coils from two distinct sources.

The connections for a calibration are then as shown in Fig. 373. It is convenient to use storage cells for supplying both the current and potential circuits. Two readings are made at each point, both the current and the potential switches being reversed. The results are averaged to eliminate the effect of the local field.

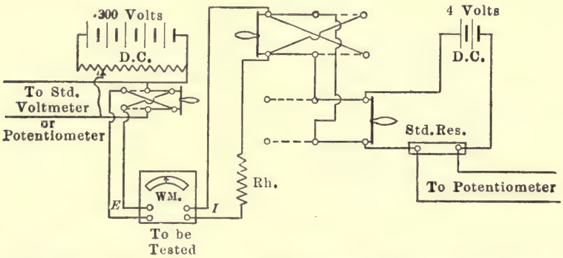


FIG. 373.—Connections for fictitious loading of wattmeter.

In this procedure eddy current and frequency errors are assumed to be negligible. If there is doubt as to this, the wattmeter must be compared with one known to be free of them using alternating currents of the proper frequency.

If an induction meter is being tested, alternating current of the proper frequency must be used and the instrument compared with an electrodynamometer wattmeter which has been calibrated with direct current.

In case it is necessary to test at different power factors, the supply may be derived from two alternating-current machines, having the same number of poles, with their armatures on the same shaft, one field being arranged so that it may be given any desired angular displacement about the axis of rotation. The wave form of both machines should be sinusoidal. A phase shifting transformer (see page 290) may also be used and is more convenient.

Reference

"Testing of Electrical Measuring Instruments," *Circular* No. 20, U. S. Bureau of Standards.

CHAPTER XIV

DETERMINATION OF WAVE FORM

To simplify the mathematical treatment of the flow of alternating currents, it is customary to assume that both the applied e.m.f. wave and the current wave are sinusoidal.

Designers now aim to produce machines with e.m.f. waves which are sinsuoidal, or nearly so, since experience has shown that, all things considered, this form of e.m.f. wave is the most advantageous in practice. As an illustration, modern metering devices, upon whose indications the charges for electric service are based, will not give results which are commercially correct if the wave form is badly distorted so that it differs greatly from a sinusoid.

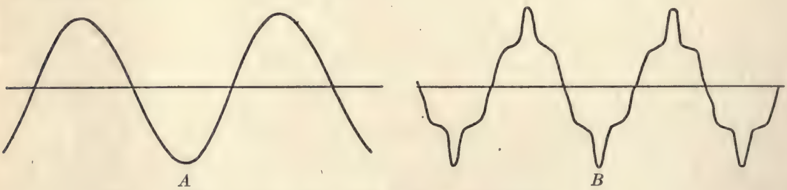


Fig. 376.—Examples of wave forms.

Fig. 376, *A*, shows the e.m.f. wave of a modern turbo-alternator. The departure from the sinusoidal form is not obvious and a careful analysis must be made before one can state what it is. On the other hand, Fig. 376, *B*, shows the e.m.f. wave of a much older type of machine; such a wave form might seriously complicate the behavior of the devices placed in circuit.

The form of the current wave is affected by the character of the circuit. If the e.m.f. wave is not a pure sine curve, the effect of its various harmonics in the current wave will be accentuated by capacity and smoothed out by inductance in the circuit.

Saturation effects in iron cores may also materially affect the form of the current wave. This is illustrated in Fig. 377,

which shows the potential difference applied to and the current in a coil with an iron core.

In engineering work, cases are continually arising where wave form determinations are of the utmost importance on account of the assistance they give in explaining the behavior of electrical apparatus.

Two cases may arise:

A. When the phenomena are periodic; for instance, the ordinary electromotive force and current waves, Fig. 377.

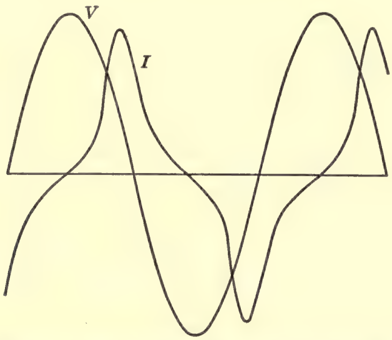


FIG. 377.—Potential difference applied to and current in a coil with iron core.

B. When the phenomena are transient; such as those occurring when the circuit conditions are suddenly altered. This is illustrated by Fig. 394, which shows the potential difference and current curves taken during a short circuit test of an enclosed fuse.

Contact Method for Determining Wave Forms.[1]—Methods for dealing with case *A* were first developed, the earliest being the contact method, used in 1849 by Lenz in investigating the wave forms of alternators. In 1880 Joubert employed it to determine the wave form of a Siemens machine and since then it has commonly been called Joubert's contact method.

The fundamental idea is to connect periodically the measuring apparatus to the circuit for a time so short that during it the current or voltage remains practically unchanged. This is accomplished by an apparatus which is the equivalent of a key operated by a rigid connection from the dynamo shaft. The

key is closed for an instant once during each revolution of the dynamo, and at a definite point on the wave.

If the voltage is high, a large non-reactive resistance, R, Fig. 378, is placed across the circuit, and by means of a tap a definite fraction of the total voltage is impressed on the apparatus.

Referring to Fig. 378, the contact wheel of hard rubber or fiber is at W. In the original arrangement this wheel was attached directly to the dynamo shaft; B_1 is a brush which rests on a collector ring and gives permanent connection to the contact point P, which projects very slightly from the periphery of the wheel, W. B_2 is a thin and very light brush which rests on the

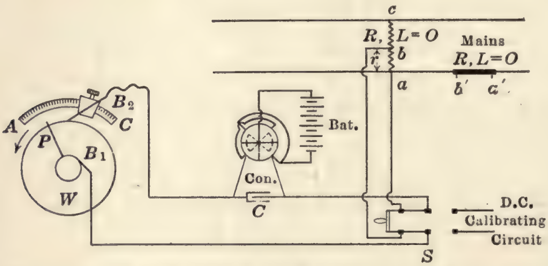


FIG. 378.—Connections for contact method for wave form, using quadrant electrometer.

contact wheel. It is supported from a movable brush holder which may be set at any desired position along a uniformly graduated arc, AC.

The measurements may be made by the aid of an electrostatic voltmeter, a quadrant electrometer or a ballistic galvanometer. In the arrangements shown in Fig. 378 the needle of the electrometer is kept charged to a high potential by the battery and consequently the deflection is sensibly proportional to the applied voltage; that is, to the potential difference between a and b at the instant of contact. The well-insulated condenser, C, adds to the capacity of the electrometer so that the voltage on the instrument will not be appreciably altered by leakage during the time between the successive contacts of B_2 and P.

The process is to set the brush at a definite position on the arc and to read the electrometer; then to move the brush forward to another position and take another reading, and so on.

The magnitude of the deflections may be controlled by means of the battery and the resistance *ba.*

The electrometer is calibrated by use of direct-current voltages as indicated.

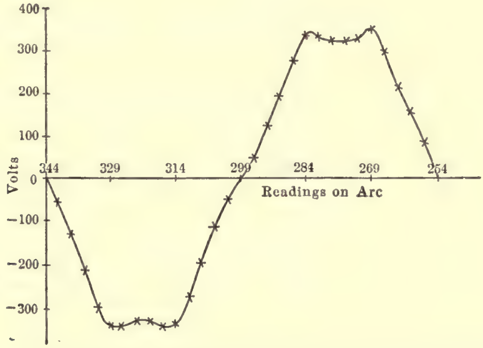


FIG. 379.—Wave form determined by contact method.

The electrometer readings, reduced to volts, are plotted against the readings on the uniformly graduated arc, as shown in Fig. 379, and a smooth curve drawn through the points.

If an electrostatic voltmeter is used in place of the electrometer, a difficulty is encountered, since the deflections depend on the square of the voltage; hence those obtained near the zero

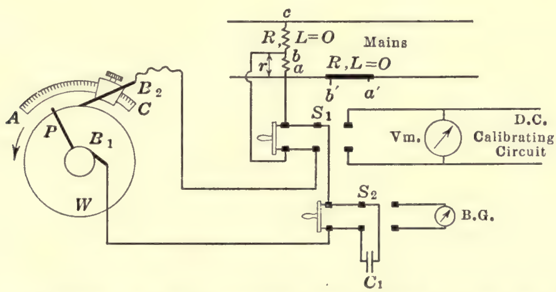


FIG. 380.—Contact method for wave form, using ballistic galvanometer.

points of the wave will be very small. This difficulty may be obviated by working from a false zero. A battery of sufficient voltage to give a large deflection is joined in series with the voltmeter; thus when readings are taken the voltage to be measured

is superposed on the battery voltage. If a reflecting instrument is used, the calibration curve is very closely a parabola, and as the upper part of it is practically a straight line the deflection from the false zero is sensibly proportional to the voltage between *a* and *b* at the instant of contact.

Fig. 380 shows the arrangement when a ballistic galvanometer is employed.

In this arrangement S_1 is a double-pole, double-throw switch, by which the apparatus may be connected to the alternating-current circuit, or to the direct-current circuit for purposes of calibration. C_1 is a variable condenser which is charged by throwing S_2 to the left, and discharged through the ballistic galvanometer, *BG*, when the switch is thrown to the right.

The deflection, which is proportional to the instantaneous voltage between *a* and *b*, may be controlled by varying the capacity.

Use of Potentiometer Principle.—These methods may be improved upon if a potentiometer arrangement is adopted, as shown in Fig. 381.

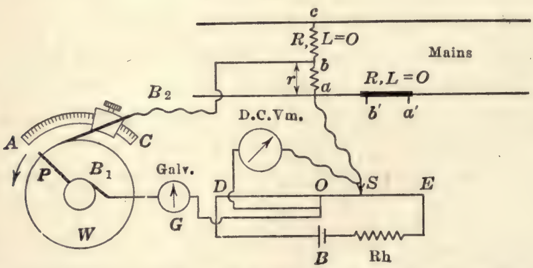


Fig. 381.—Contact method for wave form, using potentiometer principle.

Referring to the figure, *DE* is a slide wire, or its equivalent, which is supplied with direct current from *B*. The voltage from *O* to *D* or *E*, is slightly larger than the maximum occurring between *a* and *b*. A direct-current voltmeter is connected between *O* and the slider *S; G* is a detector, which may be a telephone or a moving-coil galvanometer; a Kelvin instrument is not suitable.

After setting the contact brush, the position of the slider *S* is varied until the detector *G* stands at zero. The instantaneous

voltage between a and b is then equal to the steady voltage between O and S, and the latter is read from the direct-current voltmeter.

The methods given are applied to current waves by placing a non-reactive resistance (shown at $a'b'$) directly in the circuit and determining the instantaneous potential differences between its terminals.

The time consumed in mapping a wave form by the foregoing methods is considerable; in itself this is disadvantageous, and it also necessitates holding the circuit conditions practically constant for a considerable time, for if the wave is non-sinusoidal, many points near together must be taken. Frequently in industrial testing the conditions cannot be maintained constant. In addition, much time and labor must be expended in computing and plotting the results. Obviously, simultaneous records of two or more waves cannot be obtained with a single instrument.

In many cases the necessity for directly connecting the contact disc to the dynamo shaft practically prohibits the use of the contact method in the forms previously given, for it is frequently necessary to determine the wave forms at a place which may be at a considerable distance from the generating station.

From the potentiometer method, the Rosa curve tracer, shown in Fig. 382, has been developed. The object of this machine is to reduce the time necessary for making the observations and plotting the results. In Fig. 381, if the direct-current voltmeter is omitted, and the potentiometer wire carries a definite current, the displacement, OS, of the slider from the zero position will, at balance, be proportional to the instantaneous voltage. The idea is to plot these displacements, as ordinates, on a sheet of paper carried by a drum, the abscissæ being proportional to the displacements of the contact brush along the arc AC. This is done in a semi-automatic manner, as will be seen from the following.

Referring to Fig. 382, the potentiometer wire, DE, in Fig. 381, is wound in a screw thread on an ebonite cylinder. When the cylinder is turned, another thread of the same pitch cut in the ebonite, serves to move the carriage to which are attached the contact point, S, and the stile for registering the results.

A short-period, dead-beat, moving-coil galvanometer is used

as a detector. To make an observation, the handle at the right is turned until the galvanometer stands at zero. Then the lever at the left is raised, causing the stile, through the medium of the typewriter ribbon, to imprint a dot on the paper which is carried by the large drum at the rear of the apparatus. When the lever is lowered, a ratchet and pawl turn the drum a predetermined amount, and at the same time, by means of another pawl and ratchet, actuated by an electromagnet, the contact brush, B_2, in Fig. 381, is advanced proportionally.

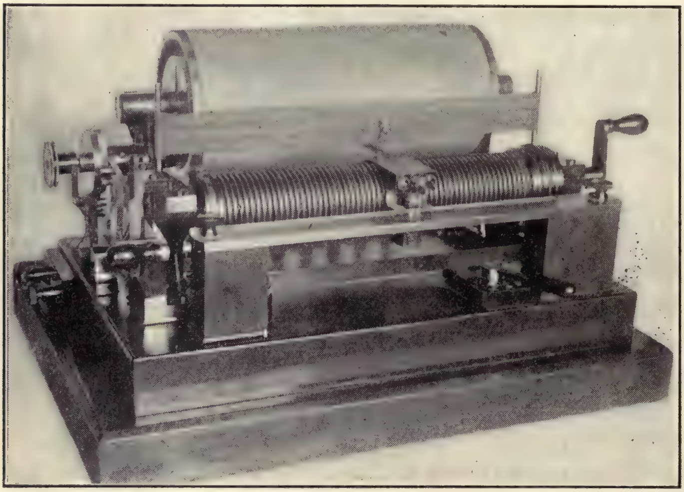


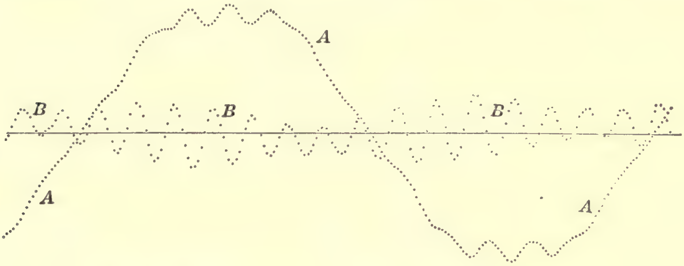
FIG. 382.—Potentiometer and registering apparatus for Rosa curve tracer.

The teeth of the ratchets of the contact maker and of the drum are numbered correspondingly, so that the brush and the recording drum may be set at any desired position. As shown in Fig. 383, the points may be taken close together, so that irregular waves may be dealt with.

For the best work, it is necessary to connect the contact wheel directly to the dynamo shaft and to keep the circuit conditions perfectly constant. If this can be done, the Rosa curve tracer furnishes the most accurate apparatus yet devised for mapping periodic electrical phenomena. There are other modifications of the contact method which reduce the time necessary for recording the waves and give results sufficiently accurate for much

engineering work. These devices are made self-registering. In that shown in Fig. 384 the trace is recorded photographically.[2]

The necessary electrical connections are shown in Fig. 385. K_1 and K_2 are two rigidly connected contact wheels of ebonite.



Curve A, Electromotive Force Wave. Curve B, Current Wave.
Resonance of Fifteenth Harmonic

FIG. 383.—Wave form taken with Rosa curve tracer.

Into the periphery of each wheel are set four brass blocks which are placed 90° apart. Upon each wheel a brush and a collector ring give permanent contact with all the blocks. Another brush, resting on the periphery of the wheel, completes the

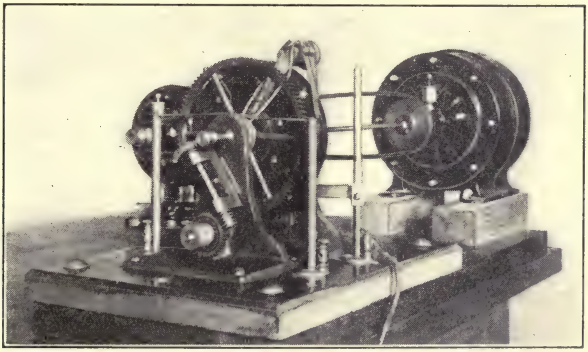


FIG. 384.—Synchronous commutator with continuously moving brushes, for use in determining wave form.

electrical connection as the blocks pass under it. The brushes are so set that contact is made and broken at K_2 before K_1 closes. The contact wheels are driven by a synchronous motor, which makes one revolution for four complete cycles of the

e.m.f. G is a dead-beat galvanometer, and C is an adjustable condenser. The leads a and b are carried to the points between which the P.D. is to be investigated. By inspection of the diagram it will be seen that once on each wave, and at a definite point, the condenser C is charged to the potential existing between a and b. As the charge is determined by the breaking of the contact, the blocks may be of sufficient width to eliminate the effect of the jumping of the brushes. Also the resistance at the contact will not be of sufficient magnitude to prevent complete charging of the condenser.

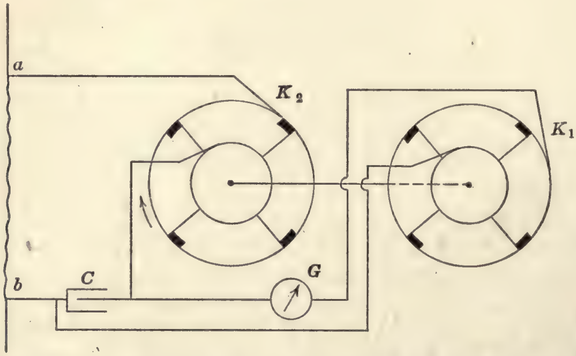


FIG. 385.—Connections for synchronous commutator.

The function of K_1 is to discharge the condenser through the galvanometer after K_2 has broken circuit. The instrument would ordinarily experience a constant deflection, but the brushes K_1 and K_2 are rigidly connected and mounted on a radial arm, which is geared to the shaft so that it moves very slowly. The effect is to gradually move the contact point over the wave. The deflection of the galvanometer will at any instant be proportional to the P.D. between a and b at the instant of breaking at K_2, or in other words, the deflection follows the wave form.

The actual arrangement is shown in Fig. 384, where the contact device, the synchronous motor, and the direct-current motor used for starting the apparatus will be seen. By use of worm gearing the wheel train necessary for moving the brushes is made very compact; the reduction for the instrument shown is 7,200 to 1.

A dead-beat galvanometer with a good law of deflection and a well-defined zero is used.

The record is made on a photographic plate which is moved vertically by a fine wire wound on a drum, seen in Fig. 384, just in front of the lower worm-wheel. This drum can be thrown in gear by a pin clutch.

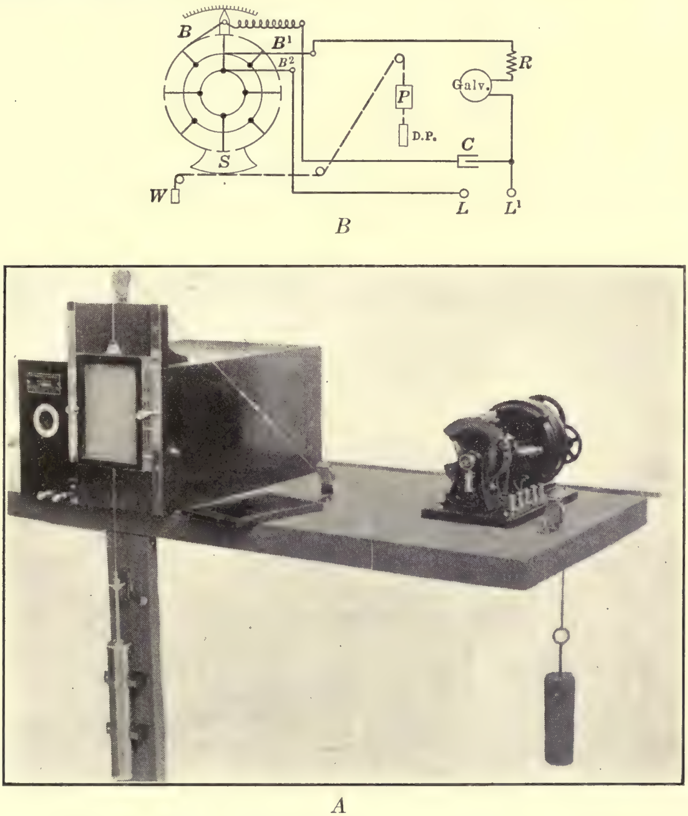
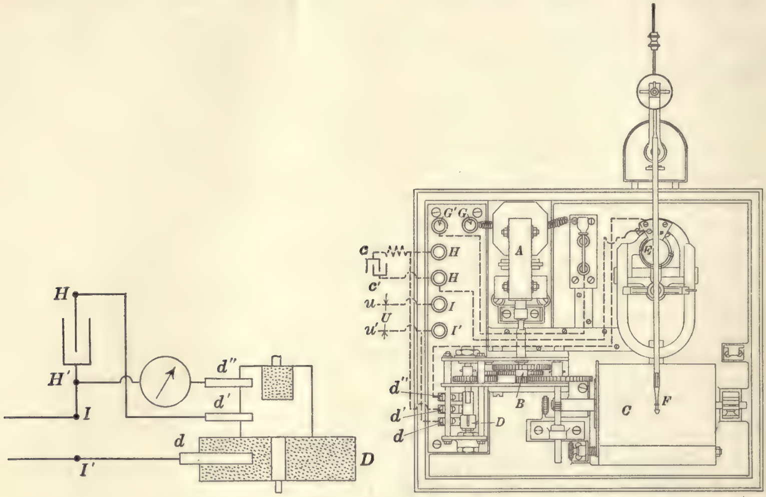


FIG. 386.—General Electric Co. wave meter.

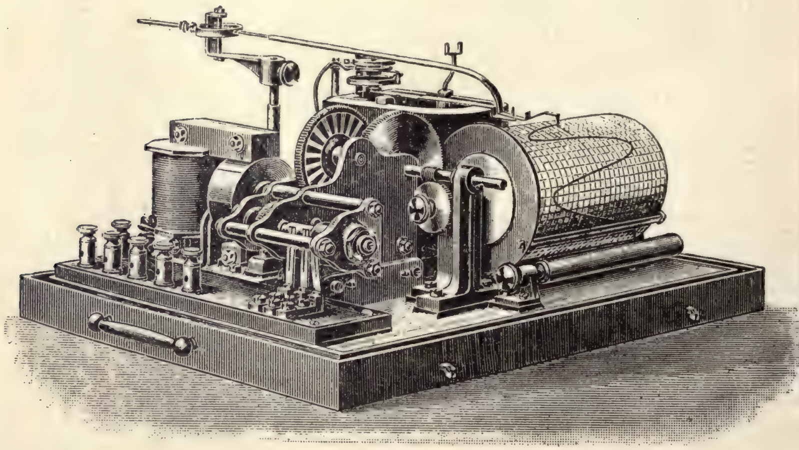
The adjustable condenser allows one to adapt the apparatus to varying conditions, so the e.m.f. curves may be taken directly, and the current curves by the use of a drop wire, as indicated in Fig. 385.

The General Electric Co.'s wave meter[5] is in effect similar to the above device. The motion of the brushes and of the pho-

tographic plate is obtained by a falling weight and controlled by means of the dash pot on the pillar below the plate holder, so that the desired speed of the contact brushes may be obtained.



A



B

FIG. 387.—Hospitalier ondograph.

The Hospitalier ondograph,[2] Fig. 387, is another arrangement of the same sort. By means of a pen actuated by a special

dead-beat moving coil galvanometer of high torque the curve is drawn on a sheet of paper carried by a revolving drum.

Referring to Fig. 387, two brushes are in metallic connection when they simultaneously rest on the unshaded portion of the

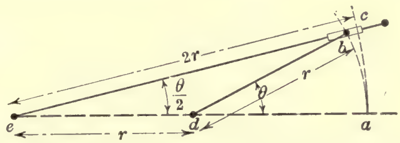


FIG. 388.—Pen mechanism of Hospitalier ondograph.

commutator; the figure, therefore, shows the connections when the condenser is discharging through the galvanometer. The condenser will be charged when d and d' both rest on the unshaded part of the commutator, the galvanometer connection then being broken.

FIG. 389.—Record obtained by Hospitalier ondograph.

The apparatus is driven by a small synchronous motor with four poles, so geared to the commutator that the latter makes 999 revolutions while the motor makes 1,000. The result is that with each revolution the brushes, though stationary, are

in effect shifted slightly with respect to the wave form, and the charge given to the condenser varies accordingly. The result is the same as that attained in the previous apparatus by shifting the brushes.

The recording drum is geared to make one complete turn while the motor makes 1,500 revolutions, so that three waves are recorded per revolution of the drum. The pen, if attached directly to the index of the galvanometer, would move on so short a radius that the diagram would be much distorted. This distortion is reduced in the manner indicated in Fig. 388.

The arm, *db*, 18 cm. long, is attached to the movable system; by means of a fork it turns the arm, *ec*, 36 cm. long and pivoted at *e*. This arm carries the pen at *c*. The arc *ac* is much nearer a straight line than is *ab* and its length is proportional to θ.

The record obtained is shown in Fig. 389. The fact that it is not on rectangular coördinates is a disadvantage, especially if it is desired to analyze the waves.

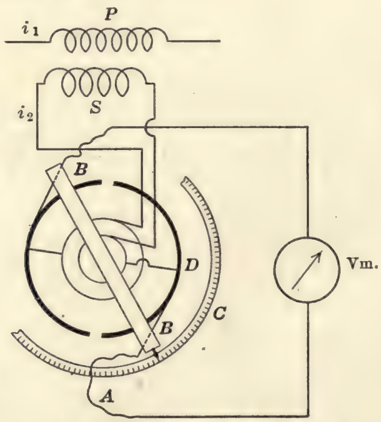


FIG. 390.—Connections for integrating method for determining wave form.

Integrating Methods for Determining Wave Form.[3]—One arrangement for determining wave form by an integrating method rather than by "instantaneous" contacts is shown diagrammatically in Fig. 390.

The current, i_1, whose wave form is to be determined passes through the primary of an air-core transformer, in the secondary of which is a low-range voltmeter, or a galvanometer, and the

commutator, D. The commutator is driven from the dynamo shaft or by a synchronous motor and is so constructed that the connections to the galvanometer are reversed at each half wave. As shown in the figure, the commutator is suitable for a two-pole machine; if there are more poles, it is merely necessary to increase the number of segments correspondingly.

The distance between the brushes is 180 electrical degrees. They are mounted in a holder which can be moved concentrically with the shaft so that they may be set at any point on the wave. Their position can be read from a graduated circle. It is convenient to have the arrangement such that the brushes can readily be moved in a succession of equal steps. Thin metallic brushes which will not short-circuit the commutator should be used. The apparatus must be well-insulated to prevent the entrance of stray currents. This form of commutator is applicable only when the two halves of the wave are alike (except for algebraic sign), that is, when only odd harmonics are present.

The galvanometer should be of the moving-coil type and one which correctly integrates a transient current.

When the commutator is in action the galvanometer experiences a deflection which is proportional to the average value of the current during a half cycle. The reading of the instrument, I_2, is given by

$$I_2 = \frac{2}{T}\int_t^{t+\frac{T}{2}} i_2 dt$$

Let R = resistance of secondary circuit of transformer.

m = mutual inductance of transformer.

f = frequency.

e_2 = instantaneous induced e.m.f. in secondary.

V_2 = reading of voltmeter.

i_1 = instantaneous current in primary.

i_2 = instantaneous current in secondary.

I_2 = average value of current in secondary.

Then
$$e_2 = m\frac{di_1}{dt}$$

$$m\int_t^{t+\frac{T}{2}} di_1 = \int_t^{t+\frac{T}{2}} e_2 dt = R\int_t^{t+\frac{T}{2}} i_2 dt = \frac{T}{2} RI$$

$$\therefore\ 2m\,(i_1)_t = \frac{T}{2}RI_2$$

$$(i_1)_t = \frac{RI_2}{4fm} = \frac{V_2}{4fm} = K \text{ times the scale reading:}$$

K is a constant of the instrument. This gives the instantaneous value of the current at a particular point on the wave, and the form is traced point by point as in the contact method.

E.m.f. Waves.—Electromotive force waves are obtained by placing the primary of a suitable air-core transformer in series with a high non-reactive resistance, which is across the line. If the resistance is so high that the circuit is practically non-reactive, a determination of the form of the current wave in it gives the wave form for the potential difference applied to its terminals. As the resistance must be large, the method is not applicable to low voltages.

With this arrangement all the measuring apparatus is entirely separated from the primary circuit. This may be advantageous if the voltage is high.

The transformer should have a variable ratio. If the mutual inductance is not known, the apparatus may be calibrated by use of a sinusoidal current. The maximum ordinate may then be calculated from the measured value of the current, and compared with the reading of the galvanometer, I_2.

Or, a wave may be plotted, using the readings of the galvanometer, its root-mean-square computed, and this compared with the same quantity determined by a standard alternating-current ammeter.

Flux Waves.—Referring to Fig. 390 the flux which threads the secondary is at every instant proportional to the current in the primary, the wave form of which has been determined. Thus the form of this flux curve has also been found. The total flux through the secondary at any instant will be $(i_1)_t m$.

This suggests that in case the form of a flux wave is desired, it is only necessary to replace the secondary of Fig. 390 by a coil of N turns wound around the core, through which the flux passes.

For the case considered above the total flux linkages at any instant, t, is given by

$$m(i_1)_t = \frac{V_2}{4f}.$$

In the case under consideration, the flux through the core is

$$(\varphi)_t = \frac{V_2}{4fN}$$

Or if V_2 is in volts,

$$(\varphi)_t = \frac{V_2 10^8}{4fN}.$$

If a non-synchronous commutator is used, the maximum value of the flux will be proportional to the maximum reading of the voltmeter, as it goes through its cycle of deflections.

Use of Condenser and Synchronous Commutator.—Another arrangement for determining the forms of potential waves is

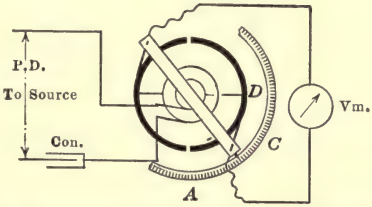


FIG. 391.—Connections for determining wave form by synchronous commutator and condenser.

shown in Fig. 391. Here a condenser, the commutator and a galvanometer, or a millivoltmeter, are joined in series across the mains.

The current through the circuit is $i = \frac{dq}{dt}$. In half a cycle the charge on the condenser changes from $+q$ to $-q$, so

$$\int_{+q}^{-q} dq = \int_{t}^{t+\frac{T}{2}} idt.$$

The reading of the galvanometer, I, is proportional to the average current, or

$$I = \frac{2}{T}\int_{t}^{t+\frac{T}{2}} idt = 2f\int_{t}^{t+\frac{T}{2}} idt$$

$$\therefore\ 2q = \frac{I}{2f}.$$

But if a good condenser is used, $q = V_tC$,

so

$$V_t = \frac{I}{4fC}.$$

The deflection may be controlled by varying C. Currents may be dealt with in the usual way by determining the P.D. between the terminals of a non-reactive resistance through which the current flows.

Determination of the Average Values of Potential Difference and Current Waves.—If the commutator is arranged as in Fig. 392, average values of the potential difference and current may be determined. To do this the brushes must be so set that the reversals are at the zero points of the wave. When the switch

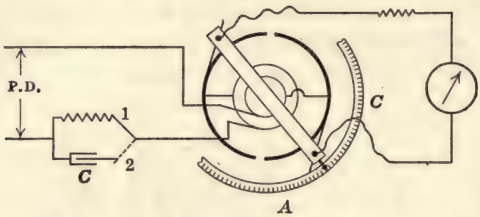


FIG. 392.—Connections for determining average value of alternating-current wave.

is on 1 the galvanometer is subjected to a series of unidirectional impulses and gives a deflection proportional to their average value. The instrument may be calibrated by transferring it to the line side of the commutator and applying a known direct-current voltage.

To properly set the brushes the switch is placed on 2 so that the condenser C is in circuit and the position of the brushes altered until the galvanometer stands at zero. Currents are dealt with by using a non-reactive resistance as in the previous method.

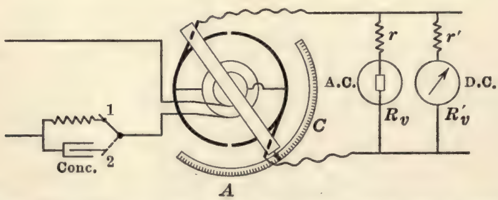


FIG. 393.—Connections for determining form factor.

Determination of Form Factor.—To determine the form factor, an electrodynamometer voltmeter and a direct-current voltmeter are placed in parallel as shown in Fig. 393.

The direct-current instrument gives the mean value while the electrodynamometer gives the root-mean-square value. The deflections are controlled by non-reactive resistances, the values of which need not be known. With the commutator running and the brushes adjusted for reversal at the zero point of the wave by the method just given, both instruments are read. Call the dynamometer reading D, and the galvanometer reading G; then

$$\underset{\text{Effective}}{\text{P.D.}} = D\left(\frac{R_v + r}{R_v}\right) \qquad \underset{\text{Average}}{\text{P.D.}} = G\left(\frac{R'_v + r'}{R'_v}\right).$$

With the commutator stationary a direct-current P.D. is now applied. Then, if D' and G' are the readings,

$$D'\left(\frac{R_v + r}{R_v}\right) = G'\left(\frac{R'_v + r}{R'_v}\right)$$

and the form factor is

$$F = \frac{\text{P.D. effective}}{\text{P.D. average}} = \frac{DG'}{D'G}.$$

The Oscillograph.[5]—All of the methods for obtaining wave form that have been given require that the phenomena be periodic and that the circuit conditions remain fixed for a considerable time. Also with some of the methods much time must be spent in calculating and plotting the results. Cases are continually arising where it is desirable to investigate phenomena which are transient in character, as for example, that illustrated in Fig. 394.

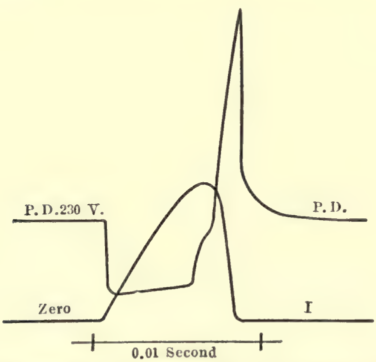


FIG. 394.—Showing variation of current and voltage during short-circuit test of 100-ampere enclosed fuse.

Nominal busbar voltage........	230 volts
Minimum busbar voltage.......	61 volts
Maximum busbar voltage.......	586 volts
Maximum current.............	6,600 amperes
Time required to open circuit....	0.0086 second
Time required to attain maximum current.................	0.006 second
Average rate of decrease of current......................	4,100,000 amperes per second.

It is obvious that in this case the previous methods are not applicable.

Blondel was the first to definitely state the conditions which must be fulfilled in order that a galvanometer may follow, with sufficient accuracy, the rapid variations of an alternating current and be capable of recording wave forms photographically with but a single traverse over the photographic plate of the spot of light which is used as an indicator. He applied the term oscillograph to such an instrument.

The conditions are as follows:

1. High free period of oscillation, as great as 50 times that of the phenomena to be investigated.
2. Damping small and in the neighborhood of the critical aperiodic value.
3. Self-induction as small as possible.
4. Negligible hysteresis and Foucault current effects.
5. Adequate sensitivity.

In addition, the design and construction must be such that the necessary adjustments and repairs may be made with ease by any one accustomed to handling electrical instruments.

The moving needle, the moving coil, the string galvanometer, and the hot-wire instrument have all been modified so that they may be used as oscillographs. Blondel's first instrument was of the moving-needle type; in its late development the needle has become a thin strip of soft iron stretched over bridge pieces, as shown in Fig. 395.

Referring to Fig. 395, the thin, soft iron strip, *S*, is drawn taut over the bridge pieces by the spring within the tube, *t*. The tension is controlled by the nut *s*. The strip is supported and protected by an insulating tube, *T*, in the sides of which soft iron

pole pieces, P, are inserted so that the middle of the iron strip is in a very narrow air gap. Midway between the bridge pieces, and behind a small window, the minute mirror, m, is attached to the strip. This movable system fits between the laminated poles of a powerful electromagnet which conforms to the outside of the tube. The middle part of the soft iron strip thus becomes, in effect, a polarized needle. The required damping is obtained by filling the tube with a transparent oil of the proper viscosity.

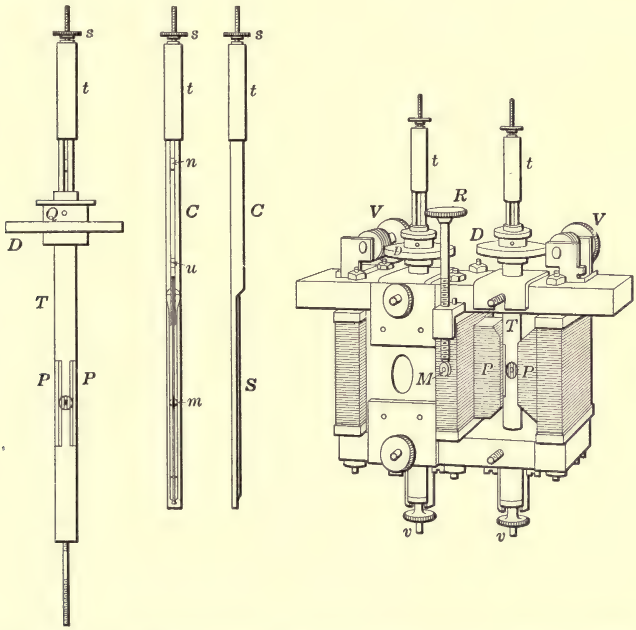


FIG. 395.—Galvanometer for Blondel moving-needle oscillograph.

The current whose wave form is to be determined is led through two coils placed with their common axis perpendicular to the magnetic axis of the needle. On the passage of the current, the needle will deflect like that of an ordinary galvanometer and the spot of light will be moved across the photographic surface.

This type of instrument may be given a very high free period of vibration, but has the disadvantage that its self-induction is comparatively large.

Blondel's suggestion for making the moving-coil galvanometer available as an oscillograph is illustrated in Fig. 396.

A single loop of a narrow and very thin conducting strip is stretched over a frame in such a manner that the part between the bridge pieces *a* and *b* is free. A very small and very light mirror is cemented to the two sides of the loop midway between the bridges. The loop is placed between the poles *NS* of a powerful electromagnet which is worked at high saturation. The poles are large enough so that the free section of the loop is in a practically uniform field.

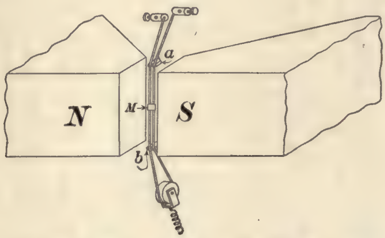


Fig. 396.—Showing principle of bifilar oscillograph.

A current passing around the loop will cause one side to advance while the other recedes; the mirror is thus turned around a vertical axis. For the very small movements which are employed, the deflections of the mirror are proportional to the current.

The movable system is immersed in oil of such a viscosity that, at the normal temperature of operation, the galvanometer is dead beat.

The material from which the strip is drawn should have a low resistivity so that there will be little heating. This avoids creeping of the spot of light due to the expansion of the strip and reduces the energy consumed by the galvanometer.

This form of instrument has the advantage that the inductance is very small and is the one to which the most attention has been given by designers in the United States and in England.

Fig. 397 shows a group of oscillograph vibrators. The corresponding galvanometers, complete, are shown in Fig. 398.

In *A*, which is intended for high-tension work, a permanent magnet is used. The disadvantage is the decreased sensitivity. In *B* the galvanometer elements are thoroughly insulated from

each other, so that it is not necessary that the vibrators be kept approximately at the same potential. With A, the potential difference between the two vibrators and also that between the vibrators and the frame of the instrument should not exceed 50 volts.

The minimum free period which it is practicable to give this form of vibrator appears to be about $\frac{1}{10,000}$ second. To attain

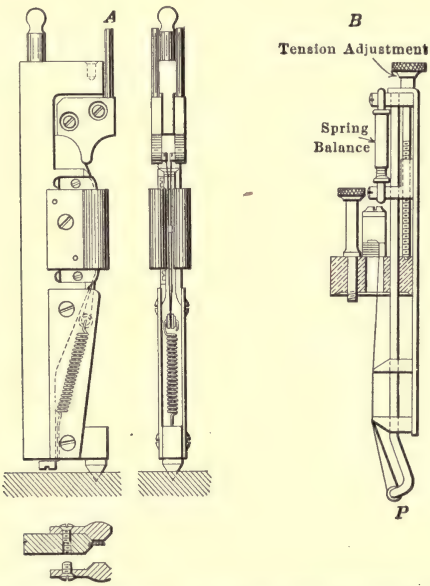
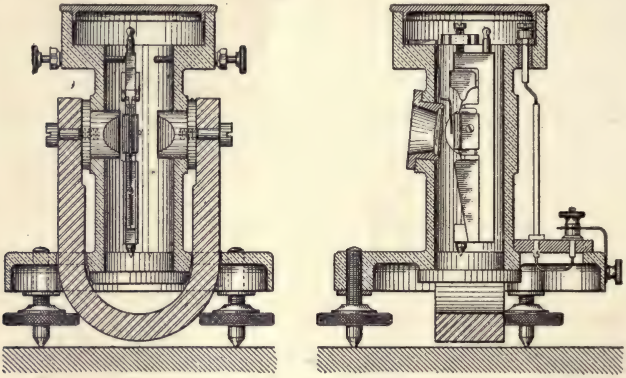


FIG. 397.—Oscillograph vibrators. *A*, Cambridge Scientific Instrument Co.; *B*, General Electric Co.

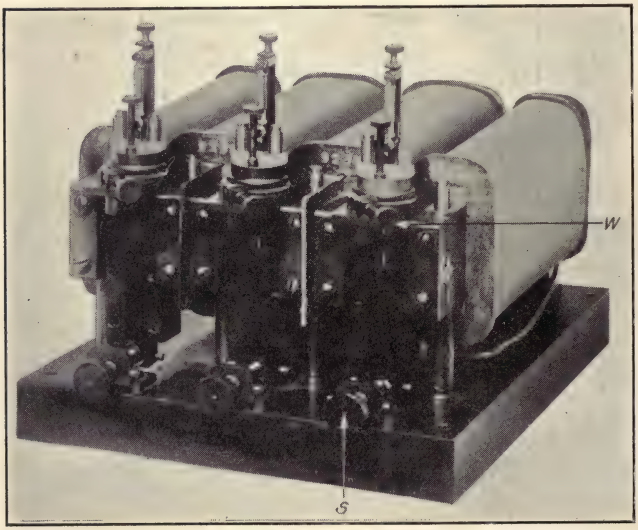
this figure the parts, especially the mirror, must be exceedingly delicate; this increases the liability to accident and the difficulty of making repairs.

Instruments with such a high rate of vibration are useful and indeed essential in certain research work, but they must be used by skilled operators. For general engineering work it is better to be content with a more robust vibrator, having a free period of about $\frac{1}{5,000}$ second. Such an instrument will follow the waves usually encountered, closely enough for practical purposes.

For demonstration purposes oscillographs are made which are capable of projecting the curves on a screen. They are provided with large mirrors and have a free period of from $\frac{1}{1{,}500}$ to



A



B

Fig. 398.—Oscillograph galvanometers. *A*, Cambridge Scientific Instrument Co.; *B*, General Electric Co.

$\frac{1}{2{,}000}$ second. Fig. 399 shows such an instrument* together with the auxiliary apparatus. It is provided with two vibrators

* Designed and built by H. G. Crane, Brookline, Mass.

(period about $\frac{1}{1,500}$ second), one for the current, the other for the potential. They have the necessary adjustments for properly locating the spots of light on the screen. No damping arrangement is employed. The rotating multisided mirror is driven by gearing from a simple synchronous motor which is easily started by giving the mirror a twist just as the switch is closed. The arc, which is enclosed, can be readily adjusted as to position and requires little attention. Small carbons are used and a very intense and well-defined spot of light is obtained.

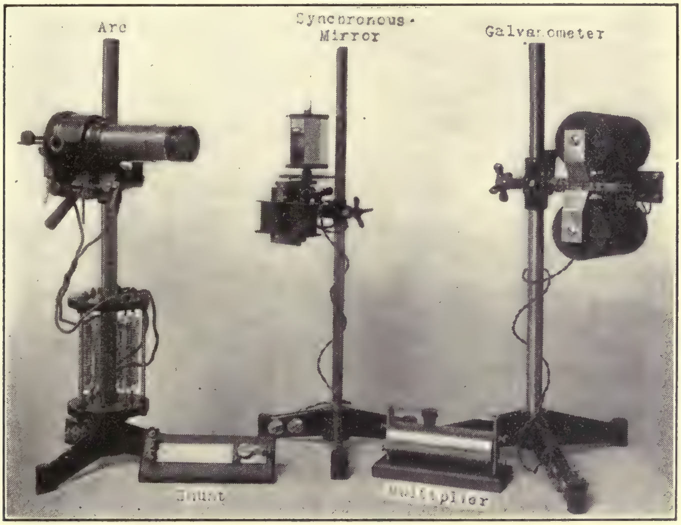


FIG. 399.—Demonstration oscillograph.

With the screen at a distance of 12 feet, the amplitude of the wave is about 2 feet, while the length of the wave is $2\frac{1}{2}$ feet. Both the shunt and the multiplier are adjustable so that the amplitude of the wave may be varied.

Naturally, as the period of an oscillograph galvanometer is very short, the sensitivity is low. In the laboratory form of instrument a deflection on the scale of from 1 to 3 mm. usually corresponds to about 0.01 ampere.

The movable mirror is very small so an intense source of light is required. A direct-current arc is commonly employed. The beam of light, after being reflected from the oscillograph mirror,

Fig. 400, passes through a long cylindrical lens which compresses the beam vertically. This focusing improves the definition of the spot of light on the screen and a still further improvement is effected by placing a narrow vertical slit between the arc and the mirror.

For visual observations the beam of light after passing through the cylindrical lens is received on a multisided mirror which is rotated at the proper speed by a synchronous motor, or else on a mirror which is tilted with a uniform angular velocity by a cam, also driven by a synchronous motor. With the tilting mirror arrangement a shutter actuated by the motor cuts off the light while the cam is returning the mirror to its initial position.

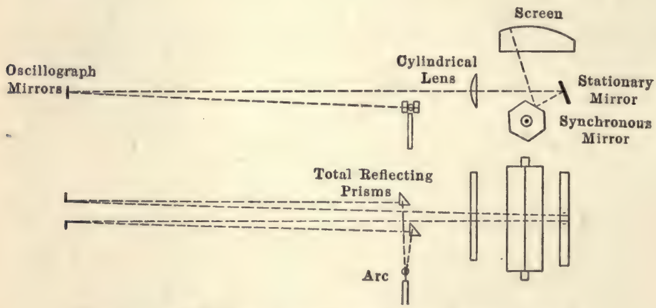


Fig. 400.—Showing optical system of oscillograph.

The revolving mirror furnishes the necessary time coördinates. From it the spot of light is reflected upon a curved translucent screen which is concentric with the axis of the mirror.

With periodic phenomena the waves appear in a fixed position on the screen, and may be traced on thin paper.

For photographic work these mirror arrangements are dispensed with and the spot of light is focused by the cylindrical lens directly on a photographic surface which is carried either by a falling plate or by a uniformly rotating drum. With the drum a shutter is employed which remains open while the drum makes one revolution. The mechanical features of these recording devices are described in the catalogues of various instrument makers.

As it is necessary to decrease the inertia of the moving parts of the oscillograph vibrator as much as possible, the principle

of the string galvanometer has been employed by some designers, for in this instrument there is no mirror to be moved.

In Fig. 401 the shell and pole pieces of an iron-clad electro-

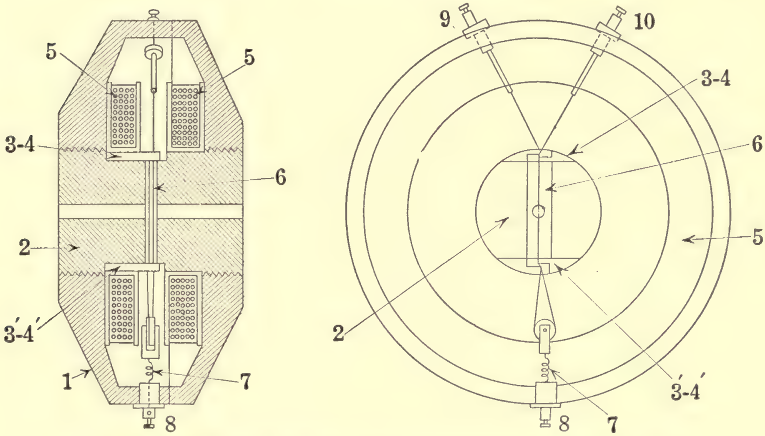


FIG. 401.—Ganz oscillograph, a modified string galvanometer.

magnet are at 1 and 2. The exciting coils are at 5. The strength of field in the air gap is from 28,000 to 30,000 c.g.s. units.

The moving parts or "strings" are at 6. Starting from the terminal 9 the thin metal strip which serves as the "string" passes down between the bridges 3 and 4, along the air gap to 3′ and 4′, around the pulley and up between the same set of bridges to the terminal 10. Both the descending and the ascending parts of the strip are thus brought into the same plane. The free vibrating length of the strip is from 3 to 5 cm. The necessary tension is given by the spring 7 and a period of from $\frac{1}{4,000}$ to $\frac{1}{5,000}$ second is obtained. At 8 is a third terminal which is common to the two parts of the strip, one of which is used for the current, the other for the potential vibrator, as is indicated in Fig. 402.

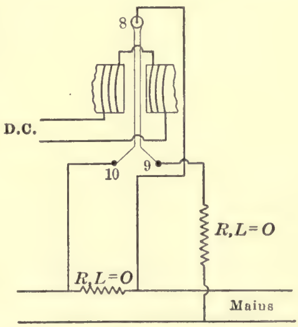


FIG. 402.—Arrangement of circuits in Ganz oscillograph.

A hole is bored axially through the pole pieces so that by means

of a projecting microscope the images of the strips may be thrown either on a uniformly moving photographic surface, for the purpose of obtaining a permanent record, or on a translucent screen for visual observations.

When permanent records are taken, the photographic surface is caused to move in a direction parallel to the length of the strips, and immediately behind a narrow transverse slit. The projections of the strips cross the slit and on the passage of the alternating current move back and forth along it. The photographic trace is, therefore, a light line on a dark ground.

For visual observations the image of the strips is focussed on a small ground-glass screen, immediately behind which is placed a stroboscopic disc with narrow radial slits. If the disc be stationary there will be a bright vertical line of light on the screen, crossing which is the dark projection of the strips. The position of this depends on the current strength; when the disc is rotated, a series of flashes sweep across the screen and as the rotation is synchronous with the current, the curves appear stationary on the screen.

Theory of the Oscillograph.—As the oscillograph is merely a damped galvanometer with a high free rate of vibration, the equation established on page 25 applies. As the current, and consequently the deflecting force, may be any function of the time, it must be expressed analytically by a Fourier series. Consequently

$$P\frac{d^2\theta}{dt^2} + k\frac{d\theta}{dt} + \tau\theta = C\sum_{n=1}^{n=\infty} I_n \sin(n\omega t - \beta_n) \qquad (1)$$

ω is 2π times the fundamental frequency of the current and n is the order of any harmonic. For the fundamental n is 1, for the third harmonic n is 3, and so on.

If the second member of 1 is zero, and the relative values of P, k and τ are such that the motion is oscillatory,

$$\theta' = K\epsilon^{-\left(\frac{2\lambda}{T}\right)t} \sin\left[\left(\frac{2\pi}{T}\right)t + \varphi\right] \qquad (2)$$

K and φ are constants which are determined by the initial conditions. λ is the logarithmic decrement and T is the time of a complete vibration (see page 29).

On account of the damping the transient portion of the deflec-

tion, represented by θ', rapidly diminishes to zero after the circuit is closed.

The particular integral to which θ' must be added to obtain the complete integral is

$$\theta = C \sum_{n=1}^{n=\infty} \frac{I_n}{\sqrt{k^2 n^2 \omega^2 + (\tau - n^2\omega^2 P)^2}} \sin\left(n\omega t - \beta_n - \tan^{-1}\frac{kn\omega}{\tau - n^2\omega^2 P}\right) \quad (3)$$

This expression for θ should be compared with the corresponding one for the flow of current in a circuit containing resistance, inductance and capacity after the steady state has been established. In both cases there are "impedance" and phase-displacement terms.

Inspection of (3) shows that after the transient term has disappeared,

1. The various harmonics do not have the same proportional effect on the deflection.

2. The harmonics suffer different phase displacements.

In consequence, the oscillograph can never give a *mathematically* correct picture of a wave form. This being so, it is necessary to find the conditions which will make the instrument sufficiently correct for practical purposes.

Obviously, if the moment of inertia and the damping were both zero the wave would be followed exactly. Therefore, the mass of the moving parts must be reduced to a minimum and an arrangement adopted which will make the moment of inertia as small as possible. At the same time the directive moment on the movable system must be increased so that the $\frac{d^2\theta}{dt^2}$ and the $\frac{d\theta}{dt}$ terms are small in comparison with it; hence the usual statement that the rate of free vibration must be high.

The instrument must closely follow sudden changes of current, therefore, the damping should be near the critical value, or mathematically,

$$k^2 = 4\tau P.$$

Then

$$\theta = C \sum_{n=1}^{n=\infty} \frac{I_n}{\tau + n^2\omega^2 P} \sin\left(n\omega t - \beta_n - \tan^{-1}\frac{2n\omega\sqrt{\tau P}}{\tau - n^2\omega^2 P}\right) \quad (4)$$

Consequently, with the critically damped oscillograph, the ratio of the amplitude of the deflection due to a given harmonic to the value it would have if the instrument were perfect, that is, without inertia and without damping, is given by

$$R_n = \frac{\tau}{\tau + n^2\omega^2 P} = \frac{1}{1 + n^2\omega^2 \dfrac{T_0{}^2}{4\pi^2}}.$$

The free period of the movable system is $T_0 = 2\pi\sqrt{\dfrac{P}{\tau}}$ and if the periodic time of the phenomena under investigation is T,

$$R_n = \frac{1}{1 + n^2\left(\dfrac{T_0}{T}\right)^2} \tag{5}$$

or

$$\frac{T_0}{T} = \frac{1}{n}\sqrt{\frac{1}{R_n} - 1} \tag{6}$$

Suppose the natural period of the movable system is $\frac{1}{6,000}$ second and that 60-cycle phenomena are being investigated. Then for the fundamental and the ninth and the twenty-first harmonics,

$$R_1 = \frac{1}{1 + 1\left(\dfrac{60}{6,000}\right)^2} = 0.9999$$

$$R_9 = \frac{1}{1 + 0.0081} = 0.992$$

$$R_{21} = \qquad 0.956$$

These departures from exact proportionality are negligible in almost all cases.

As another example, suppose that it is desired to reproduce the fifth harmonic to within 1 per cent. Then

$$R_5 = 0.99$$
$$\frac{T_0}{T} = \frac{1}{5}\sqrt{1.01 - 1} = 0.02.$$

That is, for the assigned degree of accuracy the frequency of the vibrator should be 50 times that of the phenomena under investigation, or the rate of the vibrator should be 3,000 per second when dealing with the fifth harmonic in a 60-cycle circuit.

The phase displacement of any particular harmonic is given by

$$\tan \beta'_n = \frac{kn\omega}{\tau - n^2\omega^2 P} = \frac{2n}{\dfrac{T}{T_0} - n^2 \dfrac{T_0}{T}} \tag{7}$$

If N_0 and N represent the number of vibrations per second, corresponding to T_0 and T respectively,

$$\tan \beta'_n = \frac{2n}{\dfrac{N_0}{N} - n^2 \dfrac{N}{N_0}} = \frac{2n \dfrac{N_0}{N}}{\left(\dfrac{N_0}{N}\right)^2 - n^2}. \tag{8}$$

If $\dfrac{N_0}{N} = \dfrac{6{,}000}{60} = 100$, then

$\tan \beta'_1 = \dfrac{200}{10{,}000 - 1} = 0.020 \approx 1°.15$ of the fundamental

$\tan \beta'_9 = \dfrac{1{,}800}{10{,}000 - 81} = 0.182 \approx 10°.3$ of the ninth harmonic

$\tan \beta'_{21} = 0.439 \approx 23°.7$ of the twenty-first harmonic.

If the length of one complete cycle of the curve is 2.5 in., the corresponding linear displacements are

for β'_1.......... 0.0080 in.
for β'_9.......... 0.0079 in.
for β'_{21}.......... 0.0078 in.

It will be noted that all the components are shifted from the positions they would occupy with a perfect instrument by practically the same amount, so that no error of importance is introduced by the phase displacements.

The Electrostatic Oscillograph.[6]—The electrostatic oscillograph is a form of electrometer so designed that the rate of free vibration is high and the damping of proper value. The instrument is used heterostatically. The high free period necessitates a strong controlling force, and this, together with the fact that electrostatic forces are small when low potentials are concerned, means that the electrostatic oscillograph is naturally best adapted to high-voltage work. For such work it possesses the advantage that it consumes no energy and that the current required is exceedingly small, being only that necessary

to charge it and to charge the condenser multipliers which are used at very high voltages.

In the instrument devised by Ho and Koto, Fig. 403, the potential to be measured is applied between the plates, F_1 and F_2, either directly, or through a condenser multiplier if the voltages are very high. These plates serve as "quadrants." They are 9 mm. wide and 15 mm. long, are exactly alike and placed 5 mm. apart. The mirror m is observed through one of the openings, w_1, the other opening, w_2, being added in order that the arrangement may be symmetrical.

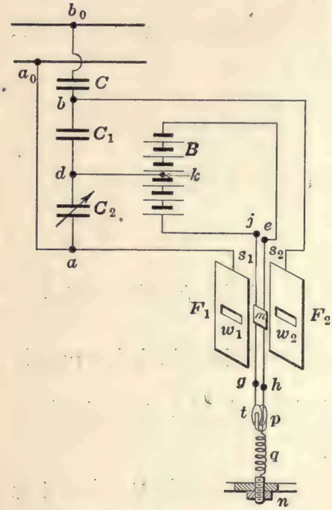


Fig. 403.—Diagram of electrostatic oscillograph of Ho and Koto.

The condensers, C_1 and C_2, serve to split the potential which is applied between the "quadrants." They are nominally of the same capacity; one of them must be adjustable. The condenser multiplier is introduced by the insertion of the condenser, C.

The moving members, or "needles," are formed by two metal strips, s_1 and s_2. As in the ordinary oscillograph they are stretched by a spring, q, between insulating bridge pieces. They are insulated from each other at their lower ends by the silk thread gph. The "needles" are charged from a 300-volt battery

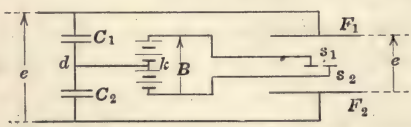


Fig. 404.—Pertaining to demonstration for electrostatic oscillograph.

of dry cells, B. The middle of the battery is connected to the point d, between the condensers C_1 and C_2.

Suppose the electrometers to be perfectly symmetrical; the arrangement is that shown in Fig. 404. Consider a fall of potential to be positive in the direction of the arrow. Let e be

the voltage between F_1 and F_2 at any instant and let B be the constant e.m.f. of the well-insulated battery. F_1, F_2 and s_1 may be regarded as forming the elements of one electrometer and F_1, F_2 and s_2, those of another. The two moving elements or needles are mechanically coupled by the mirror, m, which is cemented to both.

It has been shown (see page 250) that the force acting on the movable element of an electrometer may be represented by

$$f = K(2Vd + d^2)$$

where d is the fall (or rise) of potential from quadrant 1 to quadrant 2 and V is the fall (or rise) of potential from quadrant 2 to the needle. Applying this to the case in hand, for the electrometer F_1, F_2 and s_1,

$$f_1 = K\left[2e\left(-\frac{e}{2} + \frac{B}{2}\right) + e^2\right] = +KBe$$

and for the electrometer F_1, F_2, s_2,

$$f_2 = K\left[2e\left(-\frac{e}{2} - \frac{B}{2}\right) + e^2\right] = -KBe.$$

Therefore, the mirror which is cemented between s_1 and s_2 is acted upon by a couple proportional to Be. If the natural period of the instrument be high (a frequency of 3,500 vibrations per second may be obtained), and if proper damping is employed, the deflection at every instant is $\theta = kBe$, that is, the instrument follows the wave as does an ordinary oscillograph.

Obviously it is necessary to prevent sparking between the various parts of the instrument and across the condensers. Consequently the entire system, F_1, s_1, s_2, F_2, is immersed in a transparent oil of high dielectric strength and the condensers are similarly treated. It is essential that no dielectric losses occur, as they would introduce phase displacements.

Adjustment of Electrostatic Oscillograph.—If the voltage B is zero, the instrument should experience no deflection; it is, however, practically impossible to construct the instrument with the mathematical accuracy assumed above, so the following adjustment is necessary.

Make B equal to zero, that is, connect both strips to d and apply the full alternating voltage between F_1 and F_2. In general,

the strips will vibrate with a frequency twice that of the voltage. One of the condensers, C_1, C_2, is then adjusted until this deflection disappears. If the adjustment is not perfect, the two halves of a symmetrical alternating-current wave will appear dissimilar. If the connection at k is not at the correct point, the wave will be shifted with respect to the zero line.

The metallic vibrator case should be connected to d. The sensitivity can be varied by altering B.

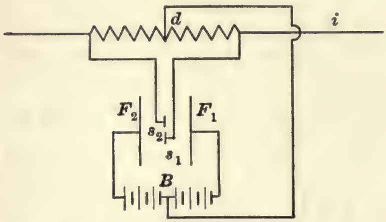


FIG. 405.—Connections for determining the wave form of a small current by electrostatic oscillograph.

The sensitivity attained is as follows: with $B = 300$ volts, $e = 2{,}000$ volts, effective, scale distance 70 cm., the amplitude of wave trace is 2 cm.

For the measurement of very small currents the connections shown in Fig. 405 may be used. The voltage B is made very large by using either a high-tension battery or two condensers in series which are continuously charged from an influence machine.

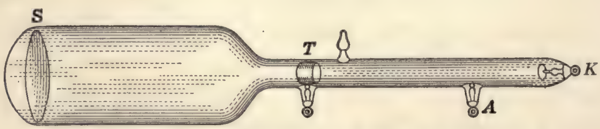


FIG. 406.—Braun tube for determining wave form.

The Braun Tube.—The Braun tube[7] is a form of vacuum tube especially designed for determining wave form.

Fig. 406 gives an idea of the tube as now made. The aluminum cathode is at K, the anode at A; they should not be less than 15 cm. apart. At T is a grounded brass target, pierced by a small hole about 0.5 mm. in diameter. The glass screen, S, is covered with zinc sulphide, or calcium tungstate.

To obtain a uniform sensitivity and a sharply defined spot where the cathode rays impinge upon the screen, it is essential that a *constant* exciting voltage be used, from 10,000 to 20,000 volts being required, unless the cathode is heated.

The cathode rays proceed perpendicularly outward from the electrode, K, and fall on the target, T, where most of them are stopped. A small pencil of rays, however, passes through the opening and falls on the screen, S, producing a bluish fluorescent spot. Just in front of the target are placed two deflecting coils

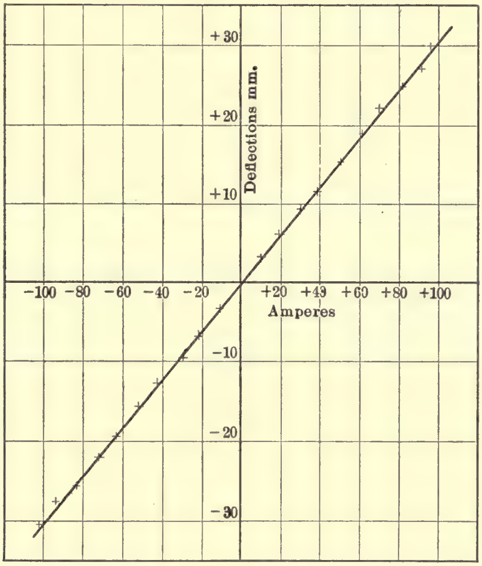


Fig. 407.—Showing proportionality of deflection and current with a Braun tube.

with their common axis transverse to that of the tube. The magnetic field set up by these coils, when traversed by a current, deflects the cathode stream and Fig. 407 illustrates the relation between the deflection of the fluorescent spot and the current. It is seen that the deflection is proportional to the current and that it reverses when the current is reversed.

When an alternating current is passed through the deflecting coils, the spot stretches out into what appears to be a fluorescent band. If this band is viewed in a revolving mirror the

wave form appears. This is the common arrangement for visual observations.

As the cathode stream is without inertia, the deflection will follow waves of the highest frequency without error.* In this the Braun tube is unique among devices for tracing wave forms.

As the cathode stream is deflected as readily in one direction as in another, the spot of light will take up a position dependent not only on the magnitude but on the direction of the resultant field due to any system of magnetizing coils. This is a second unique feature of the instrument.

Permanent Records by Braun Tubes.—To obtain a permanent record it is necessary to introduce a time coördinate in some manner. The simplest method is to photograph the fluorescent spot on a film carried by a synchronously rotating drum; the wave

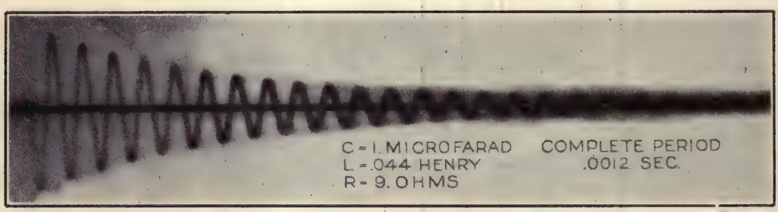


FIG. 408.—Showing oscillatory current due to the discharge of a condenser through an inductive resistance. Taken with Braun tube.

then appears on the film in rectangular coördinates. The curve shown in Fig. 408 was taken in this manner. As the photographic intensity of the fluorescent spot is not great, the Braun tube is adapted to recording periodic phenomena only. To obtain the curve shown in Fig. 408 the circuit was made and broken by a commutator attached to the axle of the drum and the spot of light traversed the same path on the film many hundred times. The inability to obtain records by a single traverse of the image of the spot over the photographic surface is a serious drawback to this form of oscillograph. Another limitation is apparent from the figure. The width of the line which is traced is considerable when compared with the amplitude of the curve.

* See paper by DR. E. L. CHAFFEE, *Proc. American Academy of Arts and Sciences*, vol. 47, 1911–12, p. 267.

Another disadvantage is that there is necessarily considerable inductance in the deflecting coils.

A second method of introducing the time coördinate is to use two sets of deflecting coils with their axes at right angles to each other and perpendicular to the axis of the tube. One set of coils is traversed by the current whose wave form is desired, the other by a current of a simple and known wave form. This auxiliary current must vary synchronously with the unknown current, and it may have a linear wave form, the current strength increasing uniformly with the time, or it may have a sinusoidal form.

Zenneck[7] uses the linear wave, which he obtains by the synchronously rotated slide-wire arrangement sketched in Fig. 409. The effect is to cause the fluorescent spot to progress regularly across the screen with every revolution of the slide wire. At the same time the regular deflecting coils cause the spot to be deflected in a perpendicular direction. The result is that the curve appears to stand still on the screen. It can be photographed by an ordinary camera or traced on a suitable transparent screen, the eye being kept in a fixed position. Perfect action of the brushes is, of course, essential.

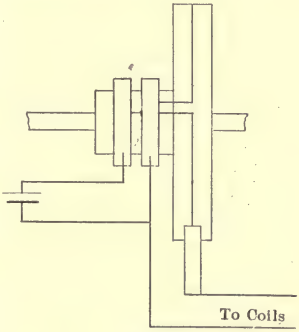


FIG. 409.—Diagram of rotary slide wire for use with Braun tube to introduce the time coördinate.

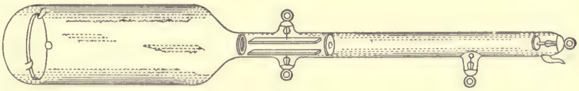


FIG. 410.—Electrostatic tube for determining wave form.

Electrostatic Tubes.—The cathode rays may also be deflected electrostatically. To this end two condenser plates are mounted so that the cathode stream passes between them, the plates being either inside or outside the tube. The deflection is proportional to the potential difference between the plates and may be adjusted by varying their distance apart. For very high voltages a condenser multiplier must be used. Ryan and Minton have em-

ployed this principle in the electrostatic power indicator, or cyclograph, see page 326.

General Considerations.—In order that the tube may operate satisfactorily it is essential that the vacuum be properly adjusted. As the degree of vacuum changes with use of the tube, (usually increasing with the time), some experimenters, in work of long duration, keep the tube attached to the air pump so that the vacuum may be adjusted at will, while others have used different forms of vacuum regulator. One such device consists of a thin platinum or palladium tube, closed at the outer end and sealed into the vacuum tube. If the vacuum be too high, the outer end of the platinum tube is heated for an instant to a dull red by a spirit lamp; a small amount of hydrogen will then pass into the tube and lower the vacuum. This arrangement gives no means of increasing the vacuum, which is sometimes necessary.

Trouble may be caused by sparking from the cathode to the glass in its immediate vicinity. Such sparking should be avoided since it causes the cathode ray stream to be unsteady. The glass becomes positively charged, and when the P.D. between the glass and the cathode becomes high enough a discharge occurs.

These two difficulties are serious and in the past have been sufficient to prevent the cathode ray tube becoming a reliable and convenient instrument, in spite of the fact that it is peculiarly adapted to certain kinds of work.

These difficulties have to do with the film of gas which adheres to the surfaces within the tube. It has been found that if the tube is exhausted at a temperature of about 350°C., and the time of exhaustion is properly adjusted, about one-half hour, a sufficient quantity of the adsorbed gas may be removed so that the vacuum will remain constant during long periods and still leave a sufficient film on the glass so that the charges on it are conducted to the cathode and neutralized.

Disturbing effects due to stray electromagnetic and electrostatic fields must be eliminated, the latter by the use of proper screens.

Electrostatic tubes with external condenser plates should be rendered non-hygroscopic in the neighborhood of the plates so that the moisture may not screen the cathode ray stream.

Cellulose nitrate made into a paste with ether and painted on the tube for a distance of a few inches on each side of the plates effectively prevents this trouble.

Soft sodium glass is the best material for the tubes as it is easily worked and gives the least trouble from electrostatic effects. Also, the proper time during which the tube must be exhausted is more readily adjusted than with other kinds of glass.

The tube may be excited by a motor-driven electrostatic machine but this becomes unreliable in damp weather. In specially equipped laboratories a storage battery of small cells, capable of giving a potential of 20,000 volts, has been used. A synchronous commutator, Fig. 411, has proved very serviceable; by it the peaks of the waves in the secondary of the small high-tension transformer T are rectified and applied to the condenser, C, of Leyden jars in parallel, around which the tube is shunted. Good contacts between the brushes and the commutators are essential. A high-resistance rod, r, of about 100,000 ohms is connected in series and directly to the cathode to prevent flash-over and to cause the tube to operate more steadily.

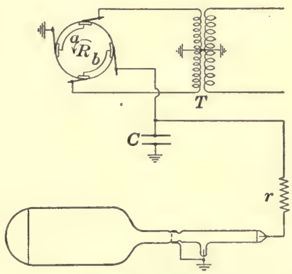


FIG. 411.—Diagram of synchronous commutator method of exciting Braun tube.

The intensity of the spot may be increased by using a focusing coil traversed by direct current. The coil is placed axially over the tube between the anode and the cathode. By this coil the sensitiveness of the tube may be varied somewhat, and the voltage necessary for the operation of the tube is reduced. Careful regulation of the direct current is necessary.

When working with high-frequency currents there will be, on account of the inductance, large differences of potential between different parts of the deflecting coils and a large electrostatic deflection of the spot may occur. This may be eliminated by surrounding the tube, inside the deflecting coils, by a split solenoid of fine insulated wire wound on a paper tube. This forms an effective electrostatic shield and has no influence on the electromagnetic deflection.

WAVE ANALYSIS

Curves taken by the foregoing methods show that in practice both the potential difference and the current waves may depart widely from the sinusoidal form. In any particular case after having obtained the graph of the wave, it is possible to write its equation as the sum of a series of sinusoidal terms of multiple frequencies which have the proper time-phase relations.

To effect such an harmonic analysis, recourse is had to the work of Fourier who, in 1812, first explicitly showed that a function which is subject to certain mathematical conditions can be represented by a constant term plus the sum of a sine and a cosine series.[8] This result he published in his "Théorie Analytique de Chaleur," 1822.

Accordingly

$$\left.\begin{aligned} f(\theta) &= A_1 \sin\theta + A_2 \sin 2\theta + A_3 \sin 3\theta + \ldots \\ &+ \frac{B_0}{2} + B_1 \cos\theta + B_2 \cos 2\theta + B_3 \cos 3\theta + \ldots \end{aligned}\right\} \quad (9)$$

The coefficients are given by the following equations:[8]

$$A_k = \frac{1}{\pi}\int_0^{2\pi} f(\theta) \sin k\theta d\theta \quad (10)$$

$$B_k = \frac{1}{\pi}\int_0^{2\pi} f(\theta) \cos k\theta d\theta \quad (11)$$

The expression (9) is called a Fourier series.

The sine and cosine terms may be combined, for

$$A \sin\theta + B \cos\theta = \sqrt{A^2 + B^2} \sin\left(\theta + \tan^{-1}\frac{B}{A}\right) = C \sin(\theta + \alpha').$$

This gives

$$f(\theta) = \frac{B_0}{2} + C_1 \sin(\theta + \alpha'_1) + C_2 \sin(2\theta + \alpha'_2) + C_3 \sin(3\theta + \alpha'_3) + \ldots \quad (12)$$

If the origin is taken at the zero of the fundamental, which is convenient if the waves are to be plotted,

$$f(\theta) = \frac{B_0}{2} + C_1 \sin\theta + C_2 \sin 2(\theta + \alpha_2) + C_3 \sin 3(\theta + \alpha_3) + \ldots \quad (12a)$$

where

$$\alpha_2 = \frac{\alpha'_2}{2} - \alpha'_1 \quad \alpha_3 = \frac{\alpha'_3}{3} - \alpha'_1; \text{ etc.}$$

Here α_n is measured on the same scale as θ and the zero point of any component is where it first passes from a negative to a positive value. A positive value of α_n indicates a leading component. The constant term is usually absent in alternating-current waves.

The effect of the odd and even harmonics and of their phase displacements is illustrated in Fig. 412. Odd harmonics produce a wave the second half of which is like the first with the algebraic signs of all the ordinates reversed. Even harmonics produce an asymmetrical wave.

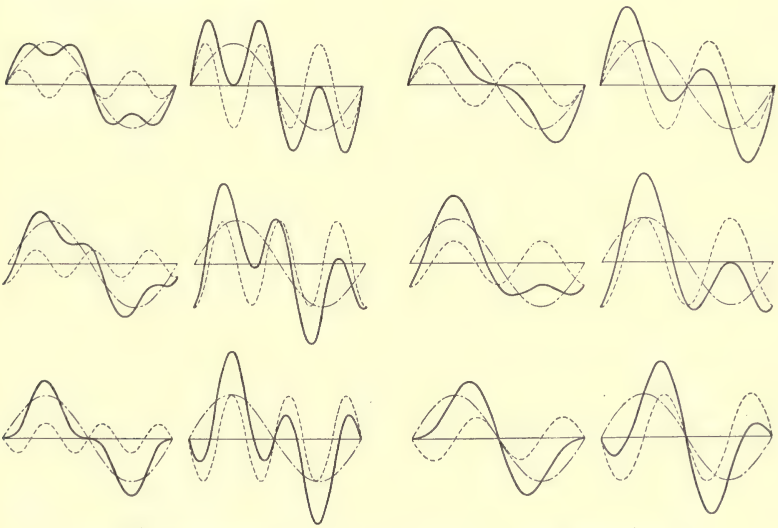


FIG. 412.—Illustrating the effects of odd and of even harmonics.

The integrals in (10) and (11) are proportional to the areas under the curves which would be obtained if each value of $f(\theta)$ were multiplied by the corresponding value of sin $k\theta$ or cos $k\theta$ and new curves plotted on the same base, 2π.

Therefore,

$$A_k = \text{twice the average ordinate of curve } f(\theta) \sin k\theta \qquad (10a)$$

$$B_k = \text{twice the average ordinate of curve } f(\theta) \cos k\theta \qquad (11a)$$

The integration could be performed by a planimeter.

The labor involved in carrying out the process just indicated

is prohibitive, but machines called harmonic analyzers have been devised, some of which practically effect the determination of A_k and B_k in this manner.

The various coefficients may be determined arithmetically as follows: A complete cycle of the curve to be analyzed is plotted. The base, (2π), is then divided into $2n$ equal spaces, and ordinates erected. Let the measured values of the $2n$ ordinates be $y_0, y_1, y_2, \ldots y_{2n-1}$. The distance between consecutive ordinates is $\Delta\theta = \frac{2\pi}{2n} = \frac{\pi}{n}$.

By (10a),

$$A_k = \frac{2}{2n}[y_0 \sin 0\,k\Delta\theta + y_1 \sin 1k\Delta\theta + y_2 \sin 2k\Delta\theta + \ldots + y_{2n-2} \sin (2n-2)\,k\Delta\theta + y_{2n-1} \sin (2n-1)\,k\Delta\theta] \quad (13)$$

If the data are furnished by the contact method, the measurements being made at equal intervals along the wave, the values of y_0, y_1, etc., are given directly and the curve need not be plotted. Equation (13) may be written

$$A_k = \frac{1}{n}\sum_{m=0}^{m=2n-1} y_m \sin km\,\Delta\theta \quad (14)$$

where m is the number of the ordinate concerned in the multiplication; similarly

$$B_k = \frac{1}{n}[y_0 \cos 0kd\theta + y_1 \cos k\Delta\theta + y_2 \cos 2k\Delta\theta + \ldots + y_{2n-2} \cos (2n-2)k\Delta\theta + y_{2n-1} \cos (2n-1)\,k\Delta\theta] \quad (15)$$

$$= \frac{1}{n}\sum_{m=0}^{m=2n-1} y_m \cos km\Delta\theta \quad (16)$$

Runge Method of Grouping Terms.—Theoretically it is easy to calculate the coefficients. The practical difficulty lies in the great expenditure of time which is necessary for carrying out the process as indicated. For this reason Runge[9] has introduced an abridged method of calculation which is carried out by aid of a systematically arranged schedule.

In the majority of cases, the two halves of an alternating-current wave are the same except for the algebraic sign; that is,

only the odd harmonics are present, and consequently the values of k are odd numbers. It is necessary, therefore, to deal with only one-half of the wave, and $2n$ spaces per *half* wave are used, making $\Delta\theta = \frac{\pi}{2n}$.

The method of grouping the terms so as to economize time may be explained as follows. Referring to (13), y_1 is multiplied by $\sin\frac{k\pi}{2n}$ and y_{2n-1} by $\sin(2n-1)\frac{k\pi}{2n}$. As k is odd,

$$\sin\frac{k\pi}{2n} = \sin\left(k\pi - \frac{k\pi}{2n}\right) = \sin(2n-1)\frac{k\pi}{2n}$$

and in general,

$$\sin(2n-m)\frac{k\pi}{2n} = \sin\frac{mk\pi}{2n} \tag{17}$$

Consequently,

1. The number of multiplications may be halved by adding, before taking the products, those values of y for which the sum of the subscripts is $2n$.

2. Again, the same products are needed in A_k and A_{2n-k}; that is, in those coefficients for which the sum of the subscripts is $2n$, since

$$\pm\sin(2n-k)\frac{m\pi}{2n} = \sin k\frac{m\pi}{2n} \tag{18}$$

If the number of the ordinate concerned in the multiplication (m) is even, the sign of the left hand member is $-$, and if m is odd the sign is $+$.

For example, suppose a half period is divided into $2n = 12$ equal parts $\left(\Delta\theta = \frac{\pi}{2n} \backsimeq 15°\right)$ and the ordinates measured; then by (13) and (17)

$A_1 = \frac{1}{6}[(y_1 + y_{11})\sin 15° + (y_2 + y_{10})\sin 30° +$
$(y_3 + y_9)\sin 45° + (y_4 + y_8)\sin 60° + (y_5 + y_7)\sin 75° + y_6\sin 90°]$
$A_{11} = \frac{1}{6}[(y_1 + y_{11})\sin 165° + (y_2 + y_{10})\sin 330° +$
$(y_3 + y_9)\sin 495° + (y_4 + y_8)\sin 660° + (y_5 + y_7)\sin 825° +$
$y_6\sin 990°]$.

For convenience the sines of all the angles may be expressed in terms of the sines of angles of 90° or less.

Accordingly, by the aid of (18) the value for A_{11} may be written,

$$A_{11} = \tfrac{1}{6}\,[(y_1 + y_{11}) \sin 15^\circ - (y_2 + y_{10}) \sin 30^\circ + (y_3 + y_9) \sin 45^\circ - (y_4 + y_8) \sin 60^\circ + (y_5 + y_7) \sin 75^\circ - y_6 \sin 90^\circ].$$

Applying the rules,

$$A_3 = \tfrac{1}{6}\,[\{(y_1 + y_{11}) + (y_3 + y_9) - (y_5 + y_7)\} \sin 45^\circ + \{(y_2 + y_{10}) - y_6\} \sin 90^\circ]$$

$$A_9 = \tfrac{1}{6}\,[\{(y_1 + y_{11}) + (y_3 + y_9) - (y_5 + y_7)\} \sin 45 - \{(y_2 + y_{10}) - y_6\} \sin 90^\circ]$$

$$A_5 = \tfrac{1}{6}\,[(y_1 + y_{11}) \sin 75^\circ + (y_2 + y_{10}) \sin 30^\circ - (y_3 + y_9) \sin 45^\circ - (y_4 + y_8) \sin 60^\circ + (y_5 + y_7) \sin 15^\circ + y_6 \sin 90^\circ]$$

$$A_7 = \tfrac{1}{6}\,[(y_1 + y_{11}) \sin 75^\circ - (y_2 + y_{10}) \sin 30^\circ - (y_3 + y_9) \sin 45^\circ + (y_4 + y_8) \sin 60^\circ + (y_5 + y_7) \sin 15^\circ - y_6 \sin 90^\circ].$$

Cosine Terms.—The cosine terms may be treated in a similar manner.

As
$$-\cos\,(2n - m)\,\frac{k\pi}{2n} = \cos\frac{mk\pi}{2n} \qquad (19)$$

the differences of the ordinates are involved.
The equation corresponding to (18) is

$$\mp \cos\,(2n - k)\,\frac{m\pi}{2n} = \cos k\,\frac{m\pi}{2n} \qquad (20)$$

The sign of the left-hand member is + if m is even and − if m is odd. Applying the above and, for convenience, expressing the results in terms of the sines of the angles, the values of B are

$$B_1 = \tfrac{1}{6}\,[y_0 \sin 90^\circ + (y_1 - y_{11}) \sin 75^\circ + (y_2 - y_{10}) \sin 60^\circ + (y_3 - y_9) \sin 45^\circ + (y_4 - y_8) \sin 30^\circ + (y_5 - y_7) \sin 15^\circ]$$

$$B_{11} = \tfrac{1}{6}\,[y_0 \sin 90^\circ - (y_1 - y_{11}) \sin 75^\circ + (y_2 - y_{10}) \sin 60^\circ - (y_3 - y_9) \sin 45^\circ + (y_4 - y_8) \sin 30^\circ - (y_5 - y_7) \sin 15^\circ]$$

$$B_3 = \tfrac{1}{6}\,[\{(y_1 - y_{11}) - (y_3 - y_9) - (y_5 - y_7)\} \sin 45^\circ + \{y_0 - (y_4 - y_8)\} \sin 90^\circ]$$

$$B_9 = \tfrac{1}{6}\,[\{-(y_1 - y_{11}) + (y_3 - y_9) + (y_5 - y_7)\} \sin 45^\circ + \{y_0 - (y_4 - y_8)\} \sin 90^\circ]$$

$$B_5 = \tfrac{1}{6}\,[y_0 \sin 90^\circ + (y_1 - y_{11}) \sin 15^\circ - (y_2 - y_{10}) \sin 60^\circ - (y_3 - y_9) \sin 45^\circ + (y_4 - y_8) \sin 30^\circ + (y_5 - y_7) \sin 75^\circ]$$

$$B_7 = \tfrac{1}{6}\,[y_0 \sin 90^\circ - (y_1 - y_{11}) \sin 15^\circ - (y_2 - y_{10}) \sin 60 + (y_3 - y_9) \sin 45^\circ + (y_4 - y_8) \sin 30^\circ - (y_5 - y_7) \sin 75^\circ].$$

The calculations may be systematized by the use of properly prepared forms such, for example, as that following.

The numerical work may be checked by using particular values of θ.

$$y_0 = (B_1 + B_{11}) + (B_3 + B_9) + (B_5 + B_7)$$
$$y_6 = (A_1 - A_{11}) - (A_3 - A_9) + (A_5 - A_7)$$
$$y_3 + y_9 = 2 \sin 45° \, [(A_1 + A_{11}) + (A_3 + A_9) - (A_5 + A_7)]$$
$$y_3 - y_9 = 2 \sin 45° \, [(B_1 - B_{11}) - (B_3 - B_9) - (B_5 - B_7)]$$
$$y_4 + y_8 = 2 \sin 60° \, [(A_1 - A_{11}) - (A_5 - A_7)]$$

Following the plan outlined above, schedules corresponding to any number of measured ordinates may be prepared. If even harmonics be present it is necessary to divide the *whole* wave into $2n$ parts.

The values of the coefficients A and B, obtained as above by the use of a definite number of measured ordinates, determine a curve which coincides with the original curve *at the measured points* and diverges from the original curve at intermediate points; consequently the more complicated the wave form, the greater the number of ordinates which must be used. A schedule based on 18 instead of 12 ordinates is frequently necessary. As a check on the sufficiency of the analysis, values of y intermediate between the measured values should be calculated and compared with the actual ordinates at the same points.

ILLUSTRATION OF THE USE OF A TWELVE-POINT SCHEDULE FOR THE ANALYSIS OF WAVES CONTAINING ONLY ODD HARMONICS

$$\Delta\theta = \frac{\pi}{2n} = \frac{\pi}{12} \approx 15°.$$

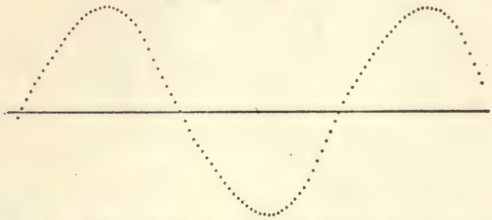


FIG. 413.—E.M.F. wave of small alternator.

The e.m.f. wave of a small alternator is shown in Fig. 413. To analyze this wave twelve equally spaced ordinates were measured; their values are entered in the form below,

$y_0 = 0.30$	$y_1 = 8.50$	$y_2 = 14.30$	$y_3 = 20.60$
	$y_{11} = 8.70$	$y_{10} = 18.40$	$y_9 = 26.00$
Sums	$y_1 + y_{11} = 17.20$	$y_2 + y_{10} = 32.70$	$y_3 + y_9 = 46.60$
Differences	$y_1 - y_{11} = -0.20$	$y_2 - y_{10} = -4.10$	$y_3 - y_9 = -5.40$
	$y_4 = 26.15$	$y_5 = 29.80$	$y_6 = 32.25$
	$y_8 = 30.70$	$y_7 = 32.90$	$y_0 = 0.30$
Sums	$y_4 + y_8 = 56.85$	$y_5 + y_7 = 62.70$	$y_0 + y_6 = 32.55$
Differences	$y_4 - y_8 = -4.55$	$y_5 - y_7 = -3.10$	

Calculation of A_1 and A_{11}

$(y_1 + y_{11}) \sin 15° = (17.2)\ 0.2588 =$ ....→	4.452	
$(y_3 + y_9) \sin 45° = (46.6)\ 0.7071 =$ ....→	32.951	
$(y_5 + y_7) \sin 75° = (62.7)\ 0.9659 =$ ....→	60.562	
$(y_2 + y_{10}) \sin 30° = (32.7)\ 0.5000 =$	........→	16.350
$(y_4 + y_8) \sin 60° = (56.85)\ 0.8660 =$	........→	49.233
$y_6 \sin 90° = (32.25)\ 1.0000 =$	........→	32.250
Sums	$(97.965)_1$	$(97.833)_2$

	$(97.965)_1$		$(97.965)_1$
	$(97.833)_2$		$(97.833)_2$
Sum	$195.798 = 6A_1$	Difference	$0.132 = 6A_{11}$
	$A_1 = 32.633$		$A_{11} = 0.022$

Calculation of A_3 and A_9

$$
\begin{aligned}
y_1 + y_{11} &= 17.20 \\
y_3 + y_9 &= 46.60 \\
\hline
\text{Sum} &= 63.80 \\
y_5 + y_7 &= 62.70 \\
\hline
\text{Difference} &= 1.10
\end{aligned}
$$

$$\{(y_1 + y_{11}) + (y_3 + y_9) - (y_5 + y_7)\} \sin 45° = (1.1)\, 0.7071 = (0.778)_1$$

$$
\begin{aligned}
y_2 + y_{10} &= 32.76 \\
y_6 &= 32.25 \\
\hline
\text{Difference} &= 0.45
\end{aligned}
$$

$$\{(y_2 + y_{10}) - y_6\} \sin 90° = \ldots\ldots\ldots\ldots (0.450)_2$$

$$
\begin{array}{llll}
 & (0.778)_1 & & (0.778)_1 \\
 & (0.450)_2 & & (0.450)_2 \\
\hline
\text{Sum} & 1.228 = 6A_3 & \text{Difference} & 0.328 = 6A_9 \\
 & A_3 = 0.205 & & A_9 = 0.055
\end{array}
$$

Calculation of A_5 and A_7

$$
\begin{aligned}
(y_1 + y_{11}) \sin 75° = (17.2)\, 0.9659 &= 16.613 \\
(y_5 + y_7) \sin 15° = (62.7)\, 0.2588 &= 16.227 \\
\hline
\text{Sum} &= 32.840 \\
(y_3 + y_9) \sin 45° = (46.6)\, 0.7071 &= 32.951 \\
\hline
\text{Difference} &= (-0.111)_1 \\
(y_2 - y_{10}) \sin 30° = (32.7)\, 0.5000 &= 16.35 \\
y_6 \sin 90° = \ldots\ldots\ldots\ldots &= 32.25 \\
\hline
\text{Sum} &= 48.600 \\
(y_4 + y_8) \sin 60° = (56.85)\, 0.8660 &= 49.232 \\
\hline
\text{Difference} &= (-0.632)_2
\end{aligned}
$$

$$
\begin{array}{llll}
 & (-0.111)_1 & & (-0.111)_1 \\
 & (-0.632)_2 & & (-0.632)_2 \\
\hline
\text{Sum} & -0.743 = 6A_5 & \text{Difference} & +0.521 = 6A_7 \\
 & A_5 = -0.124 & & A_7 = +0.087
\end{array}
$$

Calculation of B_1 and B_{11}

$y_0 = (+0.30)\ 1.000 = \ldots\ldots\rightarrow$	0.300	
$(y_2 - y_{10}) \sin 60° = (-4.10)\ 0.8660 = \ldots\ldots\rightarrow$	-3.551	
$(y_4 - y_8) \sin 30° = (-4.55)\ 0.5000 = \ldots\ldots\rightarrow$	-2.275	
$(y_1 - y_{11}) \sin 75° = (-0.20)\ 0.9659 = \ldots\ldots\rightarrow$	$\ldots\ldots\ldots\ldots$	-0.193
$(y_3 - y_9) \sin 45° = (-5.4)\ 0.7071 = \ldots\ldots$	$\ldots\ldots\ldots\rightarrow$	-3.818
$(y_5 - y_7) \sin 15° = (-3.1)\ 0.2588 = \ldots\ldots$	$\ldots\ldots\ldots\rightarrow$	-0.802
Sums $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$	$(-5.526)_1$	$(-4.813)_2$

$$\begin{array}{ll} (-5.526)_1 & \\ (-4.813)_2 & \\ \hline \text{Sum} \quad -10.339 = 6B_1 & \\ B_1 = -1.723 & \end{array} \qquad \begin{array}{l} (-5.526)_1 \\ (-4.813)_2 \\ \hline \text{Difference} \quad -0.713 = 6B_{11} \\ B_{11} = -0.119 \end{array}$$

Calculation of B_3 and B_9

$$\begin{array}{rl} y_3 - y_9 = & -5.40 \\ y_5 - y_7 = & -3.10 \\ \hline \text{Sum} = & -8.50 \\ y_1 - y_{11} = & -0.20 \\ \hline \text{Difference} = & -8.30 \end{array}$$

$$\{-(y_1 - y_{11}) + (y_3 - y_9) + (y_5 - y_7)\} \sin 45° = (-8.30)\ 0.7071 = (-5.869)_2$$

$$\begin{array}{rl} y_4 - y_8 = & -4.55 \\ y_0 = & 0.30 \\ \hline \text{Difference} = & -4.85 \end{array}$$

$$\{(y_4 - y_8) - y_0\} \sin 90° = \ldots\ldots\ldots\ldots\ldots\ldots (-4.85)_1$$

$$\begin{array}{l} (-4.850)_1 \\ (-5.869)_2 \\ \hline \text{Sum} \quad -10.719 = -6B_3 \\ B_3 = 1.786 \end{array} \qquad \begin{array}{l} (-4.850)_1 \\ (-5.869)_2 \\ \hline \text{Difference} \quad +1.019 = -6B_9 \\ B_9 = -0.170 \end{array}$$

Calculation of B_5 and B_7

y_0	=	0.300
$(y_4 - y_8)\sin 30° = (-4.55)\ 0.5000$	=	-2.275
Sum	=	-1.975
$(y_2 - y_{10})\sin 60° = (-4.1)\ 0.8660$	=	-3.551
Difference	=	$(+1.576)_1$
$(y_1 - y_{11})\sin 15° = (-0.20)\ 0.2588$	=	-0.052
$(y_5 - y_7)\sin 75° = (-3.1)\ 0.9659$	=	-2.994
Sum		-3.046
$(y_3 - y_9)\sin 45° = (-5.4)\ 0.7071$	=	-3.818
Difference		$(+0.772)_2$

	$(1.576)_1$		$(1.576)_1$
	$(0.772)_2$		$(0.772)_2$
Sum	$2.348 = 6B_5$	Difference	$0.804 = 6B_7$
	$B_5 = 0.391$		$B_7 = 0.134$

Checks, see page 655

	+	−
B_1	.	1.723
B_3	1.786	.
B_5	0.391	.
B_7	0.134	.
B_9	.	0.170
B_{11}	.	0.119
Sums	2.311	2.012
Net sum	0.299	
$y_0 =$	0.300	

A_1 =	32.633	
A_{11} =	0.022	
Sum = ...		$(32.655)_1$
Difference =	$(32.611)_2$	
A_3 =	0.204	
A_9 =	0.055	
Sum = ...		$(0.259)_3$
Difference =	$(0.149)_4$	
Sum..................		32.914
$A_5 =$	-0.124	
$A_7 =$	$+0.087$	
Sum..................		$(-0.037)_5$
Difference =	$(-0.211)_6$	
Difference............		(32.951)

$(32.951)\ (1.414) = 46.59$

From curve $y_3 + y_9 = 46.60$

	$(32.611)_2$
	$(-0.211)_6$
Diff.	$[32.822]\ 1.732 = 56.84$
From curve $y_4 + y_8 = 56.85$	
	$(32.611)_2$
	$(-0.211)_6$
Sum	32.400
	$(0.149)_4$
Diff.	$[32.251]$
From curve $y_6 = 32.25$	

$$B_3 = \quad 1.78$$
$$B_9 = -0.17$$

Difference . . . . $(+1.95)_7$

$$B_5 = 0.39$$
$$B_7 = 0.13$$

Difference . . . . $(0.26)_8$

Sum 2.21

$$B_1 = -1.720$$
$$B_{11} = -0.119$$

Difference . . . . $(-1.601)_9$

Difference . . . . $[+3.81]1.414 = +5.37$

From curve . . . . $-(y_3 - y_9) = +5.40$

$$C_1 = \sqrt{(32.633)^2 + (1.72)^2} = 32.678 \qquad \tan^{-1}\frac{B_1}{A_1} = \tan^{-1}\frac{-1.72}{+32.633} = -3.^\circ 02$$

$$C_3 = \sqrt{(0.204)^2 + (1.78)^2} = 1.798 \qquad \tan^{-1}\frac{B_3}{A_3} = \tan^{-1}\frac{+1.78}{+0.204} = +83.^\circ 5$$

$$C_5 = \sqrt{(0.124)^2 + (0.391)^2} = 0.410 \qquad \tan^{-1}\frac{B_5}{A_5} = \tan^{-1}\frac{+0.391}{-0.224} = +107.^\circ 6$$

$$C_7 = \sqrt{(0.087)^2 + (0.134)^2} = 0.160 \qquad \tan^{-1}\frac{B_7}{A_7} = \tan^{-1}\frac{+0.134}{+0.087} = +57.^\circ 0$$

$$C_9 = \sqrt{(0.055)^2 + (0.170)^2} = 0.179 \qquad \tan^{-1}\frac{B_9}{A_9} = \tan^{-1}\frac{-0.170}{+0.055} = -72.^\circ 1$$

$$C_{11} = \sqrt{(0.022)^2 + (0.119)^2} = 0.121 \qquad \tan\frac{B_{11}}{A_{11}} = \tan^{-1}\frac{-0.119}{+0.022} = -79.^\circ 5$$

Therefore the equation of the curve is

$$\begin{aligned} e = {} & 32.678 \sin (\omega t - 3.^\circ 02) + 1.798 \sin 3(\omega t + 27.^\circ 8) \\ & + 0.410 \sin 5(\omega t + 21.^\circ 52) + 0.160 \sin 7(\omega t + 8.^\circ 14) \\ & + 0.179 \sin 9(\omega t - 8.^\circ 0) + 0.121 \sin 11(\omega t - 7.^\circ 2) \end{aligned}$$

Fischer-Hinnen Method of Analysis.[10]—A convenient method of harmonic analysis and one in which the arithmetical work is reduced to a minimum is due to Fischer-Hinnen; the procedure is based on two mathematical laws which are demonstrated below.

Suppose a wave has been plotted and the length *ab* (Fig. 414), is that of a complete cycle, or 360° of the fundamental. Then between *a* and *b* there will be:

1 complete period of the fundamental
3 complete periods of the third harmonic
5 complete periods of the fifth harmonic.

Denote by k the number of complete periods of any harmonic comprised between *a* and *b*. Then

$k = 1$ for the fundamental
$k = 3$ for the third harmonic
$k = 5$ for the fifth harmonic.

. . . .

The equation of the sine curve corresponding to any particular harmonic will be

$$Y = A_k \sin k\,(\theta + \alpha)$$

where both θ and α are expressed in degrees of the fundamental. Now let *ab*, which corresponds to a *whole* wave, be divided into P equal parts and P ordinates erected, the first being coincident with *a*. In Fig. 414

$$k = 3, \text{ and } P = 7.$$

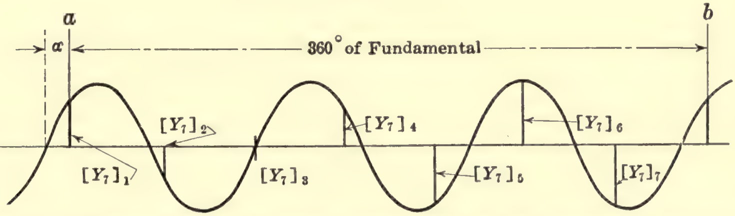


Fig. 414.—Pertaining to Fischer-Hinnen method of harmonic analysis. $k = 3$, $P = 7$

Denote the various ordinates thus $[Y_P]_1$, $[Y_P]_2$ . . . ; the subscript, P, within the bracket shows the number of sections into which the base *ab* is divided, while the subscript outside the

bracket shows the number of the particular ordinate under consideration.

$$[Y_P]_1 = A_k \sin k\alpha$$

$$[Y_P]_2 = A_k \sin\left(k\frac{360°}{P} + k\alpha\right)$$

$$[Y_P]_3 = A_k \sin\left(2k\frac{360°}{P} + k\alpha\right)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$[Y_P]_P = A_k \sin\left((P-1)k\frac{360°}{P} + k\alpha\right).$$

Then the sum of the P ordinates is

$$[Y_P]_1 + [Y_P]_2 + [Y_P]_3 + \ldots + [Y_P]_P =$$

$$A_k\Bigg[\sin k\alpha\left\{1 + \cos\left(\frac{k360°}{P}\right) + \cos 2\left(\frac{k360°}{P}\right) + \cos 3\left(\frac{k360°}{P}\right) + \ldots + \cos(P-1)\left(\frac{k360°}{P}\right)\right\} + \cos k\alpha\left\{\sin\left(\frac{k360°}{P}\right) + \sin 2\left(\frac{k360°}{P}\right) + \sin 3\left(\frac{k360°}{P}\right) + \ldots + \sin(P-1)\left(\frac{k360°}{P}\right)\right\}\Bigg]. \qquad (21)$$

Inspection shows that if $\frac{k}{P}$ is a *whole* number,

$$[Y_P]_1 + [Y_P]_2 + [Y_P]_3 + \ldots + [Y_P]_P = PA_k \sin k\alpha = P[Y_P]_1 \qquad (22)$$

That is, when $\frac{k}{P}$ is a *whole number* the sum of P equally spaced ordinates is equal to P times the first ordinate. This is the first of the laws referred to above.

If $\frac{k}{P}$ is *not* a whole number the above series can be summed by aid of the following trigonometrical formulæ.

$$\cos\theta + \cos 2\theta + \cos 3\theta + \ldots + \cos(P-1)\theta = -\tfrac{1}{2} + \frac{\cos(P-1)\theta - \cos P\theta}{2(1-\cos\theta)} \qquad (23)$$

$$\sin\theta + \sin 2\theta + \sin 3\theta + \ldots + \sin(P-1)\theta = \frac{\sin\frac{(P-1)\theta}{2}\sin\frac{P\theta}{2}}{\sin\frac{\theta}{2}} \qquad (24)$$

In this case when

$\theta = \dfrac{k360°}{P}$, and $\dfrac{k}{P}$ is not a whole number, both the series in (21) reduce to zero.

Consequently

$$[Y_P]_1 + [Y_P]_2 + [Y_P]_3 + \ . \ . \ . + [Y_P]_P = 0 \qquad (25)$$

That is, when $\dfrac{k}{P}$ is not a whole number the sum of P equally spaced ordinates is zero. This is the second of the two laws upon which this method depends.

These relations are used as follows: The wave is plotted and any point is taken as the origin.

At the origin, $t = 0$, all the sine terms are zero and all the cosine terms have their maximum values, that is, B_1, B_3, B_5 . . . so

$$Y_1 = B_1 + B_3 + B_5 + \ . \ . \ .$$

To find B_3:—

Between a and b there are three complete periods of the third harmonic, nine complete periods of the ninth harmonic, fifteen complete periods of the fifteenth harmonic, and so on.

Divide the base ab into three equal parts. Then $P = 3$ and

$$\frac{k}{P} = 1 \text{ for the third harmonic}$$

$$\frac{k}{P} = 3 \text{ for the ninth harmonic}$$

$$\frac{k}{P} = 5 \text{ for the fifteenth harmonic.}$$

. . . .

These are all whole numbers and by (22)

$$[Y_3]_1 + [Y_3]_2 + [Y_3]_3 = 3[B_3 + B_9 + B_{15} + B_{21} + \ldots]$$

To find B_5:—

Divide the base ab into five equal parts, $P = 5$; between a and b there are five complete periods of the fifth harmonic, fifteen of the fifteenth harmonic, so

$$\frac{k}{P} = 1 \text{ for the fifth harmonic}$$

$$\frac{k}{P} = 3 \text{ for the fifteenth harmonic.}$$

. . . .

Consequently

$$[Y_5]_1 + [Y_5]_2 + [Y_5]_3 + [Y_5]_4 + [Y_5]_5 = 5[B_5 + B_{15} + B_{25} + \ldots]$$

Similarly by dividing the base into seven and into nine equal parts,

$$[Y_7]_1 + [Y_7]_2 + [Y_7]_3 + \ldots + [Y_7]_7 = 7[B_7 + B_{21} + B_{35}\ldots]$$
$$[Y_9]_1 + [Y_9]_2 + [Y_9]_3 + \ldots + [Y_9]_9 = 9[B_9 + B_{27} + B_{45}\ldots]$$

It is convenient to erect the first ordinate at the point where the curve crosses the axis. In that case $Y_1 = 0$ and

$$B_1 + B_3 + B_5 + B_7 + \ldots = 0$$

In practice the process is somewhat simplified, for except in special cases the harmonics above the seventh are not important. So

$$\begin{aligned} B_3 &= \tfrac{1}{3}\,\Sigma\,[Y_3] \text{ approximately} \\ B_5 &= \tfrac{1}{5}\,\Sigma\,[Y_5] \quad \text{“} \\ B_7 &= \tfrac{1}{7}\,\Sigma\,[Y_7] \quad \text{“} \\ B_9 &= \tfrac{1}{9}\,\Sigma\,[Y_9] \quad \text{“} \\ B_1 &= -B_3 - B_5 - B_7. \end{aligned}$$

When these approximations are used, by appropriately dividing the base, tests may be applied to detect the presence of higher harmonics. If they are present the approximate values of the lower harmonics may be corrected; for instance, if the 9th be present, then

$$B_3 = \tfrac{1}{3}\,\Sigma\,[Y_3] - B_9$$

To find A_1, A_3, etc.

As these are the coefficients of the sine terms, which will have their maximum values a quarter period from the initial ordinate $[Y_P]_1$, draw the first of the new set of ordinates $[Y_P]'_1$ a quarter period from $[Y_P]_1$. At this point, all the cosine terms are zero and consequently add nothing to the value of the ordinate.

The initial ordinate of the fundamental, as well as that of the fifth and of the ninth harmonic, is positive, while the initial ordinate of the third and of the seventh harmonic is negative.

$$Y'_1 = A_1 - A_3 + A_5 - A_7 + A_9\ldots$$

When the base has been divided into three, five, seven, etc., equal parts, by the rules already given,

$$[Y_3]'_1 + [Y_3]'_2 + [Y_3]'_3 = 3\,[-A_3 + A_9 - A_{15}\ldots]$$
$$[Y_5]'_1 + [Y_5]'_2 + [Y_5]'_3 \ldots + [Y_5]'_5 = 5\,[A_5 - A_{15}\ldots]$$
$$[Y_7]'_1 + [Y_7]'_2 + [Y_7]'_3 \ldots + [Y_7]'_7 = 7\,[-A_7 + A_{21}\ldots]$$
$$[Y_9]'_1 + [Y_9]'_2 + [Y_9]'_3 + \ldots + [Y_9]'_9 = 9\,[A_9 - A_{27}\ldots]$$

Therefore,

$$A_3 = -\tfrac{1}{3}\Sigma[Y_3]' \text{ approximately}$$
$$A_5 = \tfrac{1}{5}\Sigma[Y_5]' \quad \text{“}$$
$$A_7 = -\tfrac{1}{7}\Sigma[Y_7]' \quad \text{“}$$
$$A_9 = \tfrac{1}{9}\Sigma[Y_9]' \quad \text{“}$$
$$Y_1' = A_1 - A_3 + A_5 - A_7 + A_9 \ldots .$$

When dealing with waves containing only the odd harmonics, it is necessary to plot only one-half the wave, since the second half is like the first with the algebraic sign of the ordinates reversed. Suppose the *half wave* has been divided into $2k$ parts, equivalent to dividing the whole wave into $4k$ parts, then the ordinate $[Y_k]_1$ is identical with $[Y_{4k}]_1$. The relation of the number, N_{4k}, of any ordinate when the whole base is divided into $4k$ parts to the number of the same ordinate, N_k, when the whole base has been divided into k parts is given by

$$4(N_k - 1) + 1 = 4N_k - 3 = N_{4k}.$$

Consequently

$$B_3 = \tfrac{1}{3}\left[[Y_{12}]_1 + [Y_{12}]_5 + [Y_{12}]_9\right] = \tfrac{1}{3}\left[[Y_{12}]_1 + [Y_{12}]_5 - [Y_{12}]_3\right]$$
$$B_5 = \tfrac{1}{5}\left[[Y_{20}]_1 + [Y_{20}]_5 + [Y_{20}]_9 - [Y_{20}]_3 - [Y_{20}]_7\right]$$
$$B_7 = \tfrac{1}{7}\left[[Y_{28}]_1 + [Y_{28}]_5 + [Y_{28}]_9 + [Y_{28}]_{13} - [Y_{28}]_3 - [Y_{28}]_7 - [Y_{28}]_{11}\right].$$

When the sine coefficients are determined, the ordinate $[Y_k]'_1$ is identical with $[Y_{4k}]_{k+1}$ for when the sine terms are determined the initial ordinate is transferred k spaces to the right. In this case the number of any ordinate, N_{4k}, when the whole base has been divided into $4k$ parts is related to the number of the same ordinate, N'_k, when the whole base has been divided into k parts, as follows:

$$4(N'_k - 1) + k + 1 = 4N'_k - 3 + k = N_{4k}.$$

So

$$A_3 = -\tfrac{1}{3}\left[[Y_{12}]_4 + [Y_{12}]_8 + [Y_{12}]_{12}\right] = -\tfrac{1}{3}\left[[Y_{12}]_4 - [Y_{12}]_2 - [Y_{12}]_6\right]$$
$$A_5 = \tfrac{1}{5}\left[[Y_{20}]_2 - [Y_{20}]_4 + [Y_{20}]_6 - [Y_{20}]_8 + [Y_{20}]_{10}\right]$$
$$A_7 = \tfrac{1}{7}\left[-[Y_{28}]_2 + [Y_{28}]_4 - [Y_{28}]_6 + [Y_{28}]_8 - [Y_{28}]_{10} + [Y_{28}]_{12} - [Y_{28}]_{14}\right].$$

. . . .

To combine the sine and cosine terms,

$$A_k \sin k\omega t + B_k \cos k\omega t = C_k \sin\left[k\omega t + \tan^{-1}\frac{B_k}{A_k}\right]$$
$$= C_k \sin k\left[\omega t + \frac{\varphi_k}{k}\right]$$
$$C_k = \sqrt{A_k^2 + B_k^2}$$
$$\tan \varphi_k = \frac{B_k}{A_k}$$

φ_k is positive if the ascending portion of the component curve first cuts the axis at the left of the origin.

Harmonic Analyzers.—There are other methods of harmonic analysis,[13] but it is evident that the labor involved in such work is very considerable and becomes formidable if an investigation is in hand which requires the treatment of many curves. Cases arise where curves other than those of e.m.f. and current must be analyzed and it is necessary to be able to include both the odd and the even harmonics. Hence the need in practical work of machines by which the analysis may be effected. References to descriptions of a number of these harmonic analyzers are given at the end of this chapter.

A simple form of harmonic analyzer, particularly designed for electrical engineering work has been described by Chubb.[11] Its action may be explained as follows.

On page 650 attention was called to the fact that the exact expressions for A_k and B_k are

$$A_k = \frac{1}{\pi}\int_0^{2\pi} f(\theta) \sin k\theta \, d\theta$$

and

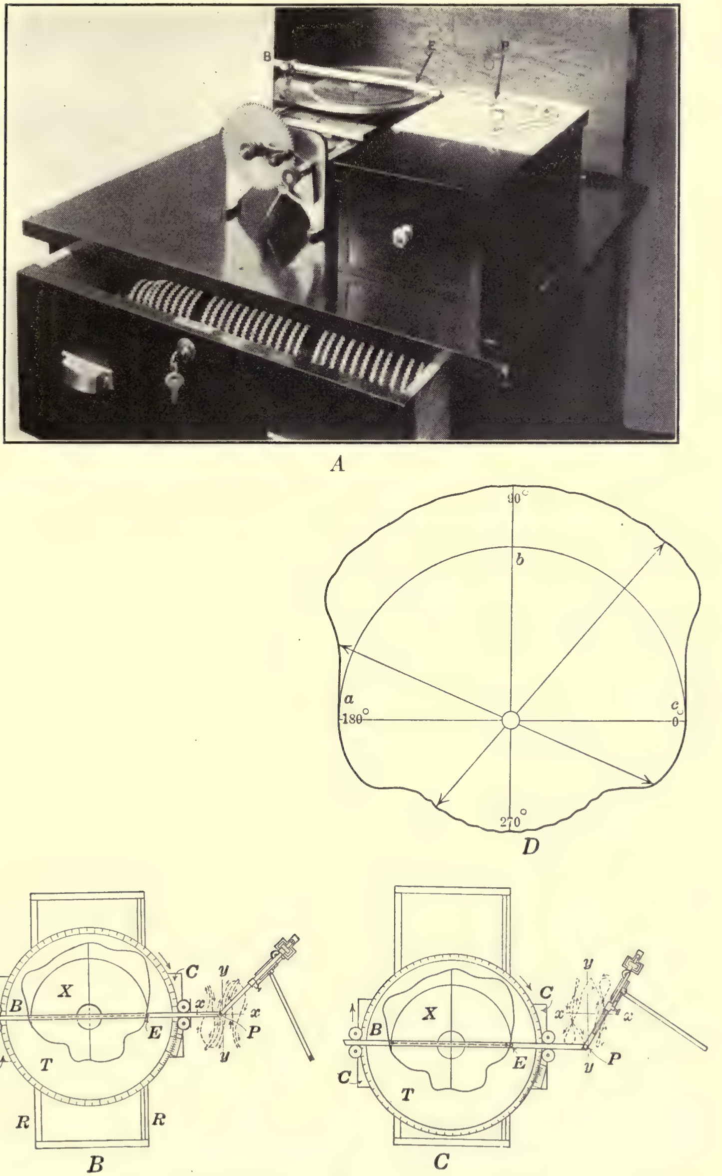
$$B_k = \frac{1}{\pi}\int_0^{2\pi} f(\theta) \cos k\theta \, d\theta.$$

Referring to Fig. 415, if the point P be given a displacement along the horizontal axis always equal to $f(\theta)$, and if at the same time it experiences a perpendicular displacement always proportional to sin $(k\theta)$, then

$x = f\theta$

$y = R \sin k\theta$, where R is the maximum displacement along y,

$dy = kR \cos k\theta \, d\theta$



B

FIG. 415.—Chubb harmonic analyzer, Westinghouse Co. *B*, analyzer set for determining cosine components; *C*, analyzer set for determining sine components.

and the area of the curve traced while θ goes through a complete cycle will be

$$\text{area} = kR\int_0^{2\pi} f\theta \ \cos\ k\theta\, d\theta.$$

Consequently by (11)

$$B_k = \frac{\text{area}}{\pi k R}.$$

Similarly if the vertical displacement of P is given by

$$y = R \sin\left(k\theta - \frac{\pi}{2}\right) = -R\cos k\theta$$

$$dy = kR \sin\ \theta\, d\theta,$$

and the area of the curve traced by P will be

$$\text{area} = Rk\int_0^{2\pi} f(\theta) \sin\ \theta\, d\theta.$$

Consequently

$$A_k = \frac{\text{area}}{\pi k R}.$$

The Chubb analyzer is a mechanism for mechanically calculating A_k and B_k in the manner just described. The areas of the curves are determined by a planimeter as is indicated in Fig. 415. The arrangement by which the point P is guided is shown in a general way in Fig. 415, B and C. The first step in using the analyzer is to cut out a bristol board template which represents $f(\theta)$, such as is shown in Fig. 415, D. The circle *abc* is the base line from which the values of $f(\theta)$ are measured, + values of $f(\theta)$ being measured radially outward and − values radially inward.

The template is mounted on the turntable T, and the pin E on the movable transverse rod BP is forced against the edge of the template by springs. Obviously if the turntable rotates, the point P experiences a displacement $x = f(\theta)$.

The frame which carries the turntable and the mounting for the rod, BP, slides on the ways, RR, and by means of a crank and slotted crosshead can be given a sinusoidal displacement along these ways.

Rotary motion is communicated to the turntable by means of a wormwheel mounted on the axis of the turntable, and a worm which slides on a splined shaft placed parallel to the ways.

By means of a system of change gears the frame carrying the

turntable may be caused to make k complete displacements along the ways while the table makes one complete revolution. The point P is thus guided in the manner already suggested.

When the sine components are to be determined the crank is placed so that the carriage is at its maximum displacement, at the lower end of the ways, when $\theta = 0$. If the cosine terms are desired the carriage is started at its mid-position when $\theta = 0$.

By the use of the proper system of change gears the coefficient of any harmonic, either odd or even, is readily determined with all the accuracy needed in electrical engineering work.

The oscillograph is readily adapted for obtaining curves plotted in the peculiar manner necessary for the construction of the template; the oscillogram is taken on a plate which is rotated about an axis perpendicular to its plane. If undeflected the spot of light traces the base circle *abc* from which the displacements are measured.

From the oscillogram one can readily detect the presence of even harmonics, for if they are absent all the "diameters" are of equal length and equal to the diameter of the base circle *abc*.

Experimental Analysis: Laws Method.[12]—When dealing with potential difference and current waves it is possible to determine the various coefficients experimentally as will be seen from the following.

The coefficient A_k is twice the mean product between $+\pi$ and $-\pi$ of the curve $f(\theta)$ and the curve $\sin k\theta$; B_k is twice the mean product of $f(\theta)$ and $\cos k\theta$, between the same limits. The deflection of an electrodynamometer is proportional to the mean product of the currents in the fixed and movable coils. Consequently, if the wave to be analyzed, of frequency f, is led through the fixed coil while a sinusoidal current wave of frequency kf and maximum value A'_k is sent through the movable coil, the deflection of the instrument will be proportional to A_k or to B_k according as the zero point of the unknown wave coincides with the zero point of the sine wave or with the maximum of the sine wave. The deflection will be proportional to C_k if the phase of the sinusoidal current is adjusted until the deflection is a maximum.

In order to carry out the analysis, the machine from which the sinusoidal current is derived must be driven from the shaft of

the generator by means of change gears which permit the speed to be varied so that the frequency may be made 1, 3, 5, 7 . . . times that of the main current. Also the machine must be so constructed that the phase of the sinusoidal current may be altered at will, a scale being provided so that the phase displacements are readily determined.

The maximum value of the sinusoidal current, A'_k, is determined from the reading of a current dynamometer. The process of making a measurement is to change the phase of the machine giving the sinusoidal current until the dynamometer stands at zero, then to shift the phase 90° and take the dynamometer reading. A contact arrangement and galvanometer which by means of a double-throw switch may be connected to the terminals of non-inductive resistances in either the main circuit or in that of the sine generator, permits the zero points on the waves to be located. The phases of the harmonics may then be determined from the readings on a scale of degrees attached to the movable field frame of the sine dynamo.

The disadvantage of this method is that it requires special apparatus.

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CHAPTER XV

CABLE TESTING

FAULT LOCATION

Continuity of service is essential to the success of any electrical undertaking, whether it be for supplying light or power. So far as the transmission lines are concerned, continuity implies that the risk of interruption has been reduced to a minimum by the use of proper methods of construction, and of suitable materials, such as cables, insulators, etc. Even when the greatest care has been exercised in these matters, cables will break down, line wires will become crossed or grounded and insulators will be punctured or broken.

Accidents which interrupt the service are often due to abnormal conditions, over which one has no control, and it is necessary to have means of locating the position of the defective parts of the lines or cables, in order that repairs may be expeditiously made. Newly installed power cables not infrequently break down during the high-voltage acceptance tests, and these breaks must be located.

The special problem of locating faults in long submarine cables is discussed in such works as Kempe's "Handbook of Electrical Testing," and will not be considered here.

In general, the methods here treated will be those employed in dealing with power cables in cities, and with telephone and telegraph lines, and it will be assumed that only one fault, or connection to ground, exists.

The theory underlying the methods of fault location is very simple, but, on account of constantly varying circuit conditions, the practical execution of the tests requires a skill and judgment which can be obtained only by actual experience.

Location of Grounds and Crosses.—An earth fault, or ground, is due to any defect in the insulation of the conductor which impairs or destroys its efficiency so that a current may pass from the wire to the earth or to the cable sheath. A cross is due

to the impairment of the insulation between two wires so that the current may pass between them.

The location of grounds and crosses is effected by the same methods. In a multiple-conductor cable, the first step is to pick out the conductors which are faulty, for, in general, some conductors remain in perfect condition.

For this purpose, both ends of the cable are disconnected from the service apparatus, and the insulation resistances between the various conductors and ground and between the conductors themselves are measured; the voltmeter method may be used. If the faults are of sufficiently low resistance, bridge measurements with reversed currents may be made. Continuity tests should also be made to determine whether any of the conductors have been burned off or broken. The above tests enable one to decide on the subsequent procedure.

If the resistance per unit length of the line is uniform, three things must be known in order that a ground or a cross may be located:

1. The total length of the faulty line.
2. The total conductor resistance of the faulty line *at the time of test.*
3. The resistance of the faulty line from the testing station to the fault.

The length of the line is given by the office records. When there are only two wires connecting the stations at the ends of the line, it is not possibe to measure the line resistance after the fault has occurred. The best approximation possible must then be made by taking the stated resistance per unit length and correcting it for temperature; in this correction there may be considerable uncertainty, for the temperature coefficient of the copper is large (0.4 per cent. per degree C.) and it is often difficult to form a just estimate of the temperature of the conductor, especially if it is in a duct near heavily loaded cables.

Blavier Test.—In case the faulty wire is the only one connecting the stations, Blavier's method furnishes the only means of locating the fault by measurements made from one end of the line. In order that the test may be carried out, it is necessary that the observer be able to send his instructions over the line to the attendant at the other end.

The total line resistance is supposed to be known from previous measurements made while the line was perfect; denote it by L.

Two measurements of the resistance to ground are made by the observer at the sending end, one, R_1, with the far end insulated and a second, R_2, with the far end grounded. Then, referring to Fig. 416,

$$L = x + y$$
$$R_1 = x + g$$
$$R_2 = x + \frac{gy}{g + y}.$$

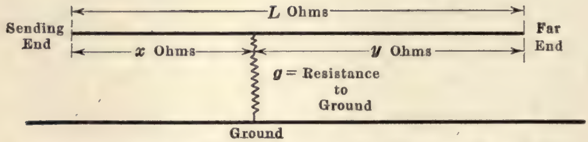


FIG. 416.—Blavier and earth overlap tests for fault location.

Eliminating g and y,

$$x = R_2 - \sqrt{(L - R_2)(R_1 - R_2)} \qquad (1)$$

This gives the resistance from the sending station to the fault. The corresponding distance is calculated by aid of the known resistance per unit length of the cable.

The resistance measurements may be made in any convenient manner, as by the volt and ammeter method. A practical difficulty is that the resistance to ground, g, is variable, being influenced by the amount of moisture present and the action of the current at the fault. Also, the resistance, g, may be so high that it exerts very little shunting action when y is placed in parallel with it by grounding the far end of the line.

The Earth Overlap Test.—In applying this test it is necessary to make resistance measurements from both ends of the line. With the far end grounded, the resistance, R_1, to ground is measured from the near end. The line is then grounded at the near end and the resistance to ground, R_2, is measured from the far end. Then

$$L = x + y.$$
$$R_1 = x + \frac{gy}{g + y}.$$
$$R_2 = y + \frac{gx}{g + x}$$

or

$$x = R_1 \left[\frac{L - R_2}{R_1 - R_2}\right] \left[1 - \sqrt{\frac{R_2 (L - R_1)}{R_1 (L - R_2)}}\right] \tag{2}$$

$$y = R_2 \left[\frac{L - R_1}{R_2 - R_1}\right] \left[1 - \sqrt{\frac{R_1 (L - R_2)}{R_2 (L - R_1)}}\right] \tag{3}$$

Practical details concerning the application of this test to long submarine cables are given in the *Journal of the Institution of Electrical Engineers*, 1885, vol. 16, page 581.

The Volt-ammeter Test.—In this method it is necessary to have between the testing stations, a second and unfaulted conductor which can be used as a potential lead to the far end of x.

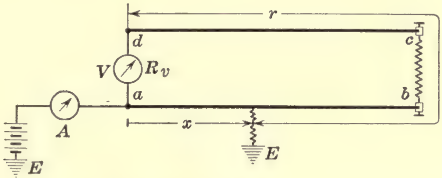


FIG. 417.—Volt-ammeter method for fault location.

In Fig. 417 the faulted wire is shown by ab, the unfaulted one by dc; V is a voltmeter and A is an ammeter. With these connections a regular fall of potential measurement of x may be made.

The P.D., V_x, between the terminals of x, will be given by

$$V_x = V_1\left(\frac{R_v + r}{R_v}\right)$$

where V_1 is the reading of the voltmeter and R_v is the voltmeter resistance. If I_1 is the reading of the ammeter, the current through x will be

$$I_x = I_1 - \frac{V_1}{R_v}.$$

If r is an appreciable fraction of R_v, it may be eliminated; to do this transfer the ammeter connection to d and make a second measurement. Call the reading of the voltmeter, V_2, and that of the ammeter. I_2. Then the voltage accross the ends of r is

$$V_r = V_2\left(\frac{R_v + x}{R_v}\right)$$

$$I_r = I_2 - \frac{V_2}{R_v}.$$

The value of x is

$$x = \frac{V_1}{I_1 - \frac{V_1}{R_v} - \frac{I_1}{I_2}\frac{V_2}{R_v}} \qquad (4)$$

If an electrostatic voltmeter is used, no allowance is necessary for the voltmeter current and

$$x = \frac{V}{I}.$$

Loop Tests.—By a loop test is meant any method of locating grounds or crosses by determining the resistances of the two sections of the loop formed by connecting the faulted conductor at its far end to an unfaulted conductor which returns to the sending station (see Fig. 418).

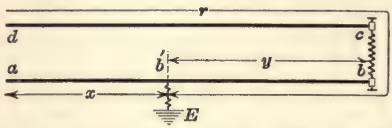


FIG. 418.—Loop test for locating faults.

The grounded conductor is represented by ab, the fault being at b'. The unfaulted conductor is shown by dc; at the far end it is electrically connected with the faulted conductor by a low-resistance jumper which must be insulated from ground. It is essential that the contacts at b and c be perfect.

The superiority of the loop tests is due to the fact that the results are independent of the resistance of the fault itself.

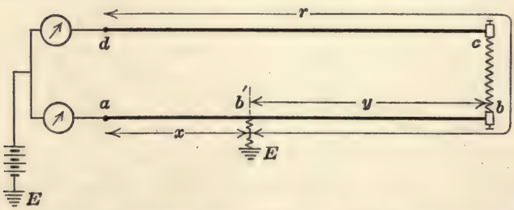


FIG. 419.—Two-ammeter loop test for locating faults.

Two-ammeter Loop Test.—In this method the two sections of the loop are fed in parallel from the same battery and the ratio $\frac{x}{r}$ is determined from the readings of two ammeters, one

placed in series with x, the other in series with r, as indicated in Fig. 419.

The resistances of the two ammeters and the connections are assumed to be negligible. The polarity of the battery should be such that the fault resistance is a minimum. If the readings of the ammeters are I_x and I_r,

$$\frac{I_r}{I_x} = \frac{x}{r} \text{ or } \frac{I_r}{I_x + I_r} = \frac{x}{x + r}$$

$$\therefore x = (x + r)\left(\frac{I_r}{I_r + I_x}\right) \tag{5}$$

The total resistance of the loop, $(x + r)$, may be determined by the volt-ammeter method.

Formula 5 gives the resistance in ohms from the sending end to the fault. If the conductors are both of the same material and size and at the same temperature, the resistance per unit length will be the same for both and

$$\text{distance to fault} = \text{total length of loop} \times \left(\frac{I_r}{I_r + I_x}\right) \tag{6}$$

The resistance, y, from the far end of the line to the fault may also be obtained, for

$$\frac{I_x}{I_r} = \frac{r}{x}$$

or

$$\frac{I_x - I_r}{I_x + I_r} = \frac{r - x}{r + x} = \frac{\frac{r - x}{2}}{\frac{r + x}{2}}$$

For a loop of uniform resistance per unit length,
distance from far end of line to fault =

$$\text{length of } \textit{one wire} \times \frac{I_x - I_r}{I_x + I_r}. \tag{7}$$

The two-ammeter method, using alternating currents, has been employed by Nicholson to locate broken or otherwise defective insulators on long high-voltage transmission lines.[1] Few of the methods of fault location are applicable to this case, for high voltages must be employed in order that the defective insulator may arc over to the metal supporting pin and thus establish the fault. Practically full-line voltage may be required.

The plant where this method was first tried transmits power at 60,000 volts, 25 cycles, 3-phase, over lines 160 miles long. The transformers are operated with a grounded neutral and the insulator pins are grounded. Consequently if an insulator breaks down a short-circuit is established and the trouble must be rectified before service can be resumed. It is desirable that special apparatus be avoided.

The connections are shown in Fig. 420. The line is opened at the far end by means of the three disconnecting switches. At the near end, the disconnecting switches in A and B are opened and jumpers are applied at both ends of the line as shown.

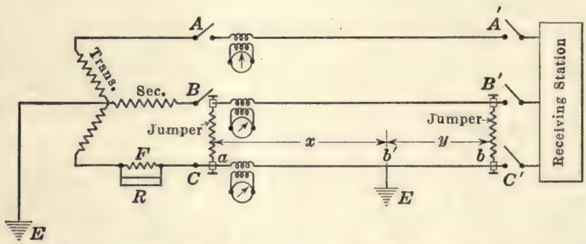


FIG. 420.—Diagram for Nicholson's application of the two-ammeter loop test to high-voltage lines.

At R is a rheostat arrangement to control the current after the arc to ground has been established; F is an expulsion fuse which short-circuits R. It permits the voltage at the fault to be raised high enough to start the arc and then blows, throwing in the controlling resistance R. For this current-limiting resistance four concrete columns 1 foot square and 12 feet long with expanded metal terminals have been used. Each has a resistance of about 2,000 ohms. They are used singly or in parallel as occasion requires. From 50 to 100 amperes are required for the test. In cases where the striking distance is several inches, the testing current does not burn the aluminum conductors if it is kept on for 40 seconds, which is sufficient time to obtain the readings.

Where a single size of conductor is employed, it is found that the currents divide inversely as the resistances of the two paths, so formula (6) is applicable.

Murray Loop Test.—In the Murray loop test the connections are such that the resistances x and r form two arms of a Wheat-

stone bridge, the other two arms being made up of resistances under control of the observer. Fig. 421 shows the scheme of connections.

The relative positions of the galvanometer and battery are important; with the connections as shown, earth currents have no effect on the readings.

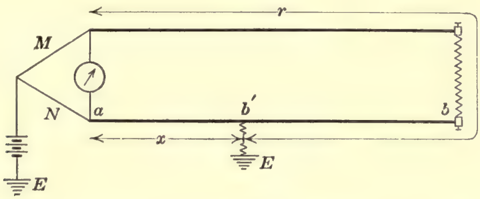


FIG. 421.—Connections for Murray loop test for fault location.

When the galvanometer stands at zero,

$$\frac{M}{N} = \frac{r}{x} \quad \text{or} \quad \frac{M+N}{N} = \frac{r+x}{x}.$$

$$\therefore\ x = (r+x)\left(\frac{N}{N+M}\right).$$

The total resistance of the loop $(r + x)$ is obtained by a bridge measurement.

If uniform wires are being dealt with,

$$\text{distance to fault} = \text{total length of the loop} \times \left(\frac{N}{M+N}\right).$$

A potential divider or a slide-wire arrangement with extension coils may be convenient for $M + N$, in which case $M + N$ is constant.

A cross between two conductors in a multiple-conductor cable is located in a similar manner, the only difference being that the battery, instead of being connected to ground, is attached to one of two faulty conductors, the other being looped with an unfaulted wire.

Varley Loop Test.—In this method a fixed bridge ratio is used and the balance obtained by adding resistance to the smaller section of the loop as indicated in Fig. 422.

With the apparatus arranged as in Fig. 422 and the switch in the position shown, the bridge will balance when

$$\frac{M}{N} = \frac{r}{x + P_1},$$

P_1 being the resistance unplugged at P,

or

$$x = \frac{(x + r)N - P_1M}{M + N} \tag{8}$$

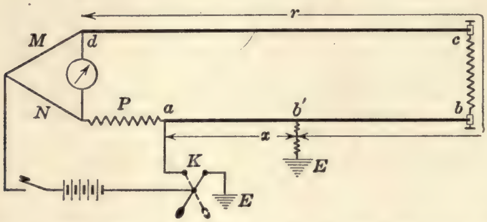


FIG. 422.—Connections for Varley loop test for fault location.

To measure $(r + x)$, the total resistance of the loop, the key K is thrown to the dotted position and a second balance obtained, using the apparatus as an ordinary Wheatstone bridge.

Determination of the Total Resistance of the Defective Conductor.—If there is only one perfect wire between the stations, the total resistance of the faulted line cannot be determined by measurements made from one end of the line.

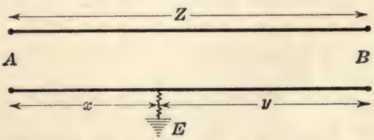


FIG. 423.—Pertaining to determination of resistance of faulty conductor.

The determination may be made by tests from both ends. The line is first looped with Z at B and x determined by one of the previous methods. Then the loop is made at A and the observer at B measures y. The total resistance is obviously $(x + y)$. (See Fig. 423.)

When there are two perfect wires between the stations, measurements from one end of the line suffice. When dealing with a multiple-conductor cable, two unfaulted wires in the same cable may be used as the auxiliary wires (see Fig 424).

The faulty wire is looped with Z and the resistance, R_1, of the loop measured by a bridge.

$$R_1 = (x + y) + Z.$$

Loop the faulty wire with W and measure the resistance, R_2, of this loop.

$$R_2 = (x + y) + W.$$

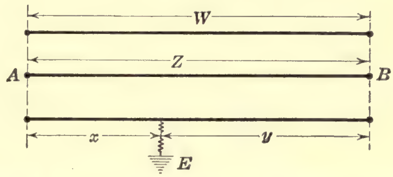


FIG. 424.—Pertaining to determination of resistance of faulty conductor.

Lastly, loop W and Z and measure the resistance, R_3, of the loop.

$$R_3 = W + Z.$$

Then

$$(x + y) = \frac{R_1 + R_2 - R_3}{2} \tag{9}$$

Another procedure is to loop the faulty wire with Z and measure the loop resistance, R_1.

$$R_1 = (x + y) + Z.$$

W and Z are then looped and grounded at B, and Z measured by one of the previous methods; then

$$(x + y) = R_1 - Z \tag{10}$$

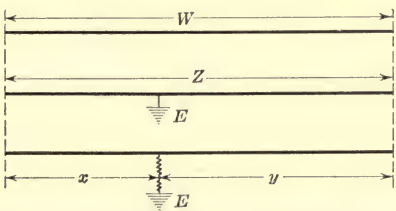


FIG. 425.—Pertaining to determination of resistance of faulty conductor.

If there are one perfect and two faulty wires of the same length and resistance connecting the stations, as indicated in Fig. 425, the total resistance of either of the faulty wires may be obtained thus:

Loop Z and W and measure the resistance, R_1, of the loop. Loop $(x + y)$ and W and measure the resistance, R_2, of this loop. Finally, connect Z and $(x + y)$ in parallel and loop the combination with W and measure the resistance, R_3.

$$R_1 = Z + W.$$
$$R_2 = (x + y) + W.$$
$$R_3 = W + \frac{Z(x + y)}{(x + y) + Z}.$$
$$(x + y) = (R_2 - R_3) + \sqrt{(R_3 - R_2)(R_3 - R_1)} \qquad (11)$$

Or,

$$Z = (R_1 - R_3) + \sqrt{(R_3 - R_2)(R_3 - R_1)}.$$

To be strictly accurate, the ratio of the resistance up to the fault to the total resistance of the wire should be the same for both defective conductors, for then there will be no flow of current through the faults.

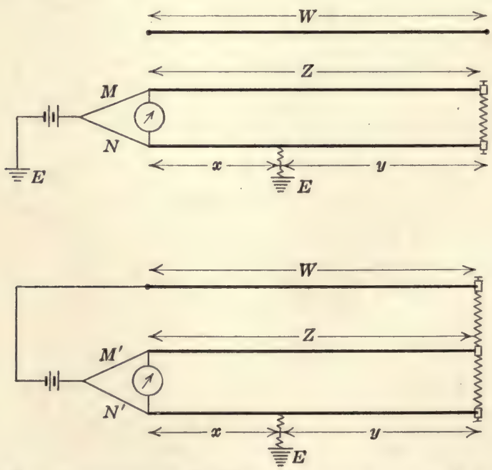


FIG. 426.—Connections for Fisher loop test for fault location.

Fisher Loop Test.—In order to make this test there must be two unfaulted wires which run from the testing station to the far end of the line. The result obtained is the same as that given by the above methods where two perfect conductors are available and both the resistance up to the fault and the total resistance of the faulty line are measured.

Two balancings are necessary, as indicated in Fig. 426.

From the first balancing,

$$\frac{M}{N} = \frac{Z + y}{x}.$$

From the second balancing,

$$\frac{M'}{N'} = \frac{Z}{x + y}.$$

Then

$$x = (x + y)\frac{\frac{M'}{N'} + 1}{\frac{M}{N} + 1} \tag{12}$$

If a slide wire or its equivalent is used for the balance arms, $M + N = M' + N'$ and

$$x = (x + y)\frac{N}{N'} \tag{12a}$$

When the resistance per unit length of conductor is uniform,

$$\text{distance to fault} = \left(\frac{\frac{M'}{N'} + 1}{\frac{M}{N} + 1}\right) \times (\text{length of cable}) \tag{13}$$

Corrections for Conductors of Different Diameters.—In the foregoing it has been assumed that the resistance per unit length of cable is uniform. In some cases, however, the conductor may be made up of a number of wires in series which have different

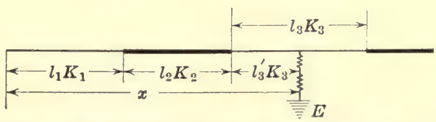


FIG. 427.—Pertaining to the location of a ground in a non-uniform conductor.

diameters. The lengths and sizes of the wires in the different sections will be known from the office records. Let the lengths be $l_1, l_2, l_3, \ . \ . \ .$ and let the corresponding resistances per unit length be $K_1, K_2, K_3, \ . \ . \ .$ The resistances of the sections will be as indicated in Fig. 427.

To locate the section in which the fault exists, compare the resistance x as found by one of the previous methods, with l_1K_1, then with $l_1K_1 + l_2K_2$ and so on. Suppose that the comparison shows the fault to be in the third section; then

$$x = l_1K_1 + l_2K_2 + l'_3K_3$$

and the distance of the fault beyond the junction of the second and third sections is

$$l'_3 = \frac{x - l_1K_1 - l_2K_2}{K_3} \tag{14}$$

Uncertainty as to the values of K introduces difficulties. The average values of K for the different sections depend on the diameters of the wires, their conductivities, the temperatures of the sections, and in underground conductors of large cross-section, where the lengths of the sections are short, on the number of joints.

In cables for large currents laid in ducts where the temperatures may be high, there may be great uncertainty as to the temperature and a consequent difficulty in correcting K_1, K_2, etc., to obtain their values at the time of test.

Locating Faults in Underground High-tension Cables.[2]—In locating faults in underground high-tension cables, special difficulties are experienced because of the low resistance of the conductor, which may be from 0.10 to 0.04 ohm per 1,000 feet. On the other hand, such cables are readily accessible at the manholes, which may be about 300 feet apart. This makes it possible, before cutting the cable, to verify the location of the fault as given by a loop test, and for this purpose special apparatus has been devised so that the particular length of cable in which the fault is located may be identified with certainty. This verification is necessary, for in a 10-mile length of cable an uncertainty of 0.3 per cent in the resistance measurements corresponds to an uncertainty of 158 feet or *half a length* of the cable between manholes.

In the long run, time will be saved by adopting an orderly procedure and the following has been found satisfactory in dealing with this class of faults:[2]

(A) Tests to diagnose the trouble and show the tester what he has to deal with.

(*B*) Reduction of the resistance of the fault (if necessary) so that current sufficient to make the location and verification tests will flow through the fault with a moderate voltage.

(*C*) Preliminary location of the fault by a loop test. If the conductor is burned off the loop test is not applicable. In this case the ground is located by use of an exploring coil, see *D* below.

(*D*) Verification of the location by use of an exploring coil.

(*A*) *Taking a three-phase cable:*

1. All three conductors are insulated at both ends and the resistance to ground of each conductor determined by the voltmeter method, using a direct-current potential of about 110 volts. This shows whether the fault is a ground or not and if it is, gives an idea of the fault resistance.

2. If a low-resistance ground is found, verify No. 1 by using a test lamp, that is, an incandescent lamp with one side of the socket attached to the 110-volt direct-current supply, the other being provided with a flexible cord so that it may be attached to any of the conductors. If the lamp glows, the resistance of the fault is to be measured by a bridge, two readings, with reversed currents, being taken. This measurement of the fault resistance shows whether it is necessary to further reduce it before making the loop and verification tests.

3. Conductors which show the same insulation to ground should now be tested to determine whether this is due to the wires being crossed. The test is made by grounding one of the wires and remeasuring the resistance to ground of the others. If the voltmeter method shows that this resistance is low, it should, for the reason stated in 2, be measured by the bridge.

4. The far ends of all three conductors should be carefully connected together and the resistances of all the uncrossed loops measured by the bridge. A comparison of these results with the resistances computed from the known size and length of the line will show if there are open faults; that is, places where the wires are broken or burned off. If an open fault exists, an idea of its resistance should be obtained. The resistance to ground of the near side of the open fault has been obtained in No. 1. The resistance to ground of the far side of the open fault is obtained by measuring it via an unfaulted conductor, which is used as a

lead. If this resistance is very low, the resistance across the open fault has practically been measured in No. 1. If it is not low, it should be determined by measuring the insulation of the open phase when the far side is grounded through one of the unfaulted conductors.

(*B*) In order to locate the ground, it is necessary that the fault resistance be low, so that sufficient current for the tests may be obtained with low voltage. This is convenient in the loop tests and is necessary in the subsequent exact localization by means of exploring coils. For, if considerable voltage is used, the charging current going to the parts of the cable beyond the fault produces confusion and renders the localization a matter of difficulty.

After having determined the nature of the fault and gained an idea of its resistance, it will probably be found necessary to reduce the fault resistance. The fundamental idea is to carbonize a sufficient amount of the paper insulation at the fault, so that a current of 1 or 2 amperes may be carried for several hours. Practice is required to accomplish this in the shortest possible time and without any approach to an explosive short-circuit at the fault, which would destroy the continuity of the path via the carbonized paper. When the fault is not submerged in water, for in water the paper cannot be carbonized, the procedure is to send a current of from 3 to 5 amperes through the fault for 10 minutes, in order to dry it out, and to follow this by a current of about 1 ampere, for 5 minutes, to carbonize the paper. High-resistance faults at a considerable distance from the testing station give trouble on account of the charging current. In such cases, the current through the fault may be determined by aid of a wattmeter.

(*C*) After having reduced the fault resistance, a Murray or a Varley loop test is used to determine the approximate location of the trouble.

In making this test, great care must be exercised in applying the jumpers at the far end and in joining the bridge to the cable, in order that extraneous resistances may not be introduced. Also allowance must be made for the leads connecting the bridge to the cable or else the bridge must be so constructed that these resistances are eliminated.

Irregularities in joint resistances, etc., render it necessary to supplement the loop tests by exploration tests which will show definitely the particular length of cable in which the fault exists. Also the cable may be burned off, in which case the loop tests are not applicable.

(*D*) The idea of the exploration tests is to send some characteristic signal into the cable and to find by means of a suitable detector the point at which the signal ceases to be heard as the exploring device is moved along the cable.

Taking the case shown in Fig. 428, when the sheaths of the various lengths of cable are bonded, to prevent electrolysis, a

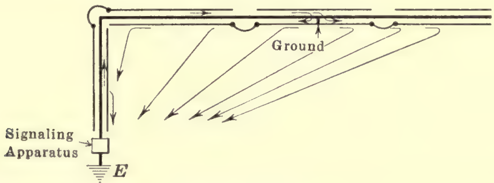


FIG. 428.—Pertaining to locating a ground by exploration tests.

diminishing portion of the current will flow in the sheath to points beyond the ground, as indicated. Currents will also flow in the sheath, which are due to inequalities of ground potential, as produced, for example, by stray currents from street-car lines.

If the detector is a simple coil of wire connected to a telephone and held with its plane parallel to the length of the cable, the sheath currents, from whatever cause, will affect it in the same manner as if they flowed in the conductor and an exact location of the trouble is not possible.

When dealing with three-phase cables, it is possible to use a longitudinal exploring coil, devised by W. A. Durgin, which is not affected by the sheath currents.

It depends for its effectiveness on the fact that the conductors in the cable are spiralled, the lay, or length of a complete spiral, being about 20 inches, for a No. 2–0 three-phase paper-insulated cable.

The exploring coil consists of a laminated iron core, of a length determined by the lay of the cable, over which is wound a coil of insulated wire with its terminals attached to a telephone. The core is placed *parallel* to the axis of the cable. Any current

which flows only in the sheath produces a field which has no longitudinal component and, therefore, stray currents cause no disturbance of the telephone.

Referring to Fig. 429, when a current flows out along the spi-

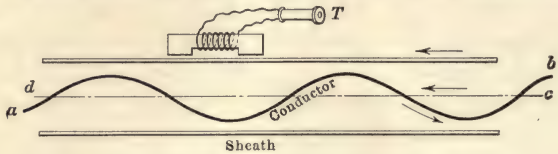


FIG. 429.—Diagram for Durgin exploring coil.

ralled conductor *ab* and returns along the sheath, in effect along *cd*, the case is entirely different for there is a loop twisted so that it alternately presents its positive and negative side to the observer as he passes along an element of the sheath; that is, positions of maximum and minimum magnetic potential succeed each other in a definite order.

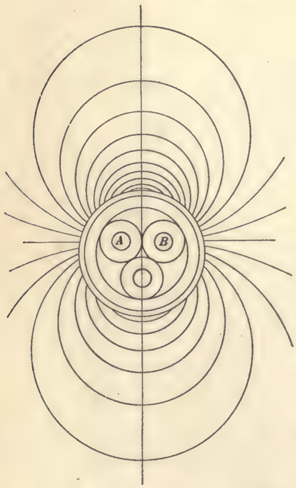


FIG. 430.—Showing magnetic equipotential lines around a three-phase cable when steady current flows in the signalling loop formed by conductors *A* and *B*.

When the longitudinal exploring coil is used in the case shown in Fig. 428, it will be found that no indication is obtained at points near the signalling apparatus, for at these points there is little return current in the sheath and the effect of a twisted loop is not obtained. On approaching the ground, more and more current flows in the sheath and the signals increase in intensity until the ground is reached; beyond this point there is silence, for only sheath currents are present and they produce no longitudinal field.

Another use of the exploring coil is in identifying a particular cable in the distributing system. When dealing with three-phase cables, there are three signalling loops which may be utilized.

I. Two conductors, the current flowing out by one and returning by the other.

II. Two conductors in parallel and in series with the third.

III. One conductor and the sheath.

For purposes of explanation take the first case; the equipotential lines on a plane perpendicular to the axis of the cable, and due to a steady current, are shown in Fig. 430.

It is evident that because of the position of the conductors in the cable, the magnetic potential varies from point to point around the circumference of the sheath and that the maximum and minimum points are 180° apart. This means, that on account of the lay of the cable, the distance between the points of the maximum and minimum magnetic potential when measured along an element of the sheath is one-half the lay of the cable. If the iron core of the exploring coil has a length equal to one-half the lay and is applied longitudinally to the cable between these points, it will be traversed by a considerable flux, and a signal sent into the cable will be audible in the telephone. Sheath currents of any sort are not effective in producing sounds in the telephone for they cause no inequalities of magnetic potential along the length of the cable.

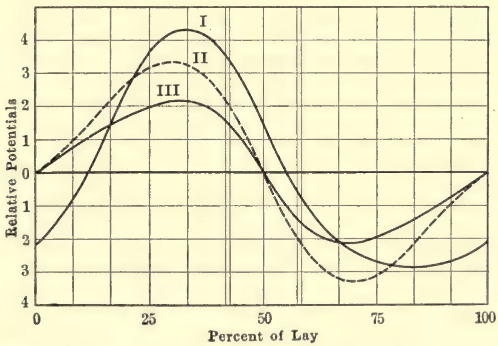


FIG. 431.—Showing variation of magnetic potential when different signalling loops are used.

The two-conductor circuit, I, is best when a cable has to be identified throughout its length because it gives the maximum difference of magnetic potential and also because the maximum and minimum points are equally spaced.

With connection II, the maximum and minimum points are spaced alternately 40 per cent and 60 per cent of the lay.

With III, the corresponding spacing is 35 and 65 per cent of the lay (see Fig. 431.)

When locating high-resistance grounds, all the conductors are connected in parallel so that the charging current to the portion of the cable beyond the fault may not produce a sound in the telephone. To send the characteristic signal into the cable, a motor-driven commutator is used which will break the circuit about 3,000 times per minute; in series with it is a make-and-break switch actuated by a cam also driven by the motor. The result is that one hears in the telephone a definite note which is interrupted in some particular manner.

Location of Total Disconnection.—A total disconnection occurs when the wire breaks inside the insulating covering and the ends are pulled so far apart that the two sections of the conductor are insulated from each other.

Theoretically, the conductors in a cable occupy definite positions with respect to each other and to the sheath. This being so, the electrostatic capacity measured between two conductors or between a conductor and sheath should be proportional to

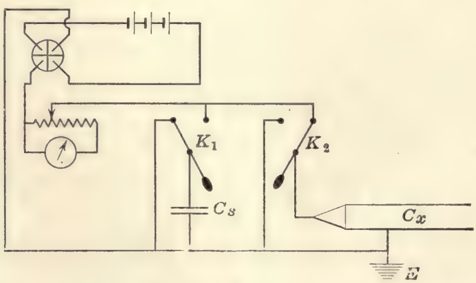


FIG. 432.—Connections for direct deflection method for measuring electrostatic capacity of cable.

the length of the cable. Consequently, when the insulation remains intact, it should be possible to locate a total disconnection by measuring the electrostatic capacity of the portion of the cable from the testing station up to the break and comparing it with the capacity of the whole cable. If the total capacity is not known, measurements must be made from both ends of the line.

The capacities may be measured by the direct deflection method, the connections for which are shown in Fig. 432. Some

definite procedure should be adopted and used throughout the test, in order that the effects of absorption may be eliminated.

The ballistic deflection of the galvanometer which occurs when the key, K_2, is thrown to the right, is read and compared with the ballistic deflection obtained when the standard condenser is substituted for the cable, the battery being kept constant. It may be necessary to change the effective sensitivity of the galvanometer, consequently an Aryton shunt may be used, as suggested by the Fig. 432. The lead from the key to the cable core should be as short as possible. If necessary, its capacity may be determined and a suitable correction made.

The capacity may also be measured by the simple bridge method (see page 392). The connections for a single-conductor cable or its equivalent are shown in Fig. 433.

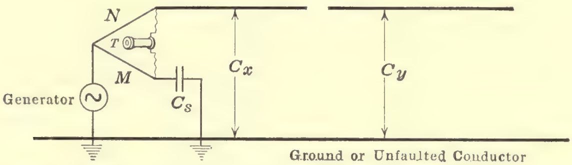


FIG. 433.—Simple bridge method for measuring electrostatic capacity of a short length of cable.

In this case it is convenient to use an alternating current or a rapidly interrupted direct current and to employ a telephone as the detector. When the ratio $\frac{M}{N}$ has been adjusted so that there is the minimum sound in the telephone, $\frac{M}{N} = \frac{C_x}{C_s}$ or

$$C_x = C_s \frac{M}{N}.$$

The capacity of the other section, C_y, may be similarly determined by tests from the far end of the line.

Then:

$$\text{Distance to the fault} = \text{total length of cable} \left(\frac{C_x}{C_x + C_y}\right) \quad (15)$$

The condensers used for C_s should be of good quality and the procedure adopted should be the same as that employed when their capacities were measured.

Where there are several pairs of wires in the cable, a pair may sometimes be used in place of the standard condenser, as indicated in Fig. 434.

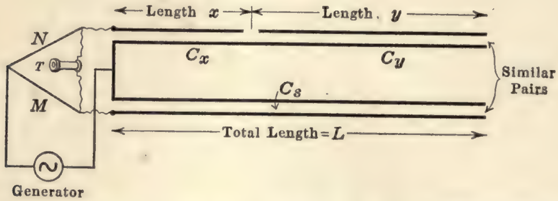


FIG. 434.—Bridge measurement of capacity to total disconnection; using the capacity of an unfaulted pair as a standard.

In this case, from the construction of the cable, the capacity per unit length for both pairs is nominally the same, and as

$$\frac{M}{N} = \frac{C_x}{C_s},$$

$$\text{distance to the fault} = \text{total length of cable} \times \left(\frac{M}{N}\right). \qquad (16)$$

These methods of location by capacity measurements are convenient when dealing with telephone cables. The difficulty in applying them to power cables is that the disconnection, due to a burn-out, may not be total and that the apparent capacity per unit length may not be uniform.

Breakdown Tests of High-voltage Cables.—In addition to the tests which are made during the process of manufacture, every high-voltage cable must be subjected to a breakdown test after it has been installed and before it is accepted, the object being to search out the weak spots due to defects of manufacture and of installation, especially defects at the joints.

In this connection it may be pointed out that laboratory tests on short samples show higher breakdown voltages than are realized in practice and are without value as indicating what may be expected of long lengths of cable.

The idea of the breakdown test is simple. A specified voltage is applied between conductors, or between conductors and sheath, for a specified time. If a breakdown occurs, the fault is located by one of the methods already described, the cable is patched and

the test repeated. A frequency of 25 cycles per second is commonly employed in these tests.

In the practical execution of breakdown tests, numerous questions arise, such as:

1. Given the size of wire, thickness of insulation and the character of the insulating material, what test voltage should be applied?

2. For how long a time should the test voltage be maintained?

3. How can the wave form of the test voltage be assured?

4. How shall the voltage be measured so that the maximum stress to which the insulation is being subjected may be known?

5. What precautions must be taken in order that high-frequency disturbances set up by spark discharges from the testing circuits may be eliminated? The cable breakdown may be due to the high-frequency disturbance rather than to the regular test voltage, so the observer is misled.

6. Is it possible to determine whether or not a cable, which has not been actually broken down, has been overstressed by the high-voltage test so that it is permanently injured?

Though breakdown tests are of the highest importance to operating companies, up to the present time no generally accepted procedure has been developed.

A high test voltage is advisable since it promotes care on the part of the manufacturer; on the other hand, it is possible to so overstress a cable, without actually breaking it down, that it is permanently injured and may in consequence fail at some future time when conditions are not abnormal.

In the past, many companies have specified that the cable must stand $2\frac{1}{2}$ times the normal working voltage for 5 minutes, but there is no rational basis for this particular requirement.

At the present time, on account of the lack of necessary data, it is not possible to specify the proper test voltage from the dimensions and properties of the insulation.

A great difficulty in applying mathematical analysis to this problem arises from the non-uniformity of the insulating materials. Air pockets in the dielectric and lack of perfect adherence of the dielectric to the conductor, especially in stranded cables, produce local overstressing and deterioration of the insulation under

prolonged application of voltage, and are factors which cannot be taken into account in the analysis.

Paragraph 684 of the Standardization Rules of the American Institute of Electrical Engineers, 1916, is as follows:

"The following test voltages shall apply unless a departure is considered necessary, in view of the above circumstances. Rubber-covered wires or cable for voltages up to 7 kv. shall be tested in accordance with the National Electric Code. Standardization for higher voltages for rubber-insulated cables is not considered possible at the present time.

"Varnished cambric and impregnated paper insulated wires or cables shall be tested at the place of manufacture for five (5) minutes in accordance with the Table XIV below.

Table XIV.—Recommended Test Kilovolts Corresponding to Operating Kilovolts

Operating kv.	Test kv.	Operating kv.	Test kv.
Below 0.5	2.5*	5	14
0.5	3.0	10	25
1.0	4.0	15	35
2.0	6.5	20	44
3.0	9.0	25	53
4.0	11.5		

"Different engineers specify different thickness of insulation for the same working voltages. Therefore, at the present time the test kv. corresponding to working kv. given in Table XIV are based on the minimum thickness of insulation specified by engineers and operating companies."†

* The minimum thickness of insulation shall be $\frac{1}{16}$ in. (1.6 mm.).

† The Standards Committee does not commit itself to the principle of basing test voltages on working voltages, but it is not yet in possession of sufficient data to base them upon the dimensions and physical properties of the insulation.

When testing insulations for dielectric strength, it is essential to employ a generator which under all conditions gives practically a sinusoidal voltage at the specimen, since the maximum stress to which the insulation is subjected should be known. For example, altering the length of cable under test, that is, changing the electrostatic capacity placed across the secondary terminals of the testing transformer must not deform the wave. The wave

form must also be independent of the particular combination of transformer windings which are used to obtain the required voltage and must be uninfluenced by the method of voltage regulation. The instruments commonly used for measuring the voltage, *i.e.*, electrostatic voltmeters and electrodynamometer voltmeters, give the effective or r.m.s. value of the voltage. If the wave is sinusoidal, the maximum value is obtained by multiplying the effective value by $\sqrt{2}$.

If the voltage wave is not sinusoidal, resonance effects due to the capacity of the cable and the reactance of the apparatus may be present, so that from this cause the wave may be distorted in a manner dependent on the length of the cable under test.

The effect resulting from taking the insulation through cycles of electrostatic stress depends not only on the maximum voltage but on the number of times the cycle is repeated in a second. If the voltage wave be very greatly distorted, this effect and the consequent weakening of the dielectric strength of the cable due to a prolonged application of the test voltage, are abnormal and the results will not be comparable with those obtained with a very different wave form.

The following series of oscillograms serve to show the necessity for the proper equipment when dielectric strengths are to be measured.[3] The generator used in the tests was a motor-driven, 25-kva, 220-volt, 4-pole, 25-cycle, *single-phase* alternator, having 10 slots per pole and a conductor belt five-eighths the pole pitch. The transformer capacity was 50 kva., 220–50,000 volts. The secondary consisted of four separate 12,500-volt coils. The high-tension winding had a total of 12,512 turns; the low-tension 55 turns. The reactance voltage was about 6 per cent. This transformer was operated at high saturation.

Fig. 435 shows that the e.m.f. of the generator is nearly enough sinusoidal and if the wave form could be maintained, satisfactory tests would be possible. Fig. 436 shows the result when the transformer, with the secondary circuit open, is attached to the generator. The exciting current is much distorted owing to the changes of permeability incident to working the core at high saturation, and a small third harmonic appears in the P.D. wave.

A cable having a capacity of 0.13 microfarad was attached to the secondary. Such a load will take a large leading current

and cause great modification of the voltage wave form. Using all the secondary coils in series, the 50,000-volt connection, the results shown in Fig. 437 were obtained. Fig. 438 shows the

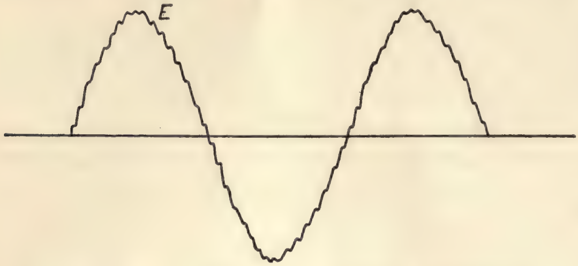


FIG. 435.—Open-circuit voltage of generator.

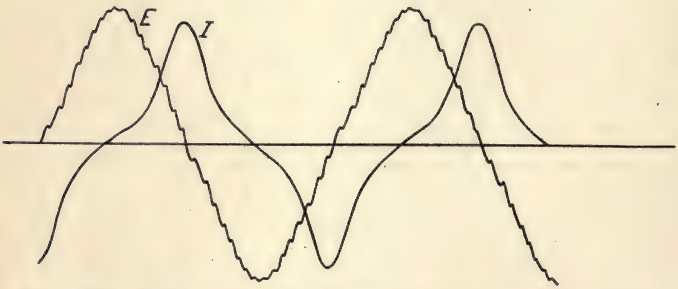


FIG. 436.—P.D. and exciting-current waves, transformer only.

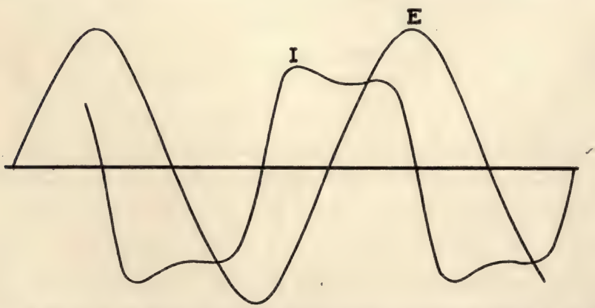


FIG. 437.—P.D. and exciting-current waves when cable having a capacity of 0.13 M.F. is attached to transformer; 50,000-volt connection used.

results when the secondary coils were used, two in series and two in parallel, the 25,000-volt connection.

When the change was made from the 50,000 to the 25,000-volt

connection, the generator voltage was practically doubled so that the effective voltage at the cable was the same in both cases.

It is seen that with the same generator, the same transformer, the same frequency and the same cable an approximately sinusoidal P.D. wave becomes badly distorted by simply changing the transformer ratio and the generator voltage.

These effects may be explained in a general way as follows: As the generator is a single-phase machine, the tendency of the armature reaction is to introduce a third harmonic in the P.D. wave. The transformer is worked at high saturation and through

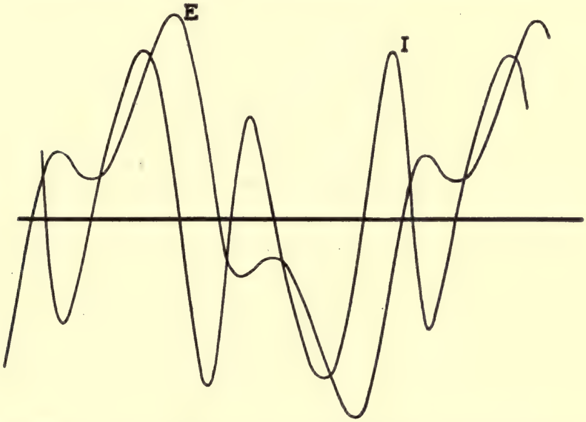


Fig. 438.—P.D. and exciting current waves when cable having a capacity of 0.13 M.F. is attached to transformer; 25,000-volt connection used.

variations in the permeability of the core introduces harmonics, especially the third, into the magnetizing current. These harmonics appear in the voltage wave and are intensified by the capacity load on the secondary of the transformer.

In addition, resonance effects are probably present. Attempts at tuning the circuit by an iron-cored reactance in parallel with the transformer were not successful, for:

1. The minimum current does not necessarily correspond to the best wave form.

2. The best wave form may occur at an abnormally large value of lagging current.

3. The wave form cannot be made sinusoidal in every case.

It is obvious that a different design of generator and transformer must be used.

To preserve a sinusoidal wave form, the generator should be a Y-wound three-phase machine with non-salient poles; it should have a distributed, fractional-pitch (5/6) winding and be provided with damping grids.

The use of the Y-wound three-phase machine eliminates the third harmonic in the e.m.f. wave; the fractional pitch greatly reduces the fifth and seventh harmonics and the damping grids tend to damp single-phase pulsating armature reaction.

The transformer should be designed to work at low saturation.

Tests of such a machine and transformer (designed by C. A. Adams) fail to show any appreciable distortion of the wave form under the most exacting conditions. No testing set should be installed which is incapable of maintaining a sinusoidal test voltage under all circumstances. Questions as to peak voltages and abnormal dielectric losses are thus eliminated.

Measurement of Peak Voltage.—Those interested in the purchase and installation of cables should be able to satisfy them-

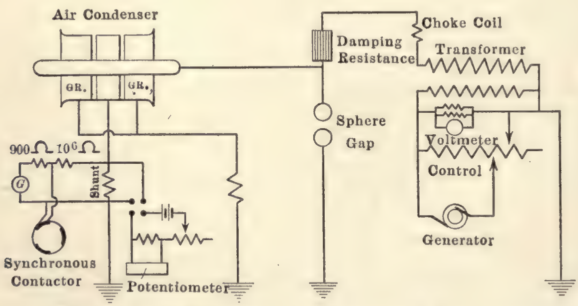


FIG. 439.—Chubb and Fortescue arrangement for measuring high peak-voltages.

selves as to the *maximum* voltage applied to the cable during the breakdown test, for as shown by the above oscillograms it may happen that the effective value of the voltage may give little information as to its peak or maximum value. Also interested parties may be skeptical as to the maintenance of the sinusoidal wave form, even when the proper testing equipment is used, and hence must be satisfied.

The spark-gap method of measuring peak voltages referred to on page 260, is not directly applicable in cable testing, for the observer should be able to follow the variation of the voltage.

Chubb and Fortescue in their calibration of the sphere spark gap arranged the rotating commutator method (page 627) so that very high voltages, up to 400 kv., could be dealt with in the laboratory.

The charging current taken by an air condenser is rectified and then measured by a critically damped D'Arsonval galvanometer. In order that the capacity of the condenser may be accurately calculated, it is provided with guard rings, GR. The two sections of the guard ring are connected together and grounded through a resistance, so that the difference of potential between them and the central or working section will be negligible at all times. The commutator is arranged to short-circuit the galvanometer every alternate half-period and as the brushes are mounted so that they can be displaced, the current may be thrown into the galvanometer for a half-period beginning at any point in the cycle.

The greatest deflection will be obtained when the brushes are so adjusted that contact begins when the voltage is a positive maximum $(+V)$ and lasts until the negative maximum $(-V)$ is reached. If C is the capacity of the condenser, the total quantity displaced by the unidirectional current will be $2CV$. Then, if f is the frequency, the average current during the contact is

$$I = 4CVf.$$

The deflection, D_2, of the galvanometer, due to a steady current, I_2, taken when the commutator is running, is

$$D_2 = KI_2$$

and

$$V = \frac{D_1}{4KCf}.$$

In order to obtain correct results, it was necessary to do away with static troubles by surrounding all wiring, instruments, switches, and resistances with grounded coverings of tin foil or with wire screens.

The condenser used by Chubb and Fortescue consisted of a central cylinder with hemispherical ends, diameter 60 cm., length 458 cm. This was the high-potential "plate." The grounded

"plate" consisted of three sections, as shown, having a total length of 240 cm. and a diameter of 162.8 cm. The central, or working section, was 47.7 cm. long, and its capacity was calculated and found to be 2.657×10^{-11} farads.

The commutator may be driven by a synchronous motor. The brushes must then be set so that the deflection of the galvanometer is a maximum. If an induction motor be used, the deflection goes through a regular cycle, corresponding to the wave form of the voltage applied to the condenser. The maximum deflection of the galvanometer may then be read.

Whitehead and Gorton in their researches on the dielectric strength of air replaced the commutator by an arrangement of mercury-arc rectifiers as shown in Fig. 440.

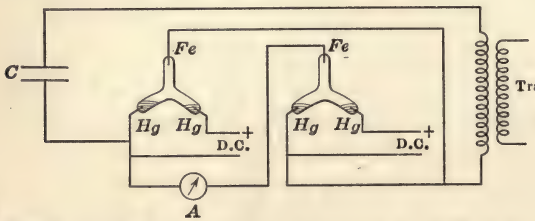


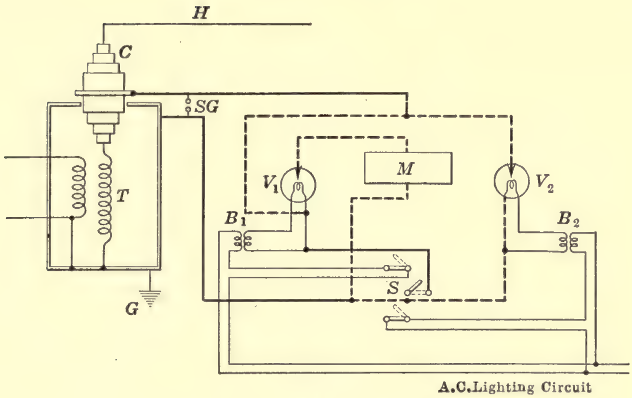
FIG. 440.—Whitehead and Gorton arrangement of electrical valves for measuring peak-voltage.

A mercury arc with a mercury and an iron electrode allows the current to flow from the iron to the mercury but effectually prevents any flow in the opposite direction.

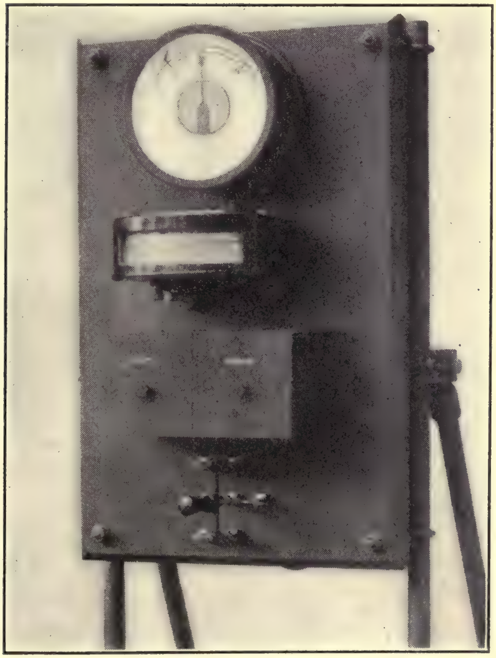
To maintain the ionization in the tubes independently of the small current whose mean value is to be measured, two sources of direct current are used.

The average value of the current is found by multiplying the reading of the ammeter A by 2.

The use of electrical valves has been further developed by Chubb in the switchboard apparatus shown in Fig. 441. The valves for suppressing alternate half-waves are placed in the drawers marked "right" and "left" and are seen at V_1 and V_2 in the diagram. The anodes are of tungsten or molybdenum, the cathodes of incandescent tungsten, the bulbs being filled with mercury vapor. The cathodes are heated by alternating currents supplied through two small transformers, B_1 and B_2.



A



B

Fig. 441.—Chubb peak voltmeter, Westinghouse Co.

The testing transformer is operated with one terminal grounded. The other terminal, of the condenser type, furnishes the capacity necessary for the operation of the arrangement. Variations of the frequency from the normal value are indicated and measured by the frequency meter at the top of the panel.

Referring to the diagram, Fig. 441, it will be seen that the full potential difference which is applied to the specimen is also impressed between the outer conducting layer of the condenser terminal C, *i.e.*, its outside flange, and the conductor H. For

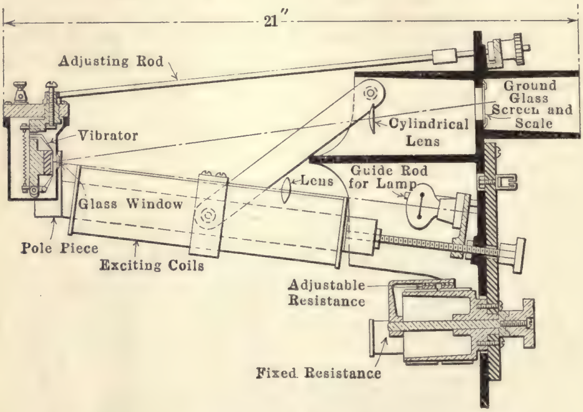


Fig. 442.—Section of Simplex Peak-voltmeter.

the first half wave the charging current flowing to the condenser C passes through V_1, and for the second half wave through V_2. The unidirectional current through V_1 is measured by the pivoted moving coil galvanometer, M, which is arranged for switchboard use.

Theoretically, the electrical valves may introduce a small error, for if the voltage wave is greatly distorted the current flowing to the condenser may have negative values during a half-cycle. In order that the net value of the current flowing through the condenser may be measured, these negative values should be included in the current which flows through the galvanometer.

Another method of measuring peak voltages is by the use of an instrument based on the oscillograph principle, such as the Simplex Vibrating Voltmeter, a section of which is shown in Fig.

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